2024

INTERNATIONAL ANTALYA MATHEMATICS OLYMPIAD

11TH GRADE QUESTION BOOKLET

NAME SURNAME : ....

SCHOOL : .... GRADE : ....

SIGNATURE : ....

EXAMINATION RULES

1. It is forbidden to take the exam with a phone. Please hand in your phone to the attendant. This exam consists of 25 multiple-choice questions and the exam duration is 120 minutes.

2. Each question has only one correct answer. Mark your correct answer by completely crossing out the relevant box on your answer sheet. No marking in the question booklet will be evaluated.

3.All questions are of equal value and four wrong answers will cancel one correct answer. Questions left blank will not have a positive or negative effect on the evaluation.

4. The questions are NOT in order of difficulty. Therefore, it is recommended that you review all questions before you start answering.

5. It is forbidden to use aids such as compasses, rulers, calculators and scratch paper. Do all your work on the question booklet.

6. During the exam, you will not talk to the staff and you will not ask them any questions. It is unlikely that there will be a mistake in the questions. If this happens, the exam academic board will take appropriate action. In this case, you should mark the option that you think is the most correct.

7. Students are not allowed to ask each other for pencils, erasers, etc.

8. It is forbidden to leave the exam for the first 60 minutes. A candidate who goes out will not be allowed to take the exam again.

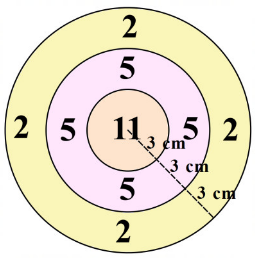
9. Do not forget to hand in your answer sheet and question booklet to the staff before leaving the exam hall.

1. For the sets $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{2, 3, 4, 5, 6, 7, 8, 9\}$, how many different sets C can be formed satisfying the conditions

$$C \subseteq B \quad \text{and} \quad s(A \backslash C) = 3\ ?$$

A) 30 B) 45 C) 15 D) 75 E) 60

2.



Berk continuously throws darts at a dartboard consisting of circles with the same centre and radii of 3, 6, 9 cm respectively. The dart always hits a region on the board for every throwing. What is Berk's average score if this throw continues for as long as desired?

A) 5 B) 4 C) 6 D) $5,5$ E) $4,5$

3. What is the numerator of the rational number

$$A \cdot B - C \cdot D$$

in its simplest form, where

$$A = \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots + \frac{1}{97} + \frac{1}{99}$$
$$B = 1 + \frac{1}{5} + \frac{1}{7} + \cdots + \frac{1}{99} + \frac{1}{101}$$
$$C = 1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{97} + \frac{1}{99}$$
$$D = \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \cdots + \frac{1}{99} + \frac{1}{101}\ ?$$

A) $\frac{98}{101}$ B) $\frac{99}{101}$ C) $\frac{98}{303}$ D) $\frac{100}{303}$ E) $\frac{100}{101}$

4. Let x be a positive integer. If

$$x^x = 2^{24} \cdot 3^x$$

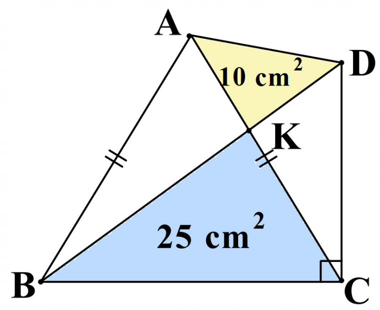
what is the value of $\left(\frac{x}{4}\right)^3$?

A) 12 B) 8 C) 64 D) 81 E) 27

5. A $\mathbf{15}$-kilogram watermelon, which is $\mathbf{97\%}$ of its weight in water, has $\mathbf{95\%}$ of its weight in water after being under the sun for a long time. How much did the watermelon weigh after being under the sun?

A) $\mathbf{9}$ B) $\mathbf{7}$ C) $\mathbf{10}$ D) $\mathbf{12}$ E) $\mathbf{13}$

6. In the convex quadrilateral $ABCD$ given in the figure below, $m(\angle BCD) = 90°$, $|AB| = |AC|$ and $AC \cap BD = K$. Since the area of the triangles ACD and BCK are $\mathbf{10}$ cm^2 and $\mathbf{25}$ cm^2 respectively. What is the area of quadrilateral $ABCD$ in cm^2?



A) $\mathbf{55}$ B) $\mathbf{60}$ C) $\mathbf{90}$ D) $\mathbf{70}$ E) $\mathbf{105}$

7. What is the sum of all $a \in \mathbb{Z}$ that guarantees the existence of two positive integer solutions of the equation

$$x^2 + ax - (4a+1) = 0\ ?$$

A) -36 B) -48 C) -44 D) -40 E) -16

8. Let a, b, c, d be the roots of the equation

$$x^4 + x + 1 = 0.$$

What is the value of the following sum?

$$S = \frac{a^2}{a^3+1} + \frac{b^2}{b^3+1} + \frac{c^2}{c^3+1} + \frac{d^2}{d^3+1}$$

A) $\mathbf{1}$ B) $\mathbf{5}$ C) $\mathbf{3}$ D) $\mathbf{7}$ E) $\mathbf{9}$

9. How many different ways can $\mathbf{10}$ identical mathematics books, $\mathbf{9}$ identical physics books and one chemistry book be arranged on a shelf so that no two adjacent books are from the same subject?

A) $\mathbf{45}$ B) $\mathbf{38}$ C) $\mathbf{36}$ D) $\mathbf{48}$ E) $\mathbf{35}$

10. For the real numbers x and y, if

$$\sqrt{x\sqrt[5]{y}} = 6^6 \quad \text{and} \quad \sqrt[3]{y\sqrt[5]{x}} = 4^4,$$

how many positive integer divisors does the integer $x \cdot y$ have?

A) $\mathbf{341}$ B) $\mathbf{300}$ C) $\mathbf{360}$ D) $\mathbf{310}$ E) $\mathbf{321}$

11. In a triangle ABC, $|AB| = 5$, $|BC| = 6$ and $|AC| = 7$. Let D and E be the feet of the height drawn from the vertices A and B, respectively. According to this, what is the radius of the circumcircle of triangle CDE ?

A) $\frac{18\sqrt{6}}{11}$ B) $\frac{25\sqrt{6}}{24}$ C) $\frac{5}{2}$ D) $\frac{7}{3}$ E) $\frac{4\sqrt{6}}{3}$

12. For $x < y < z$, how many positive integer triples (x, y, z) are there satisfying the following equality?

$$x + x{\cdot}y + x{\cdot}y{\cdot}z = 1111$$

A) $\mathbf{1}$ B) $\mathbf{7}$ C) $\mathbf{4}$ D) $\mathbf{3}$ E) $\mathbf{10}$

13. Telephone numbers in a town consist of 6 digits and are assigned according to the following three rules.

■ A telephone number must have at least 1 non-zero digit.

■ The sum of the first three digits is equal to the sum of the last three digits.

■ The sum of those in odd rows is equal to the sum of those in even rows

For example, one of the phone numbers in this town is

0	5	4	1	5	3

It can be seen that the equality

$$0+4+5=5+1+3$$

is satisfied. At most how many different phone numbers are there in this town?

A) 6624 B) 6440 C) 6400 D) 6644 E) 6699

14. Let x and y be real numbers. If

$$x^2+y^2=\frac{3}{2},$$

then what is the maximum value of $x+y-xy$?

A) $\frac{3}{4}$ B) $\frac{1}{2}$ C) $\frac{3}{2}$ D) $\frac{5}{4}$ E) $\frac{9}{4}$

15. Let $Q(x)$ be a polynomial taking integer values at integer points and

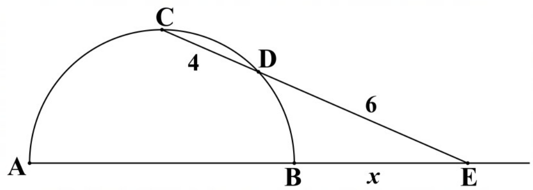
$$P(x)=3x-3+(x-1)(x-2)Q(x).$$

What is $P(6)$, if $P(x)$ is the least degree polynomial that satisfies $P(n)=n!$ for an integer $n>3$?

A) 156 B) 195 C) 183 D) 186 E) 201

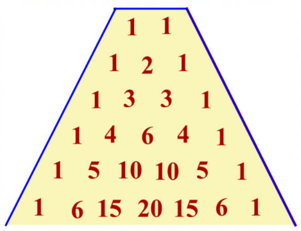
16. In the semicircle of diameter AB given in the figure below, the midpoint of arc AB is C. A point D is taken on the arc BC. What is $|BE|=x$, if

$$CD\cap AB=E,\quad |DE|=6,\ |CD|=4\,?$$



A) $3\sqrt{2}$ B) $2\sqrt{5}$ C) $2\sqrt{6}$ D) $2\sqrt{3}$ E) $3\sqrt{5}$

17. In Pascal's trapezoid, the number in each row is obtained by adding the two neighboring numbers in the previous row.



If we continue filling Pascal's trapezoid downwards, in which row are three consecutive numbers proportional to $\mathbf{2}$, $\mathbf{3}$ and $\mathbf{4}$ respectively? For example, three consecutive elements proportional to $\mathbf{2}$, $\mathbf{3}$, $\mathbf{2}$ respectively are in the fourth row: $\mathbf{4}$, $\mathbf{6}$, $\mathbf{4}$.

A) **43** B) **36** C) **42** D) **34** E) **44**

18. Let S be the number of all $\mathbf{40}$-letter words formed with the letters a, b, c in which the letter a occurs an even number of times. What is the remainder of S divided by $\mathbf{55}$? (Hint : Zero is also an even number.)

A) **1** B) **2** C) **54** D) **24** E) **15**

19. Let $a_1, a_2, a_3, \ldots, a_{100}$ be an arithmetic sequence of positive integers. Find a_{100}, if

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 = 133,$$
$$a_{a_1} + a_{a_2} + a_{a_3} + a_{a_4} + a_{a_5} + a_{a_6} + a_{a_7} = 553.$$

A) **440** B) **210** C) **403** D) **506** E) **434**

20. Points F and E are taken on the sides AC and BC of equilateral triangle ABC, respectively, such that

$$3\,|EC| = |FC| = 6.$$

What is the length of $|AD|$, if

$$EF \cap AB = D \quad \text{and} \quad BF \perp FE\,?$$

A) 8 B) 10 C) 14 D) 12 E) 15

21. Let m and n be positive integers. Find the sum of all values of n satisfying the equality

$$\sqrt[m]{7}\,\sqrt[n]{49} = \sqrt[7]{7}.$$

Note : The expression $\sqrt[p]{a^q}$ with $p, q \in \mathbb{Z}^+$ can be written as $a^{q/p}$.

A) 248 B) 232 C) 255 D) 208 E) 108

22. Let u and v be variables and a_{ij} are any constants for $i = 0, 1, ..., n$ and $j = 0, 1, ..., m$. The expression

$$P(u, v) = \sum_{i=0}^{n}\sum_{j=0}^{m} a_{ij} u^i v^j$$

is called a two-variable polynomial and the numbers a_{ij} are called the coefficients of this polynomial. If the expression

$$\left(x^{2024} + y^{2024}\right)$$

can be written as a polynomial of the variables $u = xy$ and $v = x + y$, what is the sum of the coefficients? (For example, $x^3 + y^3$ can be written as

$x^3 + y^3 = (x + y)^3 - 3xy\,(x + y) = v^3 - 3uv$,

and the sum of the coefficients is $1 + (-3) = -2$).

A) 1 B) -1 C) -3 D) 2023 E) 2024

23. If

$$f\,(a, b) = a + b + ab,$$

then what is the value of

$$f\left(\frac{1}{2}, f\left(\frac{1}{3}, f\left(\frac{1}{4}, f\left(\frac{1}{5}, f\left(\frac{1}{6}, f\left(\frac{1}{7}, \frac{1}{8}\right)\right)\right)\right)\right)\right)$$

A) $\frac{7}{2}$ B) 3 C) $\frac{9}{2}$ D) $\frac{5}{2}$ E) $\frac{3}{2}$

24. The numbers $a_1, a_2, ..., a_n$ are numbers that can take any of the integer values $-1, 0, 1, 2$. What is the maximum value of the expression

$$S = a_1^3 + a_2^3 + \cdots + a_n^3,$$

if

$$a_1 + a_2 + \cdots + a_n = 61,$$
$$a_1^2 + a_2^2 + \cdots + a_n^2 = 143\ ?$$

A) 265 B) 230 C) 250 D) 270 E) 245

25. Semicircles with diameters AB and BC are drawn inside a rectangle $ABCD$ with $|AB| = 2\,|BC|$. The circles intersect at a point F different from B. If the distance from point F to side DC is 3 cm, what is the area of rectangle $ABCD$?

A) 180 B) 210 C) 270 D) 450 E) 360