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The Effects of Thickness On Frequency Values for Rotating Circular Shells

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Abstract

The aim of the present paper is to investigate effect of thickness on frequency. For this, free vibration analysis of circular shells is made via ANSYS and numerical method. Discrete singular convolution (DSC) and differential quadrature methods have been proposed for numerical solution of vibration problem. The formulations are based on the Love's first approximation shell. The performance of the present methodology is also discussed.

Keywords: Discrete singular convolution, ANSYS, free vibration, frequencies, cylindrical shells.

1. Introduction

Cylindrical shells are widely used in many engineering applications such as mechanical, civil and aerospace engineering. Rotating circular shell structures are increasingly being used in many engineering applications like aviation, rocketry, missiles, chemical, aero-space, civil and mechanical industries. Thus, frequencies and mode shapes of such structures are important in the design of systems [1-4]. As a consequence, a number of analytical and numerical methods have been also studied on the vibration analysis of circular cylindrical shells [5-12]. In this study, free vibration analysis of rotating and non-rotating cylindrical shells is investigated by the method of DSC and DQ approaches. Also, the ANSYS program has been used for some analyses.

2. Fundamental equations

Consider a cylindrical shell rotating about its symmetrical and horizontal axis at an angular velocity ω as shown in Figure 1. The thickness of the shell, and cone length are denoted by h and L, respectively. The cylindrical shell is referred to a coordinate system (x,θ, z) as shown in Figure 1. The components of the deformation of the cylindrical shell with references to this coordinate system are denoted by u, v, w in the x, θ and z directions, respectively.

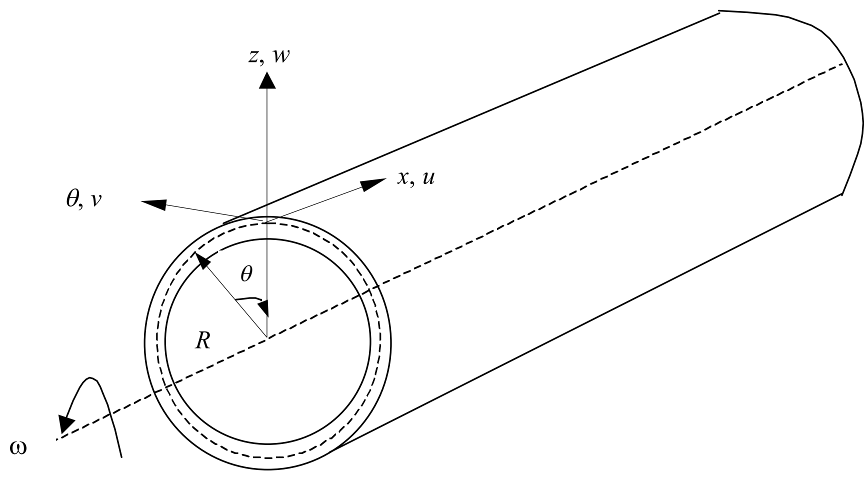

Fig.1. Geometry of a thin rotating cylindrical shell.

Following the Love's first approximation shell theory [12] governing equations for free vibration analysis of cylindrical shells can be given as [39,40];

$$L_{11}u + L_{12}v + L_{13}w - \rho h\frac{\partial^2 u}{\partial t^2} = 0, \tag{1a}$$

$$L_{21}u + L_{22}v + L_{23}w - \rho h\frac{\partial^2 v}{\partial t^2} = 0, \tag{1b}$$

$$L_{31}u + L_{32}v + L_{33}w - \rho h\frac{\partial^2 w}{\partial t^2} = 0. \tag{1c}$$

where

$$L_{11} = A_{11}\frac{\partial^2}{\partial x^2} + \frac{A_{66}}{R^2}\frac{\partial^2}{\partial \theta^2}, \tag{2}$$

$$L_{12} = \frac{(A_{12} + A_{66})}{R}\frac{\partial^2}{\partial x \partial \theta} + \frac{(B_{12} + 2B_{66})}{R^2}\frac{\partial^2}{\partial x \partial \theta} \tag{3}$$

$$L_{13} = \frac{A_{12}}{R}\frac{\partial}{\partial x} - \frac{B_{12} + 2B_{66}}{R^2}\frac{\partial^3}{\partial x \partial \theta^2} - B_{11}\frac{\partial^3}{\partial x^3} \tag{4}$$

$$L_{21} = \frac{(A_{12} + A_{66})}{R}\frac{\partial^2}{\partial x \partial \theta} + \frac{(B_{12} + 2B_{66})}{R^2}\frac{\partial^2}{\partial x \partial \theta} \tag{5}$$

$$L_{22} = A_{66}\left[\frac{\partial^2}{\partial x^2} + \frac{\sin\alpha}{R(x)}\frac{\partial}{\partial x} - \frac{\sin^2\alpha}{R^2(x)}\right] + \left[\frac{A_{22}}{R^2(x)} + \frac{D_{22}\cos^2\alpha}{R^4(x)}\right]\frac{\partial^2}{\partial\theta^2}$$

$$+2\frac{D_{66}\cos^2\alpha}{R^2(x)}\left[\frac{\partial^2}{\partial x^2} - \frac{2\sin\alpha}{R(x)}\frac{\partial}{\partial x} + \frac{2\sin^2\alpha}{R^2(x)}\right] \tag{6}$$

$$L_{23} = \left(\frac{A_{22}\cos\alpha}{R^2(x)} - \frac{4D_{66}\cos\alpha\sin^2\alpha}{R^4(x)}\right)\frac{\partial}{\partial\theta} - \frac{D_{22}\cos\alpha}{R^4(x)}\frac{\partial^3}{\partial\theta^4}$$

$$-\frac{(D_{22}-4D_{66})\sin\alpha\cos\alpha}{R^3(x)}\frac{\partial^2}{\partial x\partial\theta} - \frac{(D_{12}+2D_{66})\cos\alpha}{R^2(x)}\frac{\partial^3}{\partial x^2\partial\theta}, \tag{7}$$

$$L_{31} = -A_{12}\frac{\cos\alpha}{R(x)}\frac{\partial}{\partial x} - A_{22}\frac{\sin\alpha\cos\alpha}{R^2(x)} \tag{8}$$

$$L_{32} = -\left[A_{22}\frac{\cos\alpha}{R^2(x)} - \frac{(2D_{12}+2D_{22}+8D_{66})\cos\alpha\sin^2\alpha}{R^4(x)}\right]\frac{\partial}{\partial\theta}$$

$$+D_{22}\frac{\cos\alpha}{R^4(x)}\frac{\partial^3}{\partial\theta^4} + \frac{(D_{12}+4D_{66})\cos\alpha}{R^2(x)}\frac{\partial^3}{\partial x^2\partial\theta}$$

$$-\left[\frac{(D_{22}+2D_{12}+8D_{66})\sin\alpha\cos\alpha}{R^3(x)}\right]\frac{\partial^2}{\partial x\partial\theta}, \tag{9}$$

$$L_{33} = -A_{22}\frac{\cos^2\alpha}{R^2(x)} - D_{11}\frac{\partial^4}{\partial x^4} - \frac{2(D_{12}+2D_{66})}{R^2(x)}\frac{\partial^4}{\partial x^2\partial\theta^2} - \frac{D_{22}}{R^4(x)}\frac{\partial^4}{\partial\theta^4}$$

$$-\frac{2D_{11}\sin\alpha}{R(x)}\frac{\partial^3}{\partial x^3} + \frac{2(D_{12}+4D_{66})\sin\alpha}{}\frac{\partial^3}{\partial x\partial\theta^2} + \frac{D_{22}\sin^2\alpha}{R^2(x)}\frac{\partial^2}{\partial x^2}$$

$$-\frac{2(D_{12}+(D_{22}+4D_{66})\sin^2\alpha}{R^4(x)}\frac{\partial^2}{\partial\theta^2} - \frac{D_{22}\sin^3\alpha}{R^2(x)}\frac{\partial}{\partial x}. \tag{10}$$

3. Discrete Singular Convolution (DSC)

The discrete singular convolutions (DSC) algorithm was originally introduced by Wei [13]. Since then, applications of the DSC method to various science and engineering problems have been investigated and their successes have demonstrated the potential of the method as an attractive numerical analysis technique [14-20]. In this paper, details of the DSC method are not given; interested readers may refer to the works of [13-17]. Consider a distribution, T and $\eta(t)$ as an element of the space of the test function. A singular convolution can be defined by [15]

$$F(t) = (T * \eta)(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)dx \tag{11}$$

where $T(t-x)$ is a singular kernel. The DSC algorithm can be realized by using many approximation kernels. However, it was shown [21-28] that for many problems, the use of the regularized Shannon kernel (RSK) is very efficient. The RSK is given by [16]

$$\delta_{\Delta,\sigma}(x-x_k) = \frac{\sin[(\pi/\Delta)(x-x_k)]}{(\pi/\Delta)(x-x_k)} \exp\left[-\frac{(x-x_k)^2}{2\sigma^2}\right]; \ \sigma>0 \tag{12}$$

where $\Delta=\pi/(N-1)$ is the grid spacing and N is the number of grid points. The parameter σ determines the width of the Gaussian envelope and often varies in association with the grid spacing, i.e., $\sigma = r$h. In the DSC method, the function $f(x)$ and its derivatives with respect to the x coordinate at a grid point x_i are approximated by a linear sum of discrete values $f(x_k)$ in a narrow bandwidth $[x-x_M, x+x_M]$. This can be expressed as [21]

$$\left.\frac{d^n f(x)}{dx^n}\right|_{x=x_i} = f^{(n)}(x) \approx \sum_{k=-M}^{M} \delta^{(n)}_{\Delta,\sigma}(x_i - x_k) f(x_k); \quad (n=0,1,2,...,) \tag{13}$$

where superscript n denotes the nth-order derivative with respect to x. Beams, plates and shells have been successfully solved via DSC method by this time [22-38]. Also, some macro and nano structures analyzed by DSC [60,61].

4. Differential quadrature (DQ) method

In the DQ method, a partial derivative of a function with respect to a space variable at a discrete point is approximated as a weighted linear sum of the function values at all discrete points in the region of that variable [41-59]. For simplicity, we consider a one-dimensional function $u(x)$ in the [-1,1] domain, and N discrete points. Then the first derivatives at point i, at $x = x_i$ is given by

$$u_x(x_i) = \left.\frac{\partial u}{\partial x}\right|_{x=x_i} = \sum_{j=1}^{N} A_{ij}\, u(x_j)\ ; \qquad i = 1,2,...,N, \tag{14}$$

where x_j are the discrete points in the variable domain, $u(x_j)$ are the function values at these points and A_{ij} are the weighting coefficients for the first order derivative attached to these function values. Two methods can possible to determine the weighting coefficients. The first one is to let equation (1) be exact for the test functions

$$u_k(x) = x^{k-1}\ ; k = 1,2,...,N, \tag{15}$$

which leads to a set of linear algebraic equations

$$(k-1)\,x_i^{k-2} = \sum_{j=1}^{N} A_{ij}\,x_j^{k-1}; \text{ for } i = 1,2,...,N \text{ and } k = 1,2,...,N. \tag{16}$$

which represents N sets of N linear algebraic equations. Another way to determine the weighting coefficients is to employ harmonic functions, named the harmonic differential quadrature (HDQ). Harmonic differential quadrature has been proposed by Striz et al. [22]. Unlike the DQ that uses the polynomial functions, such as power functions, Lagrange interpolated, and Legendre polynomials as the test functions, HDQ uses harmonic or trigonometric functions as the test functions. Shu and Xue proposed an explicit means of obtaining the weighting coefficients for the HDQ [53]. When the $f(x)$ is approximated by a Fourier series expansion in the form [42-50]

$$f(x) = c_0 + \sum_{k=1}^{N/2}\left(c_k \cos\frac{k\pi x}{L} + d_k \sin\frac{k\pi x}{L}\right), \tag{17}$$

and the Lagrange interpolated trigonometric polynomials are taken as [52,53]

$$h_k(x) = \frac{\sin\frac{(x-x_0)\pi}{2}\cdots\sin\frac{(x-x_{k-1})\pi}{2}\sin\frac{(x-x_{k+1})\pi}{2}\cdots\sin\frac{(x-x_N)\pi}{2}}{\sin\frac{(x_k-x_0)\pi}{2}\cdots\sin\frac{(x_k-x_{k-1})\pi}{2}\sin\frac{(x_k-x_{k+1})\pi}{2}\cdots\sin\frac{(x_k-x_N)\pi}{2}} \tag{18}$$

for $k = 0,1,2,....,N$. According to the HDQ, the weighting coefficients of the first-order derivatives A_{ij} for $i \neq j$ can be obtained by using the following formula:

$$A_{ij} = \frac{(\pi/2)P(x_i)}{P(x_j)\sin[(x_i-x_j)/2]\pi}; \qquad i,j = 1,2,3,...,N, \tag{19}$$

where

$$P(x_i) = \prod_{j=1, j\neq i}^{N} \sin\left(\frac{x_i - x_j}{2}\pi\right); \qquad \text{for } j = 1,2,3,...,N. \tag{20}$$

The weighting coefficients of the second-order derivatives B_{ij} for $i \neq j$ can be obtained using the following formula:

$$B_{ij} = A_{ij}\left[2A_{ii}^{(1)} - \pi\cot\left(\frac{x_i - x_j}{2}\right)\pi\right]; \qquad i,j = 1,2,3,...,N, \tag{21}$$

The weighting coefficients of the first-order and second-order derivatives $A_{ij}^{(p)}$ for $i = j$ are given as

$$A_{ii}^{(p)} = -\sum_{j=1, j \neq i}^{N} A_{ij}^{(p)} \; ; \qquad p = 1 \text{ or } 2 \text{ ; and for } i = 1,2,..., N, \tag{22}$$

The weighting coefficient of the third and fourth order derivatives can be computed easily from A_{ij} and B_{ij} by

$$C_{ij} = \sum_{k=1}^{N} A_{ik} B_{kj} , \qquad D_{ij} = \sum_{k=1}^{N} B_{ik} B_{kj} . \tag{23}$$

Two different types of sampling grids are taken into consideration in this study. A natural, and often convenient, choice for sampling points is that of equally spaced grid (ES-G) points. These points are given by,

$$\text{Type-I: } x_i = \frac{i-1}{N_x - 1} \text{ and } y_i = \frac{i-1}{N_y - 1}, \tag{24}$$

in the related directions. Sometimes, the DQ solutions deliver more accurate results with unequally spaced sampling points. Another choice that is found to be even better than the Chebyshev and Legendre polynomials is the set of points proposed by Shu and Richards [52]. These points are given as

$$\text{Type-II: } \quad x_i = \frac{1}{2}\left[1 - \cos\left(\frac{2i-1}{N_x - 1}\right)\pi\right]; \text{ and } \quad y_i = \frac{1}{2}\left[1 - \cos\left(\frac{2i-1}{N_y - 1}\right)\pi\right]. \tag{25}$$

in the x- and y- directions, respectively. These type grid points are known the Chebyshev-Gauss-Lobatto or non-equally spaced grid (NES-G) points. The displacement terms are taken as

$$u = U(x) \cdot \cos(n\theta) \cdot \cos(\varpi t), \tag{26a}$$

$$v = V(x) \cdot \sin(n\theta) \cdot \cos(\varpi t), \tag{26b}$$

$$w = W(x) \cdot \cos(n\theta) \cdot \cos(\varpi t). \tag{26c}$$

where ϖ is referred to as the frequency parameter. Substituting Equations (14) into Equations (1), the governing equations can be written as

$$[G_{ij}]\{D\} = 0 \tag{27}$$

In this study, the numerical results are given by the dimensionless frequency parameter Ω, defined as

$$\Omega = R_2 \sqrt{\frac{\rho h}{A_{11}}} \varpi \ . \tag{28}$$

5. Numerical results

Some results have been presented for rotating and non-rotating shells. Firstly, frequency values for non-rotating shells have been presented in Table 1. The results produced by DSC and DQ are close agreement with the literatures. In this study, we used the classical shell theory. The differences amongst the results occurred from the different shell theory between this study and literature results (3-D elasticity and FSDT). Secondly, the effect of thickness on frequency for rotating shells is investigated and results presented in Table 2. Both the DSC and DQ produced very good results for 11 grid numbers. When we increase the same rate of thickness and length the frequency decreased interestingly. The results depicted in Fig. 2. For this graph the following values have been used : E=68.2 GPa, ρ=2700 kg/m^3, υ=0.33, L=1.7272 m, R=0.0762 m, h=0.00147 m. The results obtained via ANSYS packed programs.

Table 1. Frequency parameters of S-S cylindrical shells (h/ R=0.05;R/L=0.05;m=1)

n	Ref.6	Ref.7	Present DSC	Present DQ
2	0.039233	0.039819	0.039317	0.039319
3	0.109477	0.109898	0.109620	0.109621
4	0.209008	0.210310	0.209975	0.209978

Table 2. Frequency values ($\Omega = \omega R\sqrt{\rho\left(1-\upsilon^2\right)/E}$) of rotating isotropic cylindrical shells (L/R=10; υ=0.3; λ=0.005 rps) with C-C boundary conditions

	Present DSC Result (11×11)		Present DQ Result 11×11	
Mode numbers	h/R=0.02	h/R=0.05	h/R=0.02	h/R=0.05
2	0.04133	0.05409	0.04134	0.05409
3	0.04769	0.11043	0.04772	0.11043
4	0.08143	0.21003	0.08148	0.21001

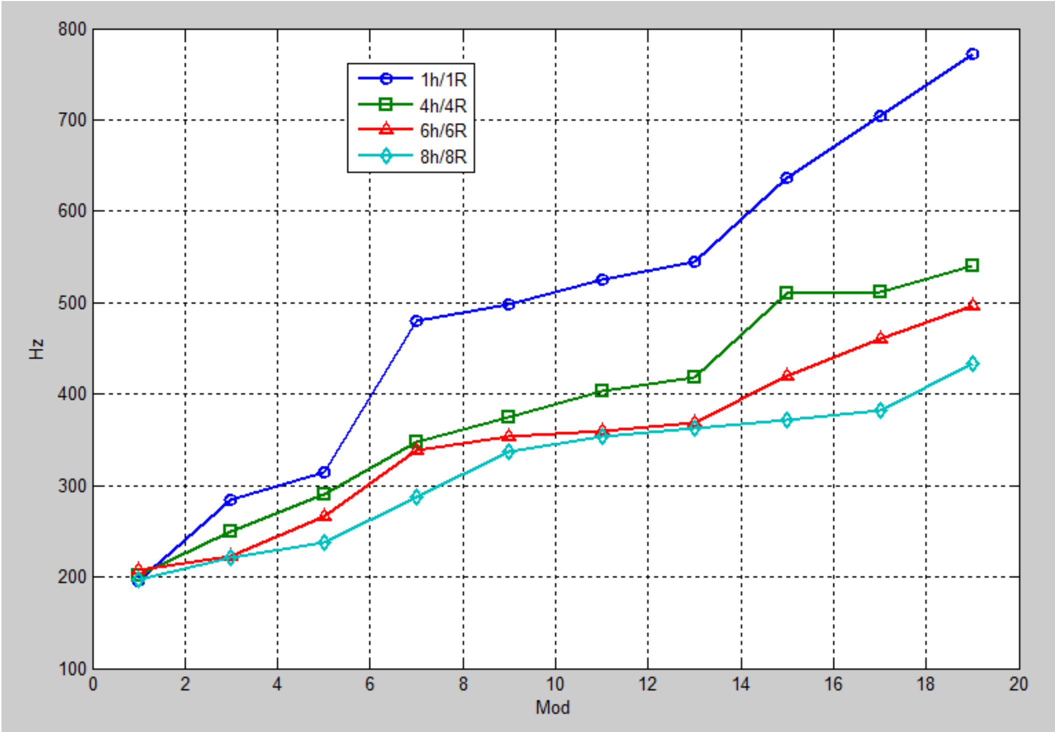

Fig. 2. The effect of thickness on frequency

6. Conclusions

It is shown that, the method of DSC and DQ have capable to give accurate results for rotating or non-rotating shells. The effect of other parameters on frequency have also been under consideration and published in the next.

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