International Journal of Engineering & Applied Sciences (IJEAS)
Vol.8, Issue 2(Special Issue: Nanomechanics) (2016) 68-86

Thermo-Mechanical Vibration of Size Dependent Shear Deformable Functionally Graded Conical Nanoshell Resting On Elastic Foundation

Fahimeh Mehralian [a], Yaghoub Tadi Beni [b]*

[a] Mechanical Engineering Department, Shahrekord University, Shahrekord, Iran
[b] Faculty of Engineering, Shahrekord University, Shahrekord, Iran
[*] Corresponding author. Tel./fax: 98 38 32324438. E-mail address: tadi@eng.sku.ac.ir

Received date: July 2016
Accepted date: August 2016

Abstract

In this paper, the size-dependent shear deformable conical shell formulation is derived based on the modified couple stress theory and first order shear deformation model to investigate the free vibration of functionally graded conical shell embedded in an elastic Pasternak medium and subjected to thermal environment. The material properties are considered temperature-dependent and graded in thickness direction according to power law distribution. The governing equations and boundary conditions are derived using Hamilton's principle. The size effect is taken into account using the modified couple stress theory, and, the free vibration of simply supported FG truncated conical nanoshell is investigated as a special case. The effects of different parameters such as dimensionless length scale parameter, temperature change and distribution of the nanoshell components on the natural frequency are investigated based on the modified couple stress theory and classical continuum theory.

Keywords: Thermo-mechanical vibration; First order shear deformation theory; FG conical shell; Modified couple stress theory.

1. Introduction

Today, nanoscale carbon-based structures such as carbon nanotubes (CNTs), fullerenes and carbon nanocones (CNCs) have attracted the attention of many researchers thanks to their diverse potential applications. Early studies on CNCs have been concurrent with studies on CNTs [1-2]; however, relatively scant attention has been paid to CNCs and their properties. CNCs were initially observed by Ge and Sattler in 1994 [3] and afterwards, Krishnan demonstrated the presence of five apex angles of CNCs [4]. In 1999, Iijima et al. built a single-walled CNC [5]. And, various experiments and studies have to date been conducted to investigate CNC structure [6-7]. So far, carbon nanocones have attracted the attention of many researchers because of their unique characteristics such as localization of electrical field in their sharp point which gives them important applications in the future as high resolution probes in scanning, tunneling, atomic force microscopy and field emission devices and optical antennas [8-10], and because of their use in drug delivery applications like micro needles, albeit under open tips, mechanical rigidity and high aspect ratios [11-12]. Since it is difficult to conduct experimental tests in the nanoscale, the majority of research has so far been done using molecular dynamics (MD) simulation [13-14]. For example, using MD simulation, Wei et al. computed the Young's modulus of a CNC and compared it with that of a corresponding CNT [13]. Today, because MD simulation is a time-consuming procedure especially for macro scale structures, it has been supplanted by the use of continuum mechanics in the modeling of structures in the nanoscale. As classical continuum models are unable to make a correct prediction of size-dependent behaviors happening in micro/nanoscale structures, researchers have attempted to use non-classical continuum theories such as non-local elasticity theory, couple stress theory and strain gradient theory, which take size effect into account in the investigation of micro/nanostructures [15-17]. The couple stress theory, which contains two higher order material length scale parameters as well as two Lame constants, was introduced by Mindlin, Toupin, Mindlin and Tiersten, and Koiter in 1960's [18-21]. Some researchers, including Anthoine have investigated the pure bending of the circular cylinder using the couple stress theory [22]. Introducing a new equilibrium equation, equilibrium of moments of couple, in addition to the classical equilibrium equations of forces and moment of forces, Yang et al. introduced the modified couple stress theory which incorporates only one higher order material parameter [23]. Applying this theory, many studies have been conducted using beam, plate and cylindrical and conical shell models [24-26]. Considering the increasing progress in nano-science which demands studying and making correct prediction of the behavior of various nano-structures, it is crucial to use theories such as the first order shear deformation theory which incorporates the effect of shear strains to make a correct prediction of the behavior of CNCs, particularly for short and rather thick CNCs. Given the stress variation in the thickness of conical shell where using FSDT the shear strains and consequently shear stresses are assumed to be constant in the thickness of the cone, in order to more accurately predict the behavior of CNCs, a correction coefficient, usually considered to be 5/6, is used [27]. Applying FSDT, many studies have to date been conducted on nanostructures using various models. For instance, using the modified couple stress theory and shear cylindrical and conical shell model, Zeighampour et al. studied the effect of parameters such as length and size effect on the natural frequency of CNTs and CNCs and demonstrated the increase in natural frequency with the increase in the small scale parameter [28-29]. Today, great attention is paid to the investigation of micro/nanostructure elements made of FG materials used in microelectronic and micromechanical structures such as shape memory alloys as thin films and, micro-and nano-electromechanical systems (MEMS and NEMS) and atomic force microscopes (AFMS) [30-33]. Functionally graded materials with their unique properties which prevent the concentration of stress, which is the primary cause of breaking in composites due to sudden inconsistency in material properties have attracted researchers' attention. To date, many studies have been conducted on FGMs [34-35]. On the other hand, since the conical shell structures have wide spread applications in nano-science, due to their unique dynamic behavior, strength and stability, the study of vibration, bending and buckling behavior is of practical interest for understanding their mechanical behavior appropriately. Moreover, the study of thermal effect on mechanical behavior is of great importance which only a limited portion of literature considered this [36-38]. Considering the above discussion, in this paper, the thermo-mechanical vibration of conical shell resting on Pasternak elastic medium are investigated by considering the first order shear deformation shell theory and the modified couple stress theory. Using Hamilton's principle, equations of motion as well as classical and non-classical boundary conditions are obtained. Besides, as a special case, the free vibration of the simply supported FG conical nanoshell is investigated using the Galerkin method. Based on the power law distribution, material properties variation of FG CNC are considered according to constituent's volume fraction along the thickness direction and temperature-dependent. Finally, the effect of some parameters on natural frequency is examined

2. Preliminaries

2.1. Modified couple stress theory

The modified couple stress theory as one of the higher order continuum theories, having one higher order material length scale parameter as well as two Lame constants, was initially developed by Yang et al. [23]. According to this theory, strain energy is expressed as:

$$U = \frac{1}{2}\int_{\Omega}\left(\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij}\right) dV \tag{1}$$

where

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i} + u_{3,i}u_{3,j}\right) \tag{2}$$

$$\chi_{ij} = \frac{1}{4}\left(e_{ipq}\eta_{jpq} + e_{jpq}\eta_{ipq}\right) \tag{3}$$

$$\sigma_{ij} = \lambda(\hat{z},T)tr(\boldsymbol{\varepsilon})\delta_{ij} + 2\mu(\hat{z},T)\varepsilon_{ij} - \left(3\lambda(\hat{z},T) + 2\mu(\hat{z},T)\right)\alpha_i(\hat{z},T)\Delta T \tag{4}$$

$$m_{ij} = 2l\mu(\hat{z},T)\chi_{ij} \tag{5}$$

In the above equations, ε_{ij}, σ_{ij}, χ_{ij}, and m_{ij} represent the components of strain tensor, Cauchy stress tensor, symmetric rotation gradient tensor and higher order stress tensor, respectively; and u_i, e_{ipq}, and η_{jpq} stand for the components of displacement vector, permutation symbol, and deviatoric stretch gradient tensor. And, in Eq. (5), l, material length scale parameter, is independent and extra.

2.2. Functionally graded material

A FG truncated conical shell at length L and thickness h resting on Pasternak elastic medium is considered according to Fig. 1. According to a simple power law distribution, volume fraction variation of metal and ceramics along the thickness of conical shell is expressed as:

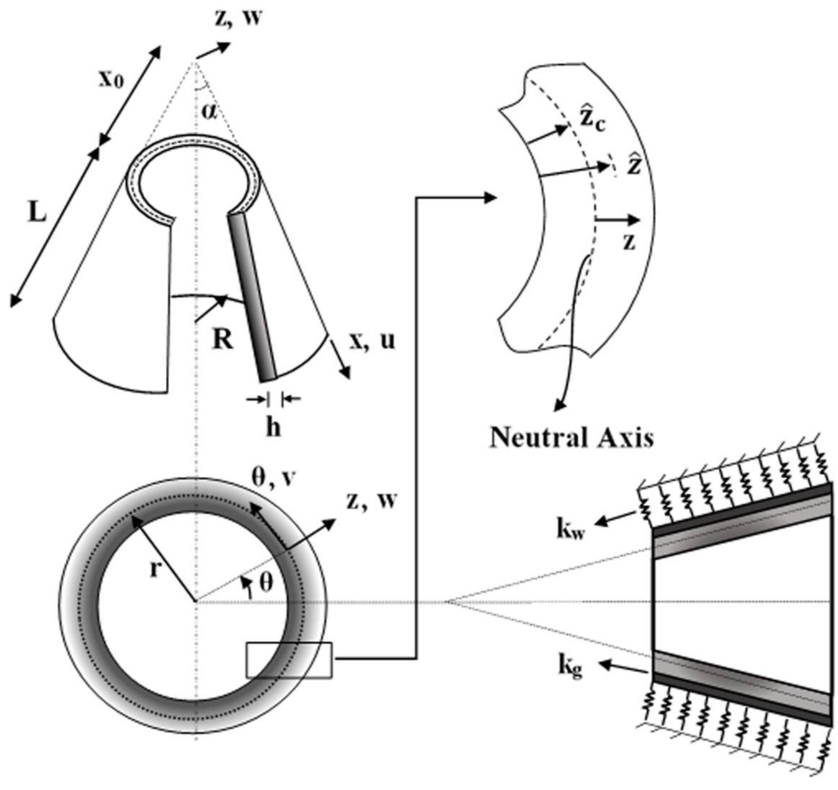

Figure 1: Schematic view of FG truncated conical shell embedded in elastic medium.

$$V_m = \left(\frac{\hat{z}}{h}\right)^{\beta}$$
$$V_c = 1 - V_m \tag{6}$$

Therefore, the material properties of this conical shell can be expressed as:

$$E(\hat{z},T) = \left(E_m(T) - E_c(T)\right)\left(\frac{\hat{z}}{h}\right)^{\beta} + E_c(T)$$
$$\rho(\hat{z},T) = \left(\rho_m(T) - \rho_c(T)\right)\left(\frac{\hat{z}}{h}\right)^{\beta} + \rho_c(T) \tag{7}$$
$$\nu(\hat{z},T) = \left(\nu_m(T) - \nu_c(T)\right)\left(\frac{\hat{z}}{h}\right)^{\beta} + \nu_c(T)$$

$$\alpha(\hat{z},T)=\left(\alpha_m(T)-\alpha_c(T)\right)\left(\frac{\hat{z}}{h}\right)^{\beta}+\alpha_c(T)$$

The material properties are considered temperature-dependent through Touloukian formula [39] as below:

$$P(T)=P_0\left(P_{-1}T^{-1}+1+P_1T+P_2T^2+P_3T^3\right) \tag{8}$$

where $T = T_0 + \Delta T$, T_0 = 300K (room temperature) and ΔT is the temperature change which is assumed to be uniform [40] and the coefficients P_i (i = 0, -1, 1, 2, 3) are unique to the component materials.

2.3. Displacement field in the conical shell

Based on the first order shear deformation theory, the three components of displacement field, u, v, and w are assumed along three x-, θ- and z-axes as follows:

$$\begin{aligned} u(x,\theta,z,t)&=U(x,\theta,t)+z\psi_x(x,\theta,t)\\ v(x,\theta,z,t)&=V(x,\theta,t)+z\psi_\theta(x,\theta,t)\\ w(x,\theta,z,t)&=W(x,\theta,t)\end{aligned} \tag{9}$$

In the above equations, $U(x,\theta,t)$, $V(x,\theta,t)$ and $W(x,\theta,t)$ are the displacement of the neutral surface in the three x, θ and z directions, and $\psi_x(x,\theta,t)$ and $\psi_\theta(x,\theta,t)$ are the rotation of a transverse normal about the axial and circumferential directions. Besides, the position of the neutral surface is determined as in Ref. [34].

3. Governing equations and boundary conditions

In order to derive the equations of motion and classical and non-classical boundary conditions of the first order shear deformable truncated conical shell using the modified couple stress theory, first, the components of classical and non-classical strains are determined by using the displacement field (Eq. (9)) and utilizing Ref. [29] as below:

$$\begin{aligned}
\varepsilon_{xx}&=\frac{\partial U}{\partial x}+\frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2+z\frac{\partial\psi_x}{\partial x}\\
\varepsilon_{\theta\theta}&=\frac{1}{x\sin\alpha}\left(\frac{\partial V}{\partial\theta}+\frac{1}{2x\sin\alpha}\left(\frac{\partial W}{\partial\theta}\right)^2+\sin\alpha\left(U+z\psi_x\right)+W\cos\alpha+z\frac{\partial\psi_\theta}{\partial\theta}\right)\\
\varepsilon_{x\theta}=\varepsilon_{\theta x}&=\frac{1}{2x\sin\alpha}\left(\frac{\partial U}{\partial\theta}+\frac{\partial W}{\partial x}\frac{\partial W}{\partial\theta}+x\sin\alpha\left(\frac{\partial V}{\partial x}+z\frac{\partial\psi_\theta}{\partial x}\right)-\sin\alpha\left(V+z\psi_\theta\right)+z\frac{\partial\psi_x}{\partial\theta}\right)\\
\varepsilon_{zx}=\varepsilon_{xz}&=\frac{1}{2}\left(\psi_x+\frac{\partial W}{\partial x}\right)\\
\varepsilon_{z\theta}=\varepsilon_{\theta z}&=\frac{1}{2x\sin\alpha}\left(\frac{\partial W}{\partial\theta}-V\cos\alpha+\psi_\theta x\sin\alpha\right)
\end{aligned} \tag{10}$$

and

$$\begin{aligned}
\chi_{xx}&=\frac{1}{2x\sin\alpha}\left(\cos\alpha\left(\frac{1}{x}\left(V+z\psi_\theta\right)-\frac{\partial V}{\partial x}\right)+\frac{\partial^2W}{\partial x\partial\theta}-\frac{1}{x}\frac{\partial W}{\partial\theta}-x\sin\alpha\frac{\partial\psi_\theta}{\partial x}\right)\\
\chi_{\theta\theta}&=\frac{1}{2x\sin\alpha}\left(\cos\alpha\frac{\partial V}{\partial x}-\frac{1}{x\tan\alpha}\frac{\partial U}{\partial\theta}-\frac{\partial^2W}{\partial x\partial\theta}+\frac{1}{x}\frac{\partial W}{\partial\theta}+\frac{\partial\psi_x}{\partial\theta}-\psi_\theta\sin\alpha+z\cos\alpha\frac{\partial\psi_\theta}{\partial x}\right)\\
\chi_{zz}&=\frac{1}{2}\left(\frac{\cos\alpha}{x^2\sin^2\alpha}\frac{\partial U}{\partial\theta}-\frac{V}{x^2\tan\alpha}+\frac{\partial\psi_\theta}{\partial x}-\frac{1}{x\sin\alpha}\frac{\partial\psi_x}{\partial\theta}+\frac{\psi_\theta}{x}\right)\\
\chi_{x\theta}=\chi_{\theta x}&=\frac{1}{4}\left(\frac{1}{x^2\sin^2\alpha}\frac{\partial^2W}{\partial\theta^2}-\frac{\partial^2W}{\partial x^2}+\frac{1}{x}\frac{\partial W}{\partial x}-\frac{\cos\alpha}{x^2\sin^2\alpha}\frac{\partial V}{\partial\theta}+\frac{\partial\psi_x}{\partial x}-\frac{1}{x\sin\alpha}\frac{\partial\psi_\theta}{\partial\theta}-\frac{\psi_x}{x}\right)\\
\chi_{z\theta}=\chi_{\theta z}&=\frac{1}{4x\sin\alpha}\left(\frac{\partial^2V}{\partial x\partial\theta}+\frac{1}{x}\frac{\partial V}{\partial\theta}-\frac{1}{x\sin\alpha}\frac{\partial^2U}{\partial\theta^2}+\cos\alpha\frac{\partial W}{\partial x}-\psi_x\cos\alpha+z\frac{\partial^2\psi_\theta}{\partial x\partial\theta}-\frac{z}{x\sin\alpha}\frac{\partial^2\psi_x}{\partial\theta^2}+\frac{z}{x}\frac{\partial\psi_\theta}{\partial\theta}\right)
\end{aligned} \tag{11}$$

$$\chi_{xz}=\chi_{zx}=\frac{1}{4x\sin\alpha}\left(\sin\alpha\frac{\partial V}{\partial x}+x\sin\alpha\frac{\partial^2 V}{\partial x^2}+\frac{V\cos 2\alpha}{x\sin\alpha}-\frac{\partial^2 U}{\partial x\partial\theta}+\frac{1}{x}\frac{\partial U}{\partial\theta}-\frac{\cos\alpha}{x\sin\alpha}\frac{\partial W}{\partial\theta}-z\frac{\partial^2\psi_x}{\partial x\partial\theta}+\frac{z}{x}\frac{\partial\psi_x}{\partial\theta}-\psi_\theta\cos\alpha\right.$$
$$\left.+z\sin\alpha\frac{\partial\psi_\theta}{\partial x}+zx\sin\alpha\frac{\partial^2\psi_\theta}{\partial x^2}\right)$$

Afterwards, classical and non-classical stresses are determined by substituting Eqs. (10) and (11) into the constitutive equations, and strain energy is determined by substituting classical and non-classical strains and stresses into Eq. (1) as follows:

$$\begin{aligned}
U_s=\frac{1}{2}\int_0^{2\pi}\int_{x_0}^{x_0+L}\Bigg[&\left(N_{xx}\right)\frac{\partial U}{\partial x}+\left(\frac{N_{x\theta}}{x\sin\alpha}-\frac{Y_{\theta\theta}\cos\alpha}{2x^2\sin^2\alpha}+\frac{Y_{zz}\cos\alpha}{2x^2\sin^2\alpha}+\frac{Y_{xz}}{2x^2\sin\alpha}\right)\frac{\partial U}{\partial\theta}-\left(\frac{Y_{z\theta}}{2x^2\sin^2\alpha}\right)\frac{\partial^2 U}{\partial\theta^2}-\left(\frac{Y_{zx}}{2x\sin\alpha}\right)\frac{\partial^2 U}{\partial x\partial\theta}\\
&+\left(\frac{N_{\theta\theta}}{x}\right)U+\left(\frac{Y_{z\theta}}{2x\sin\alpha}\right)\frac{\partial^2 V}{\partial x\partial\theta}+\left(N_{x\theta}-\frac{Y_{xx}}{2x\tan\alpha}+\frac{Y_{\theta\theta}}{2x\tan\alpha}\right)\frac{\partial V}{\partial x}+\left(\frac{N_{\theta\theta}}{x\sin\alpha}-\frac{Y_{x\theta}\cos\alpha}{2x^2\sin^2\alpha}+\frac{Y_{z\theta}}{2x^2\sin\alpha}\right)\frac{\partial V}{\partial\theta}\\
&+\left(-\frac{N_{x\theta}}{x}-\frac{Q_{z\theta}}{x\tan\alpha}+\frac{Y_{xx}}{2x^2\tan\alpha}-\frac{Y_{zz}}{2x^2\tan\alpha}+\frac{Y_{zx}\cos 2\alpha}{2x^2\sin^2\alpha}\right)V+\left(\frac{Y_{zx}}{2}\right)\frac{\partial^2 V}{\partial x^2}+\left(Q_{xz}+\frac{Y_{x\theta}}{2x}+\frac{Y_{z\theta}}{2x\tan\alpha}\right)\frac{\partial W}{\partial x}-\left(\frac{Y_{x\theta}}{2}\right)\frac{\partial^2 W}{\partial x^2}\\
&+\left(\frac{N_{\theta\theta}}{x\tan\alpha}\right)W+\frac{N_{xx}}{2}\left(\frac{\partial W}{\partial x}\right)^2+\left(\frac{Q_{z\theta}}{x\sin\alpha}-\frac{Y_{xx}}{2x^2\sin\alpha}+\frac{Y_{\theta\theta}}{2x^2\sin\alpha}-\frac{Y_{zx}\cos\alpha}{2x^2\sin^2\alpha}\right)\frac{\partial W}{\partial\theta}+\left(\frac{Y_{xx}}{2x\sin\alpha}-\frac{Y_{\theta\theta}}{2x\sin\alpha}\right)\frac{\partial^2 W}{\partial x\partial\theta}\\
&+\left(\frac{Y_{x\theta}}{2x^2\sin^2\alpha}\right)\frac{\partial^2 W}{\partial\theta^2}+\frac{N_{\theta\theta}}{2x^2\sin^2\alpha}\left(\frac{\partial W}{\partial\theta}\right)^2+\frac{N_{x\theta}}{x\sin\alpha}\left(\frac{\partial W}{\partial x}\frac{\partial W}{\partial\theta}\right)+\left(\frac{M_{x\theta}}{x\sin\alpha}+\frac{Y_{\theta\theta}}{2x\sin\alpha}-\frac{Y_{zz}}{2x\sin\alpha}+\frac{T_{xz}}{2x^2\sin\alpha}\right)\frac{\partial\psi_x}{\partial\theta}\\
&+\left(M_{xx}+\frac{Y_{x\theta}}{2}\right)\frac{\partial\psi_x}{\partial x}-\left(\frac{T_{zx}}{2x\sin\alpha}\right)\frac{\partial^2\psi_x}{\partial x\partial\theta}+\left(Q_{xz}+\frac{M_{\theta\theta}}{x}-\frac{Y_{z\theta}}{2x\tan\alpha}-\frac{Y_{x\theta}}{2x}\right)\psi_x-\left(\frac{T_{z\theta}}{2x^2\sin^2\alpha}\right)\frac{\partial^2\psi_x}{\partial\theta^2}\\
&+\left(\frac{M_{\theta\theta}}{x\sin\alpha}-\frac{Y_{x\theta}}{2x\sin\alpha}+\frac{T_{z\theta}}{2x^2\sin\alpha}\right)\frac{\partial\psi_\theta}{\partial\theta}+\left(\frac{T_{xz}}{2}\right)\frac{\partial^2\psi_\theta}{\partial x^2}+\left(M_{x\theta}-\frac{Y_{xx}}{2}+\frac{T_{\theta\theta}}{2x\tan\alpha}+\frac{Y_{zz}}{2}+\frac{T_{xz}}{2x}\right)\frac{\partial\psi_\theta}{\partial x}+\left(\frac{T_{z\theta}}{2x\sin\alpha}\right)\frac{\partial^2\psi_\theta}{\partial x\partial\theta}\\
&+\left(Q_{\theta z}-\frac{M_{x\theta}}{x}-\frac{Y_{\theta\theta}}{2x}+\frac{T_{xx}}{2x^2\tan\alpha}+\frac{Y_{zz}}{2x}-\frac{Y_{xz}}{2x\tan\alpha}\right)\psi_\theta\Bigg]x\sin\alpha\,dx\,d\theta
\end{aligned}\tag{12}$$

where the classical and non-classical forces and moments are defined as:

$$\begin{aligned}
\left(N_{xx},N_{\theta\theta},N_{x\theta}\right)&=\int_{-\hat{z}_c}^{h-\hat{z}_c}\left(\sigma_{xx},\sigma_{\theta\theta},\sigma_{x\theta}\right)dz\\
\left(M_{xx},M_{\theta\theta},M_{x\theta}\right)&=\int_{-\hat{z}_c}^{h-\hat{z}_c}\left(\sigma_{xx},\sigma_{\theta\theta},\sigma_{x\theta}\right)z\,dz\\
\left(Q_{xz},Q_{z\theta}\right)&=\int_{-\hat{z}_c}^{h-\hat{z}_c}\left(\sigma_{xz},\sigma_{z\theta}\right)dz\\
\left(Y_{xx},Y_{\theta\theta},Y_{zz},Y_{x\theta},Y_{zx},Y_{z\theta}\right)&=\int_{-\hat{z}_c}^{h-\hat{z}_c}\left(m_{xx},m_{\theta\theta},m_{zz},m_{x\theta},m_{zx},m_{z\theta}\right)dz\\
\left(T_{xx},T_{\theta\theta},T_{zz},T_{x\theta},T_{zx},T_{z\theta}\right)&=\int_{-\hat{z}_c}^{h-\hat{z}_c}\left(m_{xx},m_{\theta\theta},m_{zz},m_{x\theta},m_{zx},m_{z\theta}\right)z\,dz
\end{aligned}\tag{13}$$

Kinetic energy of the conical shell is expressed based on Eq. (9) as:

$$T=\frac{1}{2}\int_\Omega\rho\left(\hat{z},T\right)\left[\left(\frac{\partial U}{\partial t}+z\frac{\partial\psi_x}{\partial t}\right)^2+\left(\frac{\partial V}{\partial t}+z\frac{\partial\psi_\theta}{\partial t}\right)^2+\left(\frac{\partial W}{\partial t}\right)^2\right]x\sin\alpha\,dx\,d\theta\,dz \tag{14}$$

The work done by Pasternak foundation on the conical shell is computed as:

$$W_e=\frac{1}{2}\int_{x_0}^{x_0+L}\int_0^{2\pi}\left(-k_w W+k_g\left(\left(\frac{\partial W}{\partial x}\right)^2+\frac{1}{x^2\sin^2\alpha}\left(\frac{\partial W}{\partial\theta}\right)^2\right)\right)x\sin\alpha\,d\theta\,dx \tag{15}$$

where k_w is the Winkler constant and k_g is the shear modulus of subgrade.

Now, in order to derive the governing equations and boundary conditions, Hamilton's principle is used as follows:

$$\delta\int_{t_1}^{t_2}\left(T-U_s+W_e\right)dt=0 \tag{16}$$

By substituting strain energy, kinetic energy and the work done by external loads according to Eqs. (12), (14) and (15) into Eq. (16) and integrating by parts, the equations of motion as well as classical and non-classical boundary conditions are finally derived using the modified couple stress theory as follows:

$$\begin{aligned}
&\delta U: A_1\frac{\partial^2 U}{\partial x^2}+A_2\frac{\partial U}{\partial x}+A_3U+A_4\frac{\partial^2 U}{\partial\theta^2}+A_5\frac{\partial^4 U}{\partial\theta^4}+A_6\frac{\partial^4 U}{\partial x^2\partial\theta^2}+A_7\frac{\partial^3 U}{\partial x\partial\theta^2}+A_8\frac{\partial V}{\partial\theta}+A_9\frac{\partial^2 V}{\partial x\partial\theta}+A_{10}\frac{\partial^3 V}{\partial x^2\partial\theta}+A_{11}\frac{\partial^4 V}{\partial x\partial\theta^3}\\
&+A_{12}\frac{\partial^4 V}{\partial x^3\partial\theta}+A_{13}\frac{\partial^3 V}{\partial\theta^3}+A_{14}W+A_{15}\frac{\partial W}{\partial x}+A_{16}\frac{\partial^3 W}{\partial x\partial\theta^2}+A_{17}\frac{\partial^2 W}{\partial\theta^2}+A_{18}\frac{\partial^2\psi_x}{\partial x^2}+A_{19}\frac{\partial\psi_x}{\partial x}+A_{20}\psi_x+A_{21}\frac{\partial^2\psi_x}{\partial\theta^2}+A_{22}\frac{\partial^4\psi_x}{\partial\theta^4}\\
&+A_{23}\frac{\partial^3\psi_x}{\partial x\partial\theta^2}+A_{24}\frac{\partial^4\psi_x}{\partial x^2\partial\theta^2}+A_{25}\frac{\partial\psi_\theta}{\partial\theta}+A_{26}\frac{\partial^3\psi_\theta}{\partial\theta^3}+A_{27}\frac{\partial^2\psi_\theta}{\partial x\partial\theta}+A_{28}\frac{\partial^3\psi_\theta}{\partial x^2\partial\theta}+A_{29}\frac{\partial^4\psi_\theta}{\partial x\partial\theta^3}+A_{30}\frac{\partial^4\psi_\theta}{\partial x^3\partial\theta}\\
&+I_{1,1}\frac{\partial^2\psi_x}{\partial t^2}+I_{1,0}\frac{\partial^2 U}{\partial t^2}=0
\end{aligned} \tag{17}$$

$$\begin{aligned}
&\delta V: B_1\frac{\partial^4 V}{\partial x^4}+B_2\frac{\partial^3 V}{\partial x^3}+B_3\frac{\partial^2 V}{\partial x^2}+B_4\frac{\partial V}{\partial x}+B_5V+B_6\frac{\partial^2 V}{\partial\theta^2}+B_7\frac{\partial^4 V}{\partial x^2\partial\theta^2}+B_8\frac{\partial^3 V}{\partial x\partial\theta^2}+B_9\frac{\partial U}{\partial\theta}+B_{10}\frac{\partial^2 U}{\partial x\partial\theta}+B_{11}\frac{\partial^3 U}{\partial x^2\partial\theta}\\
&+B_{12}\frac{\partial^4 U}{\partial x\partial\theta^3}+B_{13}\frac{\partial^4 U}{\partial x^3\partial\theta}+B_{14}\frac{\partial^3 U}{\partial\theta^3}+B_{15}\frac{\partial W}{\partial\theta}+B_{16}\frac{\partial^2 W}{\partial x\partial\theta}+B_{17}\frac{\partial^3 W}{\partial x^2\partial\theta}+B_{18}\frac{\partial^3 W}{\partial\theta^3}+B_{19}\frac{\partial\psi_x}{\partial\theta}+B_{20}\frac{\partial^2\psi_x}{\partial x\partial\theta}+B_{21}\frac{\partial^3\psi_x}{\partial\theta^3}\\
&+B_{22}\frac{\partial^4\psi_x}{\partial x^3\partial\theta}+B_{23}\frac{\partial^3\psi_x}{\partial x^2\partial\theta}+B_{24}\frac{\partial^4\psi_x}{\partial x\partial\theta^3}+B_{25}\psi_\theta+B_{26}\frac{\partial\psi_\theta}{\partial x}+B_{27}\frac{\partial^2\psi_\theta}{\partial x^2}+B_{28}\frac{\partial^2\psi_\theta}{\partial\theta^2}+B_{29}\frac{\partial^4\psi_\theta}{\partial x^2\partial\theta^2}+B_{30}\frac{\partial^3\psi_\theta}{\partial x\partial\theta^2}+B_{31}\frac{\partial^4\psi_\theta}{\partial x^4}\\
&+B_{32}\frac{\partial^3\psi_\theta}{\partial x^3}+I_{1,1}\frac{\partial^2\psi_\theta}{\partial t^2}+I_{1,0}\frac{\partial^2 V}{\partial t^2}=0
\end{aligned} \tag{18}$$

$$\begin{aligned}
&\delta W: C_1\frac{\partial^4 W}{\partial x^4}+\left(C_2-N_{xx}\right)\frac{\partial^2 W}{\partial x^2}+\left(C_3-\frac{N_{xx}}{x}\right)\frac{\partial W}{\partial x}+C_4\frac{\partial^3 W}{\partial x^3}+C_5\frac{\partial^3 W}{\partial x\partial\theta^2}+\left(C_6-\frac{N_{\theta\theta}}{x^2\sin^2\alpha}\right)\frac{\partial^2 W}{\partial\theta^2}+C_7\frac{\partial^4 W}{\partial x^2\partial\theta^2}\\
&+C_8\frac{\partial^4 W}{\partial\theta^4}+C_9W-\frac{2N_{x\theta}}{x\sin\alpha}\frac{\partial^2 W}{\partial x\partial\theta}+k_wW-k_g\left(\frac{\partial^2 W}{\partial x^2}+\frac{1}{x}\frac{\partial W}{\partial x}+\frac{1}{x^2\sin^2\alpha}\frac{\partial^2 W}{\partial\theta^2}\right)+C_{10}U+C_{11}\frac{\partial U}{\partial x}+C_{12}\frac{\partial^3 U}{\partial x\partial\theta^2}\\
&+C_{13}\frac{\partial^2 U}{\partial\theta^2}+C_{14}\frac{\partial V}{\partial\theta}+C_{15}\frac{\partial^2 V}{\partial x\partial\theta}+C_{16}\frac{\partial^3 V}{\partial\theta^3}+C_{17}\frac{\partial^3 V}{\partial x^2\partial\theta}+C_{18}\frac{\partial^3\psi_x}{\partial x^3}+C_{19}\frac{\partial^2\psi_x}{\partial x^2}+C_{20}\frac{\partial\psi_x}{\partial x}+C_{21}\frac{\partial^2\psi_x}{\partial\theta^2}+C_{22}\frac{\partial^3\psi_x}{\partial x\partial\theta^2}\\
&+C_{23}\psi_x+C_{24}\frac{\partial^2\psi_\theta}{\partial x\partial\theta}+C_{25}\frac{\partial^3\psi_\theta}{\partial\theta^3}+C_{26}\frac{\partial^3\psi_\theta}{\partial x^2\partial\theta}+C_{27}\frac{\partial\psi_\theta}{\partial\theta}+\frac{N_{\theta\theta}^T}{x\tan\alpha}+I_{1,0}\frac{\partial^2 W}{\partial t^2}=0
\end{aligned} \tag{19}$$

$$\begin{aligned}
&\delta\psi_x: D_1\frac{\partial^2\psi_x}{\partial x^2}+D_2\frac{\partial\psi_x}{\partial x}+D_3\psi_x+D_4\frac{\partial^2\psi_x}{\partial\theta^2}+D_5\frac{\partial^3\psi_x}{\partial x\partial\theta^2}+D_6\frac{\partial^4\psi_x}{\partial\theta^4}+D_7\frac{\partial^4\psi_x}{\partial x^2\partial\theta^2}+D_8\frac{\partial\psi_\theta}{\partial\theta}+D_9\frac{\partial^2\psi_\theta}{\partial x\partial\theta}+D_{10}\frac{\partial^4\psi_\theta}{\partial x\partial\theta^3}\\
&+D_{11}\frac{\partial^3\psi_\theta}{\partial\theta^3}+D_{12}\frac{\partial^4\psi_\theta}{\partial x^3\partial\theta}+D_{13}\frac{\partial^3\psi_\theta}{\partial x^2\partial\theta}+D_{14}\frac{\partial^2 U}{\partial\theta^2}+D_{15}\frac{\partial^2 U}{\partial x^2}+D_{16}U+D_{17}\frac{\partial U}{\partial x}+D_{18}\frac{\partial^4 U}{\partial\theta^4}+D_{19}\frac{\partial^4 U}{\partial x^2\partial\theta^2}+D_{20}\frac{\partial^3 U}{\partial x\partial\theta^2}\\
&+D_{21}\frac{\partial V}{\partial\theta}+D_{22}\frac{\partial^2 V}{\partial x\partial\theta}+D_{23}\frac{\partial^4 V}{\partial x\partial\theta^3}+D_{24}\frac{\partial^4 V}{\partial x^3\partial\theta}+D_{25}\frac{\partial^3 V}{\partial\theta^3}+D_{26}\frac{\partial^3 V}{\partial x^2\partial\theta}+D_{27}\frac{\partial W}{\partial x}+D_{28}\frac{\partial^2 W}{\partial x^2}+D_{29}\frac{\partial^2 W}{\partial\theta^2}+D_{30}\frac{\partial^3 W}{\partial x^3}\\
&+D_{31}\frac{\partial^3 W}{\partial x\partial\theta^2}+D_{32}W+I_{1,1}\frac{\partial^2 U}{\partial t^2}+I_{1,2}\frac{\partial^2\psi_x}{\partial t^2}=0
\end{aligned} \tag{20}$$

$$\begin{aligned}
&\delta\psi_\theta: E_1\frac{\partial^4\psi_\theta}{\partial x^4}+E_2\frac{\partial^3\psi_\theta}{\partial x^3}+E_3\frac{\partial^4\psi_\theta}{\partial x^2\partial\theta^2}+E_4\frac{\partial^3\psi_\theta}{\partial x\partial\theta^2}+E_5\frac{\partial^2\psi_\theta}{\partial x^2}+E_6\frac{\partial\psi_\theta}{\partial x}+E_7\frac{\partial^2\psi_\theta}{\partial\theta^2}+E_8\psi_\theta+E_9\frac{\partial^4\psi_x}{\partial x^3\partial\theta}+E_{10}\frac{\partial^4\psi_x}{\partial x\partial\theta^3}\\
&+E_{11}\frac{\partial^3\psi_x}{\partial\theta^3}+E_{12}\frac{\partial^3\psi_x}{\partial x^2\partial\theta}+E_{13}\frac{\partial^2\psi_x}{\partial x\partial\theta}+E_{14}\frac{\partial\psi_x}{\partial\theta}+E_{15}\frac{\partial U}{\partial\theta}+E_{16}\frac{\partial^2 U}{\partial x\partial\theta}+E_{17}\frac{\partial^3 U}{\partial x^2\partial\theta}+E_{18}\frac{\partial^4 U}{\partial x^3\partial\theta}+E_{19}\frac{\partial^4 U}{\partial x\partial\theta^3}+E_{20}\frac{\partial^3 U}{\partial\theta^3}\\
&+E_{21}V+E_{22}\frac{\partial V}{\partial x}+E_{23}\frac{\partial^2 V}{\partial x^2}+E_{24}\frac{\partial^2 V}{\partial\theta^2}+E_{25}\frac{\partial^4 V}{\partial x^2\partial\theta^2}+E_{26}\frac{\partial^3 V}{\partial x\partial\theta^2}+E_{27}\frac{\partial^4 V}{\partial x^4}+E_{28}\frac{\partial^3 V}{\partial x^3}+E_{29}\frac{\partial W}{\partial\theta}+E_{30}\frac{\partial^2 W}{\partial x\partial\theta}\\
&+E_{31}\frac{\partial^3 W}{\partial x^2\partial\theta}+E_{32}\frac{\partial^3 W}{\partial\theta^3}+I_{1,1}\frac{\partial^2 V}{\partial t^2}+I_{1,2}\frac{\partial^2\psi_\theta}{\partial t^2}=0
\end{aligned} \tag{21}$$

where in Eq. (19), on the basis of thermal elasticity theory, the thermal force can be written as:

$$N_{xx}=N_{xx}^T=-\int_{-\hat{z}_c}^{h-\hat{z}_c}\frac{E(\hat{z},T)\alpha(\hat{z},T)\Delta T}{\left(1-2\nu(\hat{z},T)\right)}dz,\ N_{\theta\theta}=N_{\theta\theta}^T=-\int_{-\hat{z}_c}^{h-\hat{z}_c}\frac{E(\hat{z},T)\alpha(\hat{z},T)\Delta T}{\left(1-2\nu(\hat{z},T)\right)}dz,\ N_{x\theta}=0 \tag{22}$$

Coefficients A_1-A_{30}, B_1-B_{32}, C_1-C_{27}, D_1-D_{32}, and E_1-E_{32} are included in Appendix A.

4. Shear deformable simply supported FG conical shell

4.1. Governing equations of simply-supported FG conical shell

In order to investigate the free vibration of the simply supported FG conical shell, first one must determine equations of motion and boundary conditions. Due to the θ variation between zero and 2π, the equations are only relating to x = constant must exist.

$$V\big|_{x=x_0,x_0+L}=0 \tag{23}$$

$$W\big|_{x=x_0,x_0+L}=0 \tag{24}$$

$$\left(a_1\frac{\partial U}{\partial x}+a_2\frac{\partial^2 U}{\partial\theta^2}+a_3U+a_4\frac{\partial^3 U}{\partial x\partial\theta^2}+a_5\frac{\partial V}{\partial\theta}+a_6\frac{\partial^2 V}{\partial x\partial\theta}+a_7\frac{\partial^3 V}{\partial x^2\partial\theta}+a_8W+a_9\frac{\partial^2 W}{\partial\theta^2}+a_{10}\frac{\partial\psi_x}{\partial x}+a_{11}\psi_x+a_{12}\frac{\partial^2\psi_x}{\partial\theta^2}\right.$$
$$\left.+a_{13}\frac{\partial^3\psi_x}{\partial x\partial\theta^2}+a_{14}\frac{\partial\psi_\theta}{\partial\theta}+a_{15}\frac{\partial^2\psi_\theta}{\partial x\partial\theta}+a_{16}\frac{\partial^3\psi_\theta}{\partial x^2\partial\theta}\right)\Bigg|_{x=x_0,x_0+L}=0 \tag{25}$$

$$\left(b_1\frac{\partial U}{\partial\theta}+b_2\frac{\partial^2 U}{\partial x\partial\theta}+b_3\frac{\partial^2 V}{\partial x^2}+b_4\frac{\partial V}{\partial x}+b_5V+b_6\frac{\partial W}{\partial\theta}+b_7\frac{\partial\psi_x}{\partial\theta}+b_8\frac{\partial^2\psi_x}{\partial x\partial\theta}+b_9\psi_\theta+b_{10}\frac{\partial\psi_\theta}{\partial x}+b_{11}\frac{\partial^2\psi_\theta}{\partial x^2}\right)\Bigg|_{x=x_0,x_0+L}=0 \tag{26}$$

$$\left(c_1\frac{\partial V}{\partial\theta}+c_2\frac{\partial^2 W}{\partial x^2}+c_3\frac{\partial W}{\partial x}+c_4\frac{\partial^2 W}{\partial\theta^2}+c_5\frac{\partial\psi_x}{\partial x}+c_6\psi_x+c_7\frac{\partial\psi_\theta}{\partial\theta}\right)\Bigg|_{x=x_0,x_0+L}=0 \tag{27}$$

$$\left(d_1\frac{\partial U}{\partial x}+d_2U+d_3\frac{\partial^2 U}{\partial\theta^2}+d_4\frac{\partial^3 U}{\partial x\partial\theta^2}+d_5\frac{\partial V}{\partial\theta}+d_6\frac{\partial^2 V}{\partial x\partial\theta}+d_7\frac{\partial^3 V}{\partial x^2\partial\theta}+d_8\frac{\partial^2 W}{\partial x^2}+d_9\frac{\partial W}{\partial x}+d_{10}W+d_{11}\frac{\partial^2 W}{\partial\theta^2}+d_{12}\frac{\partial\psi_x}{\partial x}\right.$$
$$\left.+d_{13}\psi_x+d_{14}\frac{\partial^3\psi_x}{\partial x\partial\theta^2}+d_{15}\frac{\partial^2\psi_x}{\partial\theta^2}+d_{16}\frac{\partial\psi_\theta}{\partial\theta}+d_{17}\frac{\partial^3\psi_\theta}{\partial x^2\partial\theta}+d_{18}\frac{\partial^2\psi_\theta}{\partial x\partial\theta}\right)\Bigg|_{x=x_0,x_0+L}=0 \tag{28}$$

$$\psi_\theta\big|_{x=x_0,x_0+L}=0 \tag{29}$$

$$\left(e_1\frac{\partial U}{\partial\theta}+e_2\frac{\partial^2 U}{\partial x\partial\theta}+e_3\frac{\partial V}{\partial x}+e_4\frac{\partial^2 V}{\partial x^2}+e_5V+e_6\frac{\partial W}{\partial\theta}+e_7\frac{\partial^2\psi_x}{\partial x\partial\theta}+e_8\frac{\partial\psi_x}{\partial\theta}+e_9\frac{\partial^2\psi_\theta}{\partial x^2}+e_{10}\frac{\partial\psi_\theta}{\partial x}+e_{11}\psi_\theta\right)\Bigg|_{x=x_0,x_0+L}=0 \tag{30}$$

Coefficients a_1-a_{16}, b_1-b_{11}, c_1-c_7, d_1-d_{18}, and e_1-e_{11} are included in Appendix A.

4.2. Solution method

In order to investigate the free vibration of the simply supported FG truncated conical shell, considering the governing equations and boundary conditions, the displacement field is considered as [41].

$$\begin{aligned}
U(x,\theta,t)&=U_0\cos\left(\frac{m\pi(x-x_0)}{L}\right)\cos(n\theta)\sin(\omega t)\\
V(x,\theta,t)&=U_0\sin\left(\frac{m\pi(x-x_0)}{L}\right)\sin(n\theta)\sin(\omega t)\\
W(x,\theta,t)&=W_0\sin\left(\frac{m\pi(x-x_0)}{L}\right)\cos(n\theta)\sin(\omega t)\\
\psi_x(x,\theta,t)&=\psi_{x0}\cos\left(\frac{m\pi(x-x_0)}{L}\right)\cos(n\theta)\sin(\omega t)\\
\psi_\theta(x,\theta,t)&=\psi_{\theta 0}\sin\left(\frac{m\pi(x-x_0)}{L}\right)\sin(n\theta)\sin(\omega t)
\end{aligned} \tag{31}$$

where ω, m and n represent the natural frequency of the nanotube, and circumferential and axial wave numbers, respectively. Considering the above displacements, most boundary conditions in Eqs. (23)-(30) are satisfied and only some of them are not fully satisfied. For a complicated formulation in the references like the above formulation, not all boundary conditions are usually satisfied. Therefore, by substituting Eq. (31) into Eqs. (17)-(21) and multiplying the resulting equations by x^4

in order to do simpler integration and use the Galerkin method, the following equations will finally be obtained:

$$\begin{aligned}
&\int_{x_0}^{x_0+L}\int_0^{2\pi}\psi_1 x\cos\left(\frac{m\pi(x-x_0)}{L}\right)\sin\alpha\, dx d\theta=0\\
&\int_{x_0}^{x_0+L}\int_0^{2\pi}\psi_2 x\sin\left(\frac{m\pi(x-x_0)}{L}\right)\sin\alpha\, dx d\theta=0\\
&\int_{x_0}^{x_0+L}\int_0^{2\pi}\psi_3 x\sin\left(\frac{m\pi(x-x_0)}{L}\right)\sin\alpha\, dx d\theta=0\\
&\int_{x_0}^{x_0+L}\int_0^{2\pi}\psi_4 x\cos\left(\frac{m\pi(x-x_0)}{L}\right)\sin\alpha\, dx d\theta=0\\
&\int_{x_0}^{x_0+L}\int_0^{2\pi}\psi_5 x\sin\left(\frac{m\pi(x-x_0)}{L}\right)\sin\alpha\, dx d\theta=0
\end{aligned} \tag{32}$$

where parameters ψ_1, ψ_2, ψ_3, ψ_4, and ψ_5 are considered as follows:

$$\begin{aligned}
\psi_1&=x^4\left[A_{11}(U_0)+A_{12}(V_0)+A_{13}(W_0)+A_{14}(\psi_{x0})+A_{15}(\psi_{\theta 0})\right]\\
\psi_2&=x^4\left[A_{21}(U_0)+A_{22}(V_0)+A_{23}(W_0)+A_{24}(\psi_{x0})+A_{25}(\psi_{\theta 0})\right]\\
\psi_3&=x^4\left[A_{31}(U_0)+A_{32}(V_0)+A_{33}(W_0)+A_{34}(\psi_{x0})+A_{35}(\psi_{\theta 0})\right]\\
\psi_4&=x^4\left[A_{41}(U_0)+A_{42}(V_0)+A_{43}(W_0)+A_{44}(\psi_{x0})+A_{45}(\psi_{\theta 0})\right]\\
\psi_5&=x^4\left[A_{51}(U_0)+A_{52}(V_0)+A_{53}(W_0)+A_{54}(\psi_{x0})+A_{55}(\psi_{\theta 0})\right]
\end{aligned} \tag{33}$$

In the above equation, A_{ij} are values obtained by substituting Eq. (31) into Eqs. (17)-(21). Therefore, the matrix form of Eq. (32) is expressed as follows:

$$\left([K]-\omega^2[M]\right)\begin{Bmatrix}U_0\\V_0\\W_0\\\psi_{x0}\\\psi_{\theta 0}\end{Bmatrix}=0 \tag{34}$$

According to the eigenvalue problem, in order to obtain a non–trivial solution for Eq. (34), the determinant of coefficients must be set to zero, and, by solving the obtained equation, one can compute nanoshell frequency.

4. Results and discussion

In this section, using the obtained shear deformable FG conical shell formulation, the free vibration of simply supported FG conical nanoshell embedded in Pasternak foundation is investigated based on the modified couple stress theory and under thermal environment. The material properties of FG conical shell with dependent temperature in Eq. (8) are listed in Table 1. and are assumed a blend of Si_3N_4 (Ceramic), and SUS304 (metal) [42]. Moreover, the coefficients of thermal expansion are negative at low temperature and are positive in high temperature [37-38] and the temperature change at high temperature are assumed to be ΔT = 50 (K) [36]. The dimensionless natural frequency is computed based on the $\Omega=\omega R\sqrt{\rho_m(1-\nu_m{}^2)/E_m}$ equation.

Table 1: Tempreature-dependent coefficients for Si_3N_4 and SUS304

Material	Property	P_{-1}	P_0	P_1	P_2	P_3
SUS304	α_m (1/K)	0	12.33e-6	8.086e-4	0	0
	E_m (Pa)	0	201.04e9	3.079e-4	-6.534e-7	0
	ν_m	0	0.3262	-2.002e-4	3.797e-7	0
	ρ_m (kg/m^3)	0	8166	0	0	0
Si_3N_4	α_c (1/K)	0	5.8723e-6	9.095e-4	0	0
	E_c (Pa)	0	348.43e9	-3.07e-4	2.16e-7	-8.946e-11
	ν_c	0	0.24	0	0	0
	ρ_c (kg/m^3)	0	2370	0	0	0

5.1. Influence of dimensionless length scale parameter

The effects of dimensionless length scale parameter on dimensionless natural frequency based on both modified couple stress theory and classical continuum theory for three apex angles at high temperature are shown in Fig. (2). As illustrated, according to both modified couple stress theory and classical continuum theory with the increase in apex angle, the dimensionless natural frequency decreases. Besides, according to the modified couple stress theory, decrease in dimensionless length scale parameter h/l, which is in fact equivalent to increase in length scale parameter, leads to greater stiffness in the nanoshell, and, finally, results in increased natural frequency in the entire gradient index (β). Increase in gradient index and decrease in apex angle intensify the effect of this parameter on natural frequency; in contrast, based on the classical continuum theory, length scale parameter's variation has no effect on natural frequency. In addition, as ceramic has a higher modulus than metal, an increase in gradient index, where $\beta = 0$ is the metal shell and $\beta = \infty$ is the ceramic shell, leads to an increase in natural frequency in all apex angles.

5.2. Influence of dimensionless CNC length parameter

Fig. (3) demonstrates the effects of dimensionless length parameters on the dimensionless natural frequency for three apex angles in different gradient index in high temperature. In this figure, $l = 5h$ is considered, and, as illustrated, with the increase in dimensionless length parameter which means increased length parameter, due to increased nanoshell instability and hence increased deformation, the natural frequency is decreased, and this decrease is intensified by decrease in apex angle and increase in gradient index, such that in $\beta = 1$, with the change of the length parameter from $L/R = 2$ to $L/R = 6$, the dimensionless natural frequency decreases from 2.9 to 0.6 in $2\alpha = 60$, and from 2.3 to 0.5 in $2\alpha = 123.6$. Besides, in $2\alpha = 86.6$, with the change of the size parameter from $L/R = 2$ to $L/R = 6$, the dimensionless natural frequency is reduced from 2.5 to 0.53 in $\beta = 1$ and from 3.1 to 0.65 in $\beta = 3$.

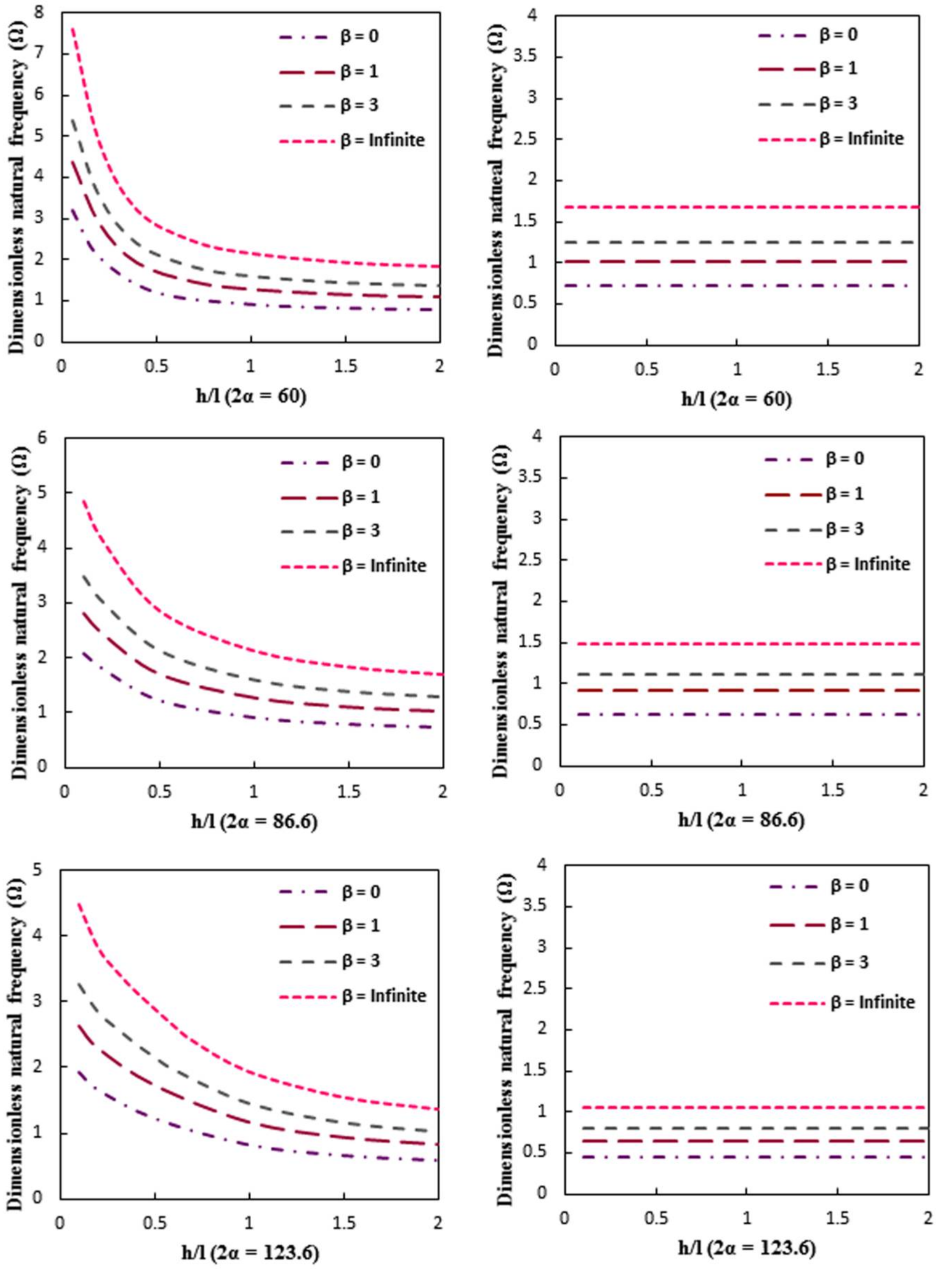

Figure 2: Effect of dimensionless length scale parameter *h*/*l* on dimensionless natural frequency in three apex angles $2\alpha = 60$, $2\alpha = 86.6$ and $2\alpha = 123.6$ in the case of high temperature, a) Classical continuum theory, b) Modified couple stress theory

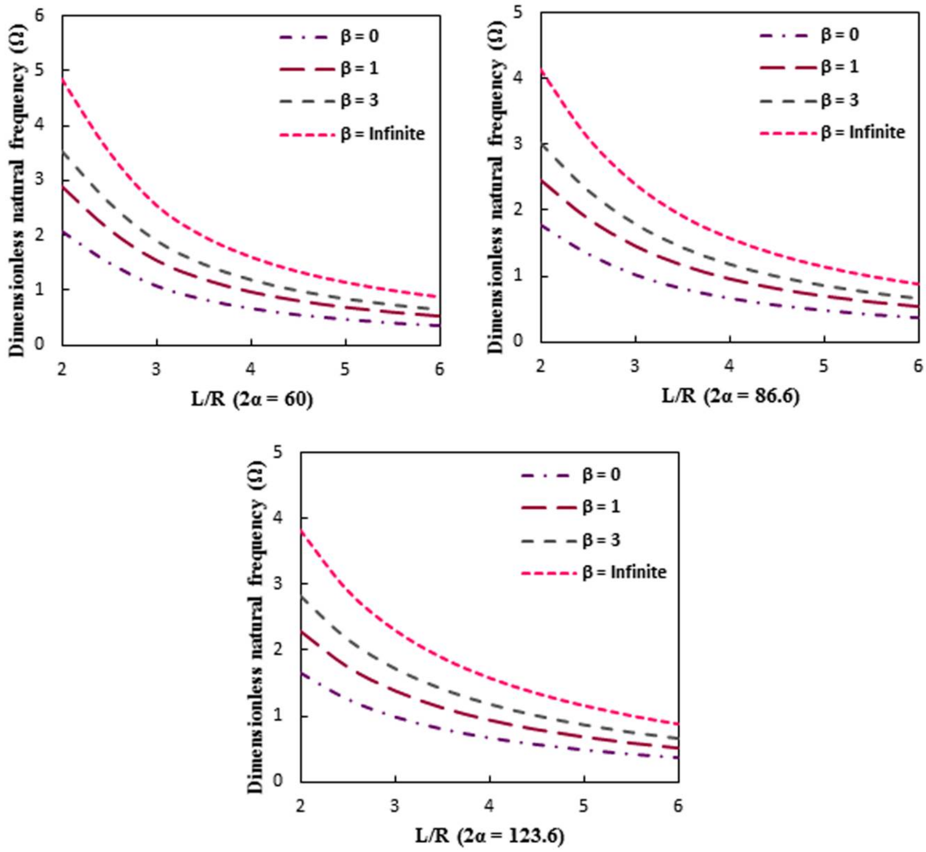

Figure 3: Effect of dimensionless length parameter L/R on dimensionless natural frequency in three apex angles $2\alpha = 60$, $2\alpha = 86.6$ and $2\alpha = 123.6$ in the case of high temperature.

5.3. Influence of circumferential and axial wave numbers

Figs. (4-5) shows the effects of conical nanoshell thickness as well as circumferential and axial wavenumbers on dimensionless natural frequency based on the modified couple stress theory and classical continuum theory in $h/l = 2$, $L/R = 2$ and $\beta = 1$ in three apex angles for $m = 1$ and $m = 2$ for the case of high temperature. As it is indicated, an increase in circumferential and axial wavenumbers leads to an increase in natural frequency, which is intensified by increased thickness and decreased apex angle. The effects of variation of circumferential and axial wavenumbers on natural frequency is greater based on the modified couples stress theory than that based on the classical continuum theory. As is clear from the illustration, the natural frequencies predicted by the modified couple stress theory in all values of circumferential and axial wavenumbers are greater than those predicted by the classical continuum theory. Hence, the modified couple stress theory is known to predict greater stiffness than the classical continuum theory. In addition, as illustrated, according to the previous illustration, the effects of increase in apex angle on the decrease in dimensionless natural frequency, with the increase in circumferential and axial wave numbers become more considerable.

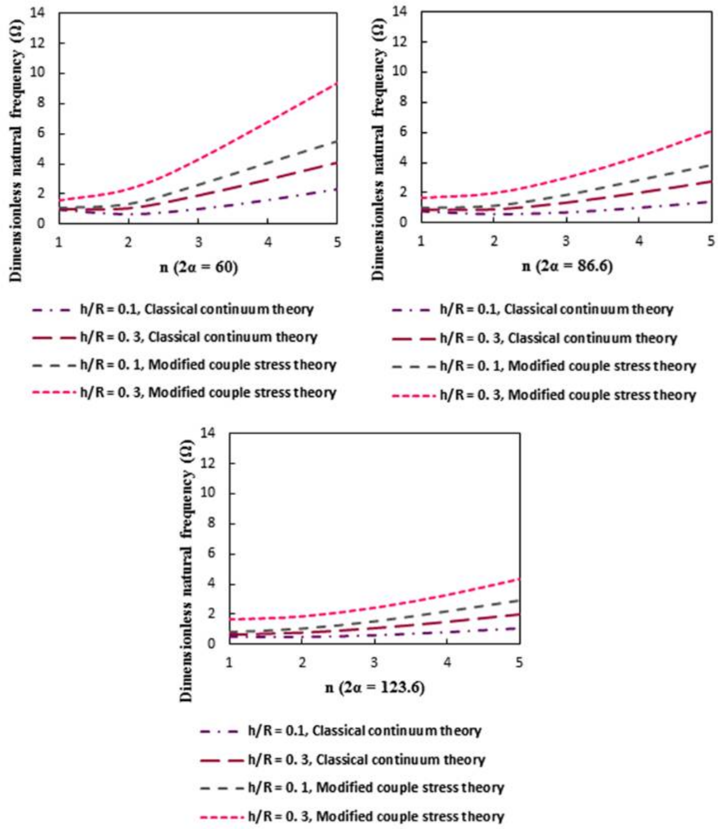

Figure 4: Effect of circumferential wave numbers n on dimensionless natural frequency in the case of high temperature. ($m = 1$)

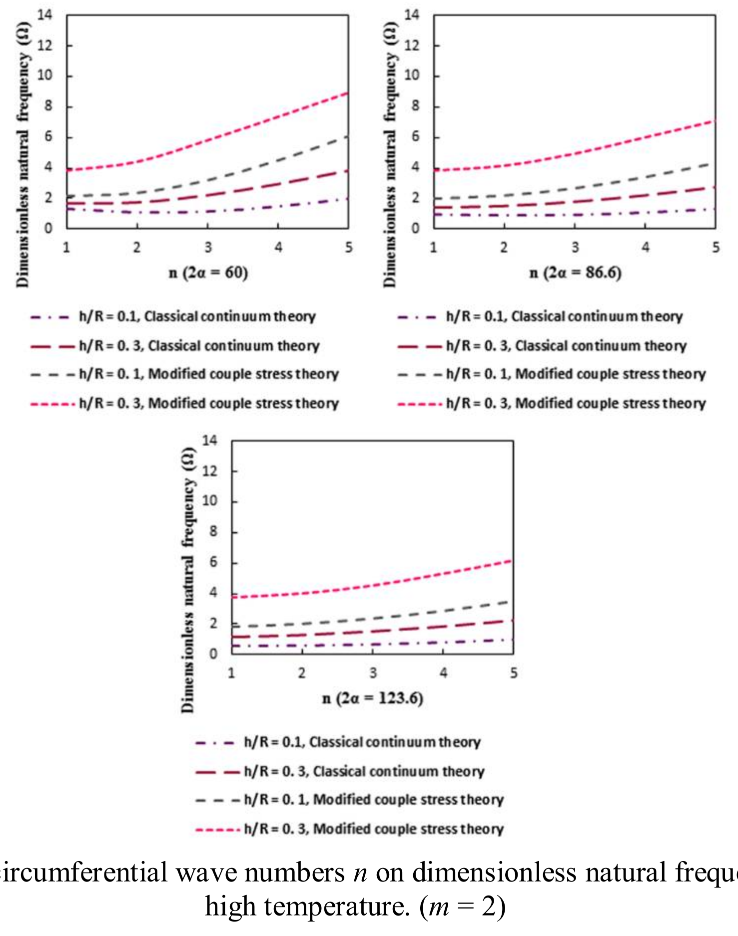

Figure 5: Effect of circumferential wave numbers n on dimensionless natural frequency in the case of high temperature. ($m = 2$)

5.4. Influence of temperature change

The influence of temperature change on dimensionless natural frequency is shown in Fig. (6), at two cases of low temperature and high temperature environment on the basis of modified couple stress theory and classical continuum theory. As is illustrated, the increase in temperature, in high temperature case, leads to decrease in dimensionless natural frequency while as ΔT is increased for the case of low temperature, the dimensionless natural frequency increases based on both theories, modified couple stress theory and classical continuum theory. Besides, it is clear that the small scale plays an important role in thermo-mechanical vibration analysis. It is illustrated that the natural frequency considering the modified couple stress theory are always higher than that of classical continuum theory for two cases of high and low temperature. Consequently, the present study clearly indicates the importance of utilizing modified couple stress theory in thermo-mechanical analysis since the modified couple stress theory predicts the natural frequency higher than that of classical continuum theory at all temperatures and in both low and high temperature environments.

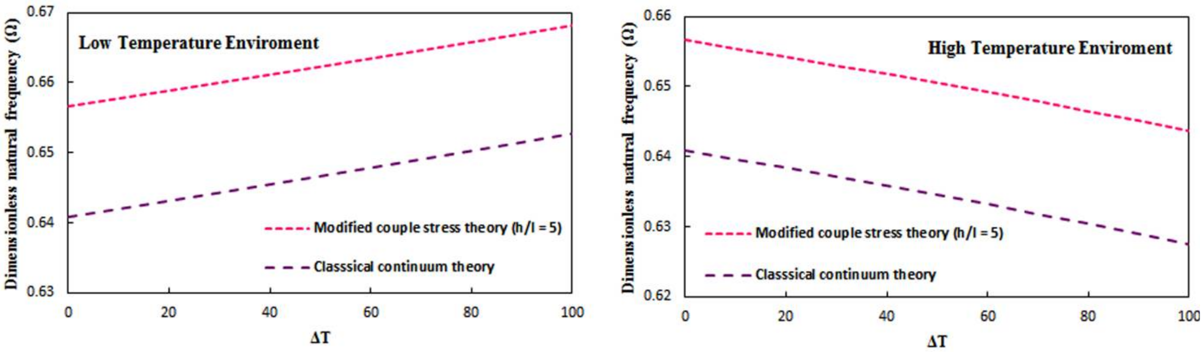

Figure 6: Effect of temperature change on dimensionless natural frequency for two cases of high temperature and low temperature environments.

5.5. Influence of foundation stiffness

Figs. (7-8) indicates the effect of elastic medium k_w, k_g on vibration behavior of FG conical shell. As is clear, the increase in Winkler and Pasternak elastic foundation results in increasing dimensionless natural frequency since elastic foundation constants increase the stiffness of nanoshell. Moreover, the influence of coefficient k_g is illustrated more significant than Winkler foundation with coefficient k_w.

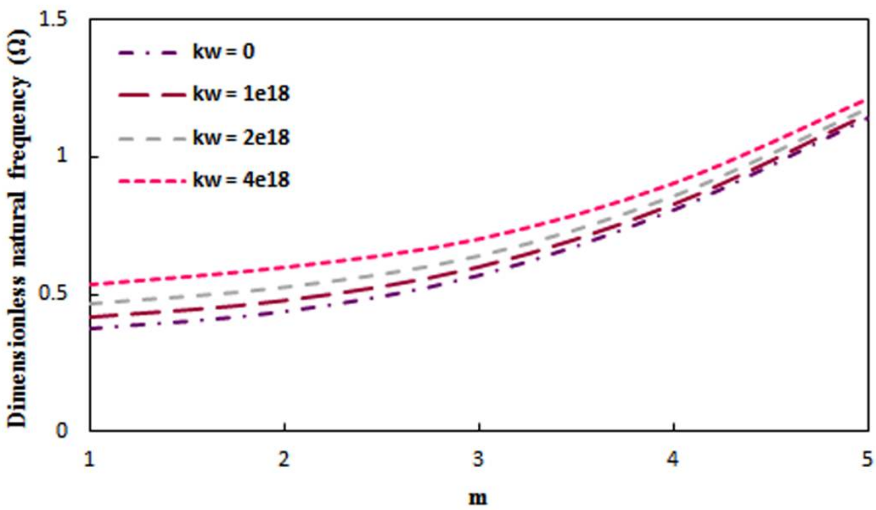

Figure 7: Effect of Winkler parameter on the dimensionless natural frequency in the case of high temperature.

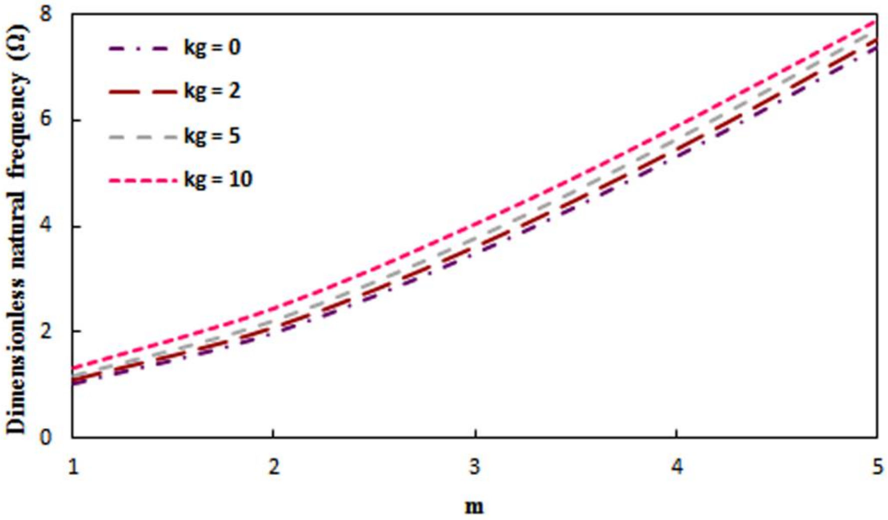

Figure 8: Effect of Pasternak shear parameter on the dimensionless natural frequency in the case of hig temperature.

6. Conclusion

In this paper, using the first order shear deformation theory as well as the modified couple stress theory, the formulation was derived to examine the thermo-mechanical vibration of shear deformable FG truncated conical shell embedded in an elastic medium. According to power law distribution and based on the volume fractions of constituents, FG truncated conical shell properties were considered variable along the thickness direction of nanoshell and temperature-dependent, and governing equations as well as classical and non-classical boundary conditions were derived using the Hamilton's principle. Finally, using these equations, the free vibration of the simply supported FG conical shell resting on elastic medium subjected to thermal environment is investigated as a special case. Afterwards, the effects of parameters such as length scale parameter, length parameter, apex angle, temperature change, gradient index, Winkler and Pasternak constants of elastic medium and circumferential and axial wave numbers on the natural frequency of FG conical shell based on the modified couple stress theory and the classical continuum theory was investigated.

Appendix A:

$$\begin{aligned}
\left(D_{1,i}\right) &= \int_{-\hat{z}_c}^{h-\hat{z}_c} \frac{E\left(\hat{z},T\right)}{1-\nu^2\left(\hat{z},T\right)}\left(z^i\right)dz, \ \left(i=0,1,2\right) \\
\left(D_{3,i}\right) &= \int_{-\hat{z}_c}^{h-\hat{z}_c} \frac{E\left(\hat{z},t\right)\nu\left(\hat{z},T\right)}{1-\nu^2\left(\hat{z},T\right)}\left(z^i\right)dz, \ \left(i=0,1,2\right) \\
\left(D_{5,i}\right) &= \int_{-\hat{z}_c}^{h-\hat{z}_c} \mu\left(\hat{z},T\right)\left(z^i\right)dz, \ \left(i=0,1,2\right) \\
\left(I_{1,i}\right) &= \int_{-\hat{z}_c}^{h-\hat{z}_c} \rho\left(\hat{z},T\right)\left(z^i\right)dz, \ \left(i=0,1,2\right)
\end{aligned} \tag{A1}$$

$$
\begin{aligned}
&A_1=-D_{1,0},A_2=-\frac{D_{1,0}}{x},A_3=\frac{D_{1,0}}{x^2},A_4=\frac{D_{5,0}}{x^2\sin^2\alpha}\left(-1+\frac{l^2}{4x^2}-\frac{l^2}{x^2\tan^2\alpha}\right),A_5=\frac{D_{5,0}l^2}{4x^4\sin^4\alpha},A_6=\frac{D_{5,0}l^2}{4x^2\sin^2\alpha},\\
&A_7=-\frac{D_{5,0}l^2}{4x^3\sin^2\alpha},A_8=\left(\frac{D_{1,0}}{x^2\sin\alpha}+\frac{D_{5,0}}{x^2\sin\alpha}\left(1+\frac{3l^2}{4x^2\tan^2\alpha}-\frac{l^2}{4x^2}\right)\right),\\
&A_9=-\left(\frac{D_{3,0}}{x\sin\alpha}+\frac{D_{5,0}}{x\sin\alpha}\left(1-\frac{l^2}{4x^2}-\frac{l^2}{4x^2\tan^2\alpha}\right)\right),A_{10}=-\frac{D_{5,0}l^2}{2x^2\sin\alpha},A_{11}=-\frac{D_{5,0}l^2}{4x^3\sin^3\alpha},A_{12}=-\frac{D_{5,0}l^2}{4x\sin\alpha},\\
&A_{13}=-\frac{D_{5,0}l^2}{4x^4\sin^3\alpha},A_{14}=\frac{D_{1,0}}{x^2\tan\alpha},A_{15}=-\frac{D_{3,0}}{x\tan\alpha},A_{16}=-\frac{D_{5,0}l^2\cos\alpha}{2x^3\sin^3\alpha},A_{17}=\frac{D_{5,0}l^2\cos\alpha}{4x^4\sin^3\alpha},A_{18}=-D_{1,1},\\
&A_{19}=-\frac{D_{1,1}}{x},A_{20}=\frac{D_{1,1}}{x^2},A_{21}=\frac{1}{x^2\sin^2\alpha}\left(\frac{5D_{5,0}l^2}{4x\tan\alpha}-D_{5,1}+\frac{D_{5,1}l^2}{4x^2}\right),A_{22}=\frac{D_{5,1}l^2}{4x^4\sin^4\alpha},A_{23}=-\frac{D_{5,1}l^2}{4x^3\sin^2\alpha},\\
&A_{24}=\frac{D_{5,1}l^2}{4x^2\sin^2\alpha},A_{25}=\frac{1}{x^2\sin\alpha}\left(-\frac{D_{5,0}l^2}{x\tan\alpha}+D_{1,1}+D_{5,1}\right),A_{26}=-\frac{D_{5,1}l^2}{4x^4\sin^3\alpha},\\
&A_{27}=\frac{1}{x\sin\alpha}\left(-\frac{D_{5,0}l^2}{4x\tan\alpha}-D_{3,1}-D_{5,1}+\frac{D_{5,1}l^2}{2x^2\tan^2\alpha}\right),A_{28}=-\frac{D_{5,1}l^2}{2x^2\sin\alpha},A_{29}=-\frac{D_{5,1}l^2}{4x^3\sin^3\alpha},A_{30}=-\frac{D_{5,1}l^2}{4x\sin\alpha},\\
&B_1=\frac{D_{5,0}l^2}{4},B_2=\frac{D_{5,0}l^2}{2x},B_3=-D_{5,0}\left(1+\frac{l^2}{2x^2\tan^2\alpha}+\frac{3l^2}{4x^2}\right),B_4=-\frac{D_{5,0}}{x}\left(1-\frac{l^2}{2x^2\tan^2\alpha}-\frac{3l^2}{4x^2}\right),\\
&B_5=\frac{D_{5,0}}{x^2}\left(1+\frac{k_s}{\tan^2\alpha}+\frac{l^2\cos^2 2\alpha}{4x^2\sin^4\alpha}+\frac{l^2\cos 2\alpha}{x^2\sin^2\alpha}\right),B_6=-\frac{1}{x^2\sin^2\alpha}\left(D_{1,0}+\frac{D_{5,0}l^2}{x^2}\left(\frac{1}{4\tan^2\alpha}+\frac{3}{4}\right)\right),\\
&B_7=\frac{D_{5,0}l^2}{4x^2\sin^2\alpha},B_8=-\frac{D_{5,0}l^2}{4x^3\sin^2\alpha},B_9=-\frac{1}{x^2\sin\alpha}\left(D_{1,0}+D_{5,0}\left(1-\frac{3l^2}{4x^2}+\frac{5l^2}{4x^2\tan^2\alpha}\right)\right),\\
&B_{10}=-\frac{1}{x\sin\alpha}\left(D_{3,0}+D_{5,0}\left(1+\frac{3l^2}{4x^2}-\frac{l^2}{4x^2\tan^2\alpha}\right)\right),B_{11}=\frac{D_{5,0}l^2}{2x^2\sin\alpha},B_{12}=-\frac{D_{5,0}l^2}{4x^3\sin^3\alpha},\\
&B_{13}=-\frac{D_{5,0}l^2}{4x\sin\alpha},B_{14}=\frac{3D_{5,0}l^2}{4x^4\sin^3\alpha},B_{15}=-\frac{\cos\alpha}{x^2\sin^2\alpha}\left(D_{1,0}+D_{5,0}\left(k_s+\frac{l^2}{4x^2\tan^2\alpha}-\frac{3l^2}{4x^2}\right)\right),\\
&B_{16}=-\frac{D_{5,0}l^2\cos\alpha}{x^3\sin^2\alpha},B_{17}=\frac{3D_{5,0}l^2\cos\alpha}{4x^2\sin^2\alpha},B_{18}=\frac{D_{5,0}l^2\cos\alpha}{4x^4\sin^4\alpha},\\
&B_{19}=\frac{1}{x^2\sin\alpha}\left(\frac{5D_{5,0}l^2}{4x\tan\alpha}-D_{1,1}-D_{5,1}+\frac{D_{5,1}l^2\cos 2\alpha}{4x^2\sin^2\alpha}+\frac{D_{5,1}l^2}{x^2}\right),\\
&B_{20}=-\frac{1}{x\sin\alpha}\left(\frac{D_{5,0}l^2}{2x\tan\alpha}+D_{3,1}+D_{5,1}+\frac{D_{5,1}l^2\cos 2\alpha}{4x^2\sin^2\alpha}+\frac{D_{5,1}l^2}{x^2}\right),B_{21}=\frac{3D_{5,1}l^2}{4x^4\sin^3\alpha},B_{22}=-\frac{D_{5,1}l^2}{4x\sin\alpha},\\
&B_{23}=\frac{D_{5,1}l^2}{2x^2\sin\alpha},B_{24}=-\frac{D_{5,1}l^2}{4x^3\sin^3\alpha},B_{25}=-\left(\frac{D_{5,0}}{x\tan\alpha}\left(k_s+\frac{l^2}{4x^2\tan^2\alpha}+\frac{l^2}{x^2}\right)-\frac{D_{5,1}}{x^2}+\frac{D_{5,1}l^2}{2x^4\tan^2\alpha}\right),\\
&B_{26}=-\left(\frac{D_{5,0}l^2}{4x^2\tan\alpha}+\frac{D_{5,1}}{x}-\frac{D_{5,1}l^2}{x^3\tan^2\alpha}-\frac{D_{5,1}l^2\cos 2\alpha}{4x^3\sin^2\alpha}-\frac{D_{5,1}l^2}{4x^3}\right),B_{27}=-\left(\frac{3D_{5,0}l^2}{4x\tan\alpha}+D_{5,1}+\frac{D_{5,1}l^2}{4x^2\tan^2\alpha}+\frac{D_{5,1}l^2}{2x^2}\right),\\
&B_{28}=-\frac{1}{x^2\sin^2\alpha}\left(\frac{D_{5,0}l^2}{4x\tan\alpha}+D_{1,1}+\frac{3D_{5,1}l^2}{4x^2}\right),B_{29}=\frac{D_{5,1}l^2}{4x^2\sin^2\alpha},B_{30}=-\frac{D_{5,1}l^2}{4x^3\sin^2\alpha},B_{31}=\frac{D_{5,1}l^2}{4},B_{32}=\frac{D_{5,1}l^2}{2x}\\
&C_1=\frac{D_{5,0}l^2}{4},C_2=-D_{5,0}\left(k_s+\frac{l^2}{4x^2\tan^2\alpha}+\frac{l^2}{4x^2}\right),C_3=-\frac{D_{5,0}}{x}\left(k_s-\frac{l^2}{4x^2\tan^2\alpha}-\frac{l^2}{4x^2}\right),C_4=\frac{D_{5,0}l^2}{2x},C_5=-\frac{D_{5,0}l^2}{2x^3\sin^2\alpha}\\
&,C_6=-\frac{D_{5,0}}{x^2\sin^2\alpha}\left(k_s+\frac{l^2}{4x^2\tan^2\alpha}-\frac{l^2}{x^2}\right),C_7=\frac{D_{5,0}l^2}{2x^2\sin^2\alpha},C_8=\frac{D_{5,0}l^2}{4x^4\sin^4\alpha},C_9=\frac{D_{1,0}}{x^2\tan^2\alpha},C_{10}=\frac{D_{1,0}}{x^2\tan\alpha},C_{11}=\frac{D_{3,0}}{x\tan\alpha}\\
&,C_{12}=\frac{D_{5,0}l^2\cos\alpha}{2x^3\sin^3\alpha},C_{13}=-\frac{3D_{5,0}l^2\cos\alpha}{4x^4\sin^3\alpha},C_{14}=\frac{\cos\alpha}{x^2\sin^2\alpha}\left(D_{1,0}+D_{5,0}\left(k_s+\frac{l^2\cos 2\alpha}{4x^2\sin^2\alpha}\right)\right),C_{15}=\frac{D_{5,0}l^2\cos\alpha}{2x^3\sin^2\alpha}\\
&,C_{16}=-\frac{D_{5,0}l^2\cos\alpha}{4x^4\sin^4\alpha},C_{17}=-\frac{3D_{5,0}l^2\cos\alpha}{4x^2\sin^2\alpha},C_{18}=-\frac{D_{5,0}l^2}{4},C_{19}=-\frac{D_{5,0}l^2}{2x},C_{20}=-\left(k_sD_{5,0}-\frac{D_{5,0}l^2}{4x^2}-\frac{D_{5,0}l^2}{4x^2\tan^2\alpha}-\frac{D_{3,1}}{x\tan\alpha}\right)
\end{aligned}
\qquad \text{(A2)}
$$

$$,C_{21}=-\frac{l^2}{4x^3\sin^2\alpha}\left(D_{5,0}+\frac{D_{5,1}}{x\tan\alpha}\right),C_{22}=-\frac{D_{5,0}l^2}{4x^2\sin^2\alpha},C_{23}=-\frac{1}{x}\left(D_{5,0}k_s+\frac{D_{5,0}l^2}{4x^2\tan^2\alpha}+\frac{D_{5,0}l^2}{4x^2}-\frac{D_{1,1}}{x\tan\alpha}\right)$$

$$,C_{24}=\frac{l^2}{4x^2\sin\alpha}\left(D_{5,0}+\frac{3D_{5,1}}{x\tan\alpha}\right),C_{25}=-\frac{D_{5,0}l^2}{4x^3\sin^3\alpha},C_{26}=-\frac{l^2}{2x\sin\alpha}\left(\frac{D_{5,0}}{2}+\frac{D_{5,1}}{x\tan\alpha}\right)$$

$$,C_{27}=-\frac{1}{x\sin\alpha}\left(D_{5,0}k_s+\frac{D_{5,0}l^2}{4x^2\tan^2\alpha}+\frac{D_{5,0}l^2}{4x^2}-\frac{D_{1,1}}{x\tan\alpha}\right),D_1=-\left(D_{1,2}+\frac{D_{5,0}l^2}{4}\right),D_2=-\left(\frac{D_{1,2}}{x}+\frac{D_{5,0}l^2}{4x}\right),$$

$$D_3=\left(\frac{D_{1,2}}{x^2}+D_{5,0}\left(k_s+\frac{l^2}{4x^2\tan^2\alpha}+\frac{l^2}{4x^2}\right)\right),D_4=-\frac{1}{x^2\sin^2\alpha}\left(D_{5,2}+D_{5,0}l^2-\frac{D_{5,2}l^2}{4x^2}-\frac{D_{5,1}l^2}{2x\tan\alpha}\right),$$

$$D_5=-\frac{D_{5,2}l^2}{4x^3\sin^2\alpha},D_6=\frac{D_{5,2}l^2}{4x^4\sin^4\alpha},D_7=\frac{D_{5,2}l^2}{4x^2\sin^2\alpha},D_8=\frac{1}{x^2\sin\alpha}\left(D_{1,2}+D_{5,2}+\frac{5D_{5,0}l^2}{4}-\frac{D_{5,1}l^2}{4x\tan\alpha}\right),$$

$$D_9=-\frac{1}{x\sin\alpha}\left(D_{3,2}+D_{5,2}-\frac{3D_{5,0}l^2}{4}+\frac{D_{5,1}l^2}{2x\tan\alpha}\right),D_{10}=-\frac{D_{5,2}l^2}{4x^3\sin^3\alpha},D_{11}=-\frac{D_{5,2}l^2}{4x^4\sin^3\alpha},D_{12}=-\frac{D_{5,2}l^2}{4x\sin\alpha},$$

$$D_{13}=-\frac{D_{5,2}l^2}{2x^2\sin\alpha},D_{14}=\frac{1}{x^2\sin^2\alpha}\left(\frac{5D_{5,0}l^2\cos\alpha}{4x\sin\alpha}-D_{5,1}+\frac{D_{5,1}l^2}{4x^2}\right),D_{15}=-D_{1,1},D_{16}=\frac{D_{1,1}}{x^2},D_{17}=-\frac{D_{1,1}}{x},$$

$$D_{18}=\frac{D_{5,1}l^2}{4x^4\sin^4\alpha},D_{19}=\frac{D_{5,1}l^2}{4x^2\sin^2\alpha},D_{20}=-\frac{D_{5,1}l^2}{4x^3\sin^2\alpha},D_{21}=\frac{1}{x^2\sin\alpha}\left(-\frac{3D_{5,0}l^2\cos\alpha}{4x\sin\alpha}+D_{1,1}+D_{5,1}+\frac{D_{5,1}l^2\cos2\alpha}{4x^2\sin^2\alpha}\right),$$

$$D_{22}=-\frac{1}{x\sin\alpha}\left(\frac{D_{5,0}l^2}{2x\tan\alpha}+D_{3,1}+D_{5,1}+\frac{D_{5,1}l^2\cos2\alpha}{4x^2\sin^2\alpha}\right),D_{23}=-\frac{D_{5,1}l^2}{4x^3\sin^3\alpha},D_{24}=-\frac{D_{5,1}l^2}{4x\sin\alpha},D_{25}=-\frac{D_{5,1}l^2}{4x^4\sin^3\alpha},$$

$$D_{26}=-\frac{D_{5,1}l^2}{2x^2\sin\alpha},D_{27}=\left(D_{5,0}k_s-\frac{D_{5,0}l^2}{4x^2\tan^2\alpha}-\frac{D_{5,0}l^2}{4x^2}-\frac{D_{3,1}}{x\tan\alpha}\right),D_{28}=\frac{D_{5,0}l^2}{4x},D_{29}=-\frac{l^2}{2x^3\sin^2\alpha}\left(D_{5,0}+\frac{D_{5,1}}{2x\tan\alpha}\right),$$

$$D_{30}=\frac{D_{5,0}l^2}{4},D_{31}=\frac{D_{5,0}l^2}{4x^2\sin^2\alpha},D_{32}=\frac{D_{1,1}}{x^2\tan\alpha},E_1=\frac{D_{5,2}l^2}{4},E_2=\frac{D_{5,2}l^2}{2x},E_3=\frac{D_{5,2}l^2}{4x^2\sin^2\alpha},E_4=-\frac{D_{5,2}l^2}{4x^3\sin^2\alpha}$$

$$E_5=-\left(D_{5,2}+D_{5,0}l^2+\frac{D_{5,2}l^2}{4x^2}+\frac{D_{5,2}l^2}{2x^2\tan^2\alpha}+\frac{D_{5,1}l^2}{2x\tan\alpha}\right),E_6=-\frac{1}{x}\left(D_{5,2}+D_{5,0}l^2-\frac{D_{5,2}l^2}{4x^2}-\frac{D_{5,2}l^2}{2x^2\tan^2\alpha}\right),$$

$$E_7=-\frac{1}{x^2\sin^2\alpha}\left(D_{1,2}+\frac{D_{5,0}l^2}{4}+\frac{3D_{5,2}l^2}{4x^2}\right),E_8=\left(\frac{D_{5,2}}{x^2}+D_{5,0}k_s+\frac{D_{5,0}l^2}{x^2}+\frac{D_{5,0}l^2}{4x^2\tan^2\alpha}+\frac{D_{5,2}l^2}{2x^4\tan^2\alpha}-\frac{5D_{5,1}l^2}{4x^3\tan\alpha}\right),$$

$$E_9=-\frac{D_{5,2}l^2}{4x\sin\alpha},E_{10}=-\frac{D_{5,2}l^2}{4x^3\sin^3\alpha},E_{11}=\frac{3D_{5,2}l^2}{4x^4\sin^3\alpha},E_{12}=\frac{D_{5,2}l^2}{2x^2\sin\alpha},$$

$$E_{13}=-\frac{1}{x\sin\alpha}\left(D_{5,2}+D_{3,2}-\frac{3D_{5,0}l^2}{4}+\frac{D_{5,2}l^2}{x^2}+\frac{D_{5,1}l^2}{2x\tan\alpha}\right),E_{14}=-\frac{1}{x^2\sin\alpha}\left(D_{5,2}+D_{1,2}+\frac{5D_{5,0}l^2}{4}-\frac{D_{5,2}l^2}{x^2}-\frac{3D_{5,1}l^2}{4x\tan\alpha}\right),$$

$$E_{15}=\frac{1}{x^2\sin\alpha}\left(\frac{5D_{5,0}l^2}{4x\tan\alpha}-D_{1,1}-D_{5,1}-\frac{D_{5,1}l^2}{x^2\tan^2\alpha}+\frac{D_{5,1}l^2}{x^2}\right),E_{16}=-\frac{1}{x\sin\alpha}\left(\frac{D_{5,0}l^2}{4x\tan\alpha}+D_{3,1}+D_{5,1}-\frac{D_{5,1}l^2}{2x^2\tan^2\alpha}+\frac{D_{5,1}l^2}{x^2}\right),$$

$$E_{17}=\frac{D_{5,1}l^2}{2x^2\sin\alpha},E_{18}=-\frac{D_{5,1}l^2}{4x\sin\alpha},E_{19}=-\frac{D_{5,1}l^2}{4x^3\sin^3\alpha},E_{20}=\frac{3D_{5,1}l^2}{4x^4\sin^3\alpha},$$

$$E_{21}=-\left(\frac{D_{5,0}k_s}{x\tan\alpha}+\frac{5D_{5,0}l^2}{4x^3\tan\alpha}+\frac{D_{5,0}l^2}{4x^3\tan^3\alpha}-\frac{D_{5,1}}{x^2}-\frac{D_{5,1}l^2}{2x^4\tan^2\alpha}-\frac{D_{5,1}l^2\cos2\alpha}{x^4\sin^2\alpha}\right),$$

$$E_{22}=\left(\frac{D_{5,0}l^2}{4x^2\tan\alpha}-\frac{D_{5,1}}{x}-\frac{3D_{5,1}l^2\cos2\alpha}{4x^3\sin^2\alpha}+\frac{D_{5,1}l^2}{4x^3}\right),E_{23}=-\left(\frac{3D_{5,0}l^2}{4x\tan\alpha}+D_{5,1}+\frac{D_{5,1}l^2}{2x^2\tan^2\alpha}-\frac{D_{5,1}l^2\cos2\alpha}{4x^2\sin^2\alpha}+\frac{D_{5,1}l^2}{4x^2}\right),$$

$$E_{24}=-\frac{1}{x^2\sin^2\alpha}\left(\frac{D_{5,0}l^2}{4x\tan\alpha}+D_{1,1}+\frac{3D_{5,1}l^2}{4x^2}\right),E_{25}=\frac{D_{5,1}l^2}{4x^2\sin^2\alpha},E_{26}=-\frac{D_{5,1}l^2}{4x^3\sin^2\alpha},E_{27}=\frac{D_{5,1}l^2}{4},E_{28}=\frac{D_{5,1}l^2}{2x},$$

$$E_{29}=\frac{1}{x\sin\alpha}\left(D_{5,0}k_s+\frac{D_{5,0}l^2}{4x^2\tan^2\alpha}-\frac{D_{1,1}}{x\tan\alpha}-\frac{D_{5,1}l^2}{x^3\tan\alpha}\right),E_{30}=\frac{l^2}{4x^2\sin\alpha}\left(D_{5,0}-\frac{D_{5,1}}{x\tan\alpha}\right),$$

$$E_{31}=\frac{l^2}{2x\sin\alpha}\left(\frac{D_{5,0}}{2}+\frac{D_{5,1}}{x\tan\alpha}\right),E_{32}=\frac{D_{5,0}l^2}{4x^3\sin^3\alpha}$$

The coefficients of the boundary conditions equations, Eqs. (23)-(30), are expressed as:

$$
\begin{gathered}
a_1 = D_{1,0}, a_2 = \frac{D_{5,0} l^2}{4x^3 \sin^2\alpha}, a_3 = \frac{D_{3,0}}{x}, a_4 = -\frac{D_{5,0} l^2}{4x^2 \sin^2\alpha}, a_5 = \frac{1}{x\sin\alpha}\left(D_{3,0} + \frac{D_{5,0} l^2 \cos 2\alpha}{4x^2 \sin^2\alpha}\right), a_6 = \frac{D_{5,0} l^2}{4x^2 \sin\alpha}, \\
a_7 = \frac{D_{5,0} l^2}{4x\sin\alpha}, a_8 = \frac{D_{3,0}}{x\tan\alpha}, a_9 = -\frac{D_{5,0} l^2 \cos\alpha}{4x^3 \sin^3\alpha}, a_{10} = D_{1,1}, a_{11} = \frac{D_{3,1}}{x}, a_{12} = \frac{D_{5,1} l^2}{4x^3 \sin^2\alpha}, a_{13} = -\frac{D_{5,1} l^2}{4x^2 \sin^2\alpha}, \\
a_{14} = \frac{1}{x\sin\alpha}\left(D_{3,1} - \frac{D_{5,0} l^2}{4x\tan\alpha}\right), a_{15} = \frac{D_{5,1} l^2}{4x^2 \sin\alpha}, a_{16} = \frac{D_{5,1} l^2}{4x\sin\alpha}, b_1 = \frac{D_{5,0} l^2}{4x^2 \sin\alpha}, b_2 = -\frac{D_{5,0} l^2}{4x\sin\alpha}, b_3 = \frac{D_{5,0} l^2}{4}, \\
b_4 = \frac{D_{5,0} l^2}{4x}, b_5 = \frac{D_{5,0} l^2 \cos 2\alpha}{4x^2 \sin^2\alpha}, b_6 = -\frac{D_{5,0} l^2 \cos\alpha}{4x^2 \sin^2\alpha}, b_7 = \frac{D_{5,1} l^2}{4x^2 \sin\alpha}, b_8 = -\frac{D_{5,1} l^2}{4x\sin\alpha}, b_9 = -\frac{D_{5,0} l^2}{4x\tan\alpha}, b_{10} = \frac{D_{5,1} l^2}{4x}, \\
b_{11} = \frac{D_{5,1} l^2}{4}, c_1 = -\frac{D_{5,0} l^2 \cos\alpha}{4x^2 \sin^2\alpha}, c_2 = -\frac{D_{5,0} l^2}{4}, c_3 = \frac{D_{5,0} l^2}{4x}, c_4 = \frac{D_{5,0} l^2}{4x^2 \sin^2\alpha}, c_5 = \frac{D_{5,0} l^2}{4}, c_6 = -\frac{D_{5,0} l^2}{4x}, c_7 = -\frac{D_{5,0} l^2}{4x\sin\alpha} \\
d_1 = D_{1,1}, d_2 = \frac{D_{3,1}}{x}, d_3 = \frac{D_{5,1} l^2}{4x^3 \sin^2\alpha}, d_4 = -\frac{D_{5,1} l^2}{4x^2 \sin^2\alpha}, d_5 = \frac{1}{x\sin\alpha}\left(D_{3,1} - \frac{D_{5,0} l^2}{4x\tan\alpha} + \frac{D_{5,1} l^2 \cos 2\alpha}{4x^2 \sin^2\alpha}\right), d_6 = \frac{D_{5,1} l^2}{4x^2 \sin\alpha}, \\
d_7 = \frac{D_{5,1} l^2}{4x\sin\alpha}, d_8 = -\frac{D_{5,0} l^2}{4}, d_9 = \frac{D_{5,0} l^2}{4x}, d_{10} = \frac{D_{3,1}}{x\tan\alpha}, d_{11} = \frac{l^2}{4x^2 \sin^2\alpha}\left(D_{5,0} - \frac{D_{5,1}}{x\tan\alpha}\right), d_{12} = \left(D_{1,2} + \frac{D_{5,0} l^2}{4}\right), \\
d_{13} = \frac{1}{x}\left(D_{3,2} - \frac{D_{5,0} l^2}{4}\right), d_{14} = -\frac{D_{5,2} l^2}{4x^2 \sin^2\alpha}, d_{15} = \frac{D_{5,2} l^2}{4x^3 \sin^2\alpha}, d_{16} = \frac{1}{x\sin\alpha}\left(D_{3,2} - \frac{D_{5,0} l^2}{4} - \frac{D_{5,1} l^2}{4x\tan\alpha}\right), \\
d_{17} = \frac{D_{5,2} l^2}{4x\sin\alpha}, d_{18} = \frac{D_{5,2} l^2}{4x^2 \sin\alpha}, e_1 = \frac{D_{5,1} l^2}{4x^2 \sin\alpha}, e_2 = -\frac{D_{5,1} l^2}{4x\sin\alpha}, e_3 = \frac{D_{5,1} l^2}{4x}, e_4 = \frac{D_{5,1} l^2}{4}, e_5 = \frac{D_{5,1} l^2 \cos 2\alpha}{4x^2 \sin^2\alpha}, \\
e_6 = -\frac{D_{5,1} l^2 \cos\alpha}{4x^2 \sin^2\alpha}, e_7 = -\frac{D_{5,2} l^2}{4x\sin\alpha}, e_8 = \frac{D_{5,2} l^2}{4x^2 \sin\alpha}, e_9 = \frac{D_{5,2} l^2}{4}, e_{10} = \frac{D_{5,2} l^2}{4x}, e_{11} = -\frac{D_{5,1} l^2}{4x\tan\alpha}
\end{gathered} \tag{A3}
$$

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