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Thermal Vibration of Zinc Oxide Nanowires by using Nonlocal Finite Element Method

Hayri Metin Numanoğlu

Giresun University, Civil Engineering Department, Division of Mechanics, 28200, Giresun, Turkey
*E-mail address: metin_numanoglu@hotmail.com

ORCID numbers of author:
0000-0003-0556-7850

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Abstract

Zinc oxide nanowires (ZnO NWs) can be used in some NEMS applications due to their remarkable chemical, physical, mechanical and thermal resistance properties. In terms of the suitability of such NEMS organizations, a correct mechanical model and design of ZnO NWs should also be established under different effects. In this study, thermal vibration analyses of elastic beam models of ZnO NWs are examined based on Eringen's nonlocal elasticity theory. The resulting equation of motion is solved with a finite element formulation developed for the atomic size-effect and thermal environment. The vibration frequencies of ZnO NWs with different boundary conditions are calculated under nonlocal parameter and temperature change values and numerical results were discussed.

Keywords: Finite element method, nonlocal elasticity, thermal environment, vibration, Zinc Oxide nanowire.

1. Introduction

It is seen that people use products with stronger physical, chemical, thermal, mechanical, optical, etc. properties. This is possible with the science of nanotechnology that is today's pioneer technology. Nanotechnology is a science that aims to investigate the properties of materials with dimensions from 1 nm to 100 nm and to integrate these materials into classical applications of science and engineering disciplines. It can be stated that nanotechnology, which started its adventure with gave a conference by R. Feynman [1] in 1959, gained a serious importance with the discovery of the carbon nanotube material [2,3]. Additionally, properties of wide range of nanomaterials such as boron nitride nanotube [4], graphene [5] and metallic or molecular nanowires [6-8] are fundamental topics of this discipline. It can be expressed that such nanomaterials show their effect in different applications such as sensor, switch, actuator, bridge, transistor.

The structural-electronic applications containing nanomaterials are generally collected under the name of nanoelectromechanical systems (NEMS). To perform the accurate mechanical analyses of NEMS is essential for NEMS applications to work properly in terms of engineering. To perform mechanical analysis via experimental methods requires high operation costs, professional expert approaches and long processes. Also, it is a well-known fact that the results obtained by experimental methods do not present results in accordance with the classical elasticity theory. These difficulties have been overcome by adapting the mathematical approaches developed in different periods to the classical elasticity theory. The new elasticity theories, namely, higher-order continuum theories, generally include parameters related to the atomic dimensions of nanomaterials. It can be said that nonlocal elasticity theory [9-10], couple stress elasticity theory [11,12], strain gradient elasticity theory [13,14], surface energy elasticity theory [15,16] and doublet mechanics elasticity theory [17] exemplify for higher-order continuum theories.

The nonlocal elasticity theory states that the stress and strain of other regions adjacent to that region must also be taken into consideration in order to calculate the stress and strain in a certain region of the atomic structure. Thus, the uncertainty in the strain energy that goes to infinity due to atomic factors is resolved. In the 1960s, the studies of Eringen et al. enabled the establishment of the nonlocal elasticity theory and the determination of its main principles. It can be stated that approximately 45 years later, analyses of continuous mechanical models of nanoscaled structures started to be handled with the nonlocal elasticity theory [18-20]. Following these, vibration, buckling and bending analyses of nonlocal Euler–Bernoulli nano beams are given [21-23]. Lu et al. studied the nonlocal vibration phase velocities of single and multi-walled carbon nanotubes by using Euler-Bernoulli and Timoshenko beam theories [24]. Numanoğlu examined axial and flexural vibration analyses of different nanowires and nanotubes [25]. Axial and torsional vibration analyses of nonlocal nanorods are also available in the literature [26-32]. Jalaei and Civalek studied the nonlocal elasticity dynamic instability of functionally graded porous beam under magnetic effects resting on viscoelastic foundation by employing Navier's technique and Bolotins's approach [33]. Apart from these, vibration and bending of some nanomaterials are tackled based on the classical theory [34-36]. Civalek presented the finite element formulations of plates and shells [37]. On the other hand, it can be stated that studies on the use of finite element formulation in mechanical analysis of nanostructures with nonlocal elasticity have taken place in the literature [27,28,38-52]. Additionally, the free vibration behavior of a functionally graded beam is researched for Euler-Bernoulli, Timoshenko, Shear and Rayleigh beam theories [53]. Moreover, mechanical analyses of different continuous structures have been performed via novel numerical approaches such as discrete singular convolution and differential quadrature [54-60].

In this article, vibration analyses of nanobeams modeled by using zinc oxide nanowires (ZnO NWs), which has an important area in the applications of nanotechnology science, are carried out with the nonlocal elasticity theory. The temperature effect is considered in the vibration analysis. A nonlocal finite element formulation (NL-FEM) is presented for the solution of equation of motion. Then, the vibration frequencies of simply supported ZnO NWs are calculated via analytical method and NL-FEM and compared. Also, thermal vibration frequency results are presented by using NL-FEM for beam models with boundary condition that is not possible to be solve analytically. In the solution of nonlocal free vibration, the accuracy of the proposed formulation is discussed. Finally, the most general results are summarized.

2. Nonlocal Finite Element Analysis for Thermal Vibration of Nanobeams

The equation of motion of nonlocal thermal vibration of nano scaled beams according to Euler-Bernoulli beam theory can be presented as follows:

$$\left[EI-(e_0a)^2 EA\alpha\Delta T\right]\frac{\partial^4 w}{\partial x^4}+EA\alpha\Delta T\frac{\partial^2 w}{\partial x^2}-f+\rho A\frac{\partial^2 w}{\partial t^2}-(e_0a)^2\rho A\frac{\partial^4 w}{\partial x^2\partial t^2}$$
$$+(e_0a)^2\frac{\partial^2 f}{\partial x^2}=0 \tag{1}$$

where EI is bending rigidity, e_0a is nonlocal parameter and EA is axial rigidity. α defines the thermal expansion coefficient. ΔT is temperature change and w is transverse displacement. On the other hand, ρA explains volume of unit length and f is transverse distributed force.

The solution of Eq. (1) will be performed in this current study by using finite element. The fundamental of this solution based on weighted residual method [49]. According to this, average weighted residue is written as

$$\mathrm{I}=\int_0^l h\left(\left[EI-(e_0a)^2 EA\alpha\Delta T\right]\frac{\partial^4 w}{\partial x^4}+EA\alpha\Delta T\frac{\partial^2 w}{\partial x^2}-f+\rho A\frac{\partial^2 w}{\partial t^2}-(e_0a)^2\rho A\frac{\partial^4 w}{\partial x^2\partial t^2}\right.$$
$$\left.+(e_0a)^2\frac{\partial^2 f}{\partial x^2}=0\right)\mathrm{d}x \tag{2}$$

here, h is weighting function and l is length of finite element. The transverse motion of bending finite element is described as

$$w=\phi\mathbf{w} \tag{3}$$

where ϕ is shape function of beam finite element and $\mathbf{w}$ is displacement vector. Additionally, the first derivation of displacement of bending finite element can be written as

$$\frac{\partial w}{\partial x}=\mathrm{D}^{\mathrm{k}}w=\mathbf{Bw} \tag{4}$$

where $\mathrm{D}^{\mathrm{k}}\phi=\mathbf{B}$ and D^{k} is defined as kinematic operator.

The partial integrations of all terms seen in Eq. (2) can be written as

$$\mathrm{I}_1=\int_0^l EIh\frac{\partial^4 w}{\partial x^4}\mathrm{d}x=EIh\frac{\partial^3 w}{\partial x^3}\bigg|_0^l-EI\frac{\partial h}{\partial x}\frac{\partial^2 w}{\partial x^2}\bigg|_0^l+\int_0^l EI\frac{\partial^2 h}{\partial x^2}\frac{\partial^2 w}{\partial x^2}\mathrm{d}x,$$
$$\mathrm{I}_2=\int_0^l (e_0a)^2 EA\alpha\Delta Th\frac{\partial^4 w}{\partial x^4}\mathrm{d}x=(e_0a)^2 EA\alpha\Delta Th\frac{\partial^3 w}{\partial x^3}\bigg|_0^l-(e_0a)^2 EA\alpha\Delta T\frac{\partial h}{\partial x}\frac{\partial^2 w}{\partial x^2}\bigg|_0^l$$
$$+\int_0^l (e_0a)^2 EA\alpha\Delta T\frac{\partial^2 h}{\partial x^2}\frac{\partial^2 w}{\partial x^2}\mathrm{d}x,$$
$$\mathrm{I}_3=\int_0^l EA\alpha\Delta Th\frac{\partial^2 w}{\partial x^2}\mathrm{d}x=EA\alpha\Delta Th\frac{\partial w}{\partial x}\bigg|_0^l-\int_0^l EA\alpha\Delta T\frac{\partial h}{\partial x}\frac{\partial w}{\partial x}\mathrm{d}x,$$

$$\mathrm{I}_4=\int_0^l hf\,\mathrm{d}x,\ \mathrm{I}_5=\int_0^l \rho A h\frac{\partial^2 w}{\partial t^2}\mathrm{d}x,$$

$$\mathrm{I}_6=\int_0^l (e_0a)^2\rho A h\frac{\partial^4 w}{\partial x^2\partial t^2}\mathrm{d}x=(e_0a)^2\rho A h\frac{\partial^3 w}{\partial x\partial t^2}\bigg|_0^l+\int_0^l (e_0a)^2\rho A\frac{\partial h}{\partial x}\frac{\partial^3 w}{\partial x\partial t^2}\mathrm{d}x,$$

$$\mathrm{I}_7=\int_0^l (e_0a)^2 h\frac{\partial^2 f}{\partial x^2}\mathrm{d}x=(e_0a)^2 h\frac{\partial f}{\partial x}\bigg|_0^l-\int_0^l (e_0a)^2\frac{\partial h}{\partial x}\frac{\partial f}{\partial x}\mathrm{d}x, \tag{5}$$

If above equations are substituted into Eq. (2) and weighted residual is evanished, the weak formulation is attained as follows

$$\int_0^l EI\frac{\partial^2 h}{\partial x^2}\frac{\partial^2 w}{\partial x^2}\mathrm{d}x-\int_0^l (e_0a)^2 EA\alpha\Delta T\frac{\partial^2 h}{\partial x^2}\frac{\partial^2 w}{\partial x^2}\mathrm{d}x-\int_0^l EA\alpha\Delta T\frac{\partial h}{\partial x}\frac{\partial w}{\partial x}\mathrm{d}x$$
$$-\int_0^l hf\,\mathrm{d}x+\int_0^l \rho A h\frac{\partial^2 w}{\partial t^2}\mathrm{d}x+\int_0^l (e_0a)^2\rho A\frac{\partial h}{\partial x}\frac{\partial^3 w}{\partial x\partial t^2}-\int_0^l (e_0a)^2\frac{\partial h}{\partial x}\frac{\partial f}{\partial x}\mathrm{d}x=0 \tag{6}$$

To rearrange Eq. (6), following expressions can be used:

$$h=\phi^{\mathrm{T}},\quad \frac{\partial h}{\partial x}=\left(\phi^{\mathrm{T}}\right)'=\mathbf{B}^{\mathrm{T}},\quad \frac{\partial^2 w}{\partial x^2}=\mathbf{B}'\mathbf{w},\quad \frac{\partial^2 w}{\partial t^2}=\phi\ddot{\mathbf{w}} \tag{7}$$

Substituting of Eq. (7) into Eq. (6) yields following equation

$$\int_0^l EI\left(\mathbf{B}'^{\mathrm{T}}\mathbf{B}'\right)\mathbf{w}\mathrm{d}x-\int_0^l (e_0a)^2 EA\alpha\Delta T\left(\mathbf{B}'^{\mathrm{T}}\mathbf{B}'\right)\mathbf{w}\mathrm{d}x-\int_0^l EA\alpha\Delta T\left(\mathbf{B}^{\mathrm{T}}\mathbf{B}\right)\mathbf{w}\mathrm{d}x-\int_0^l \phi^{\mathrm{T}} f\mathrm{d}x$$
$$+\int_0^l \rho A\left(\phi^{\mathrm{T}}\phi\right)\ddot{\mathbf{w}}\mathrm{d}x+\int_0^l (e_0a)^2\rho A\left(\mathbf{B}^{\mathrm{T}}\mathbf{B}\right)\ddot{\mathbf{w}}\mathrm{d}x-\int_0^l (e_0a)^2\mathbf{B}^{\mathrm{T}} f'\mathrm{d}x=0 \tag{8}$$

this equation can be written as follows in the matrix form:

$$\left(K-K_{T,c}-K_{T,nl}\right)\mathbf{w}+\left(M_c+M_{nl}\right)\ddot{\mathbf{w}}=\mathbf{f}_c+\mathbf{f}_{nl} \tag{9}$$

In here,

$$K=\int_0^l EI\left(\mathbf{B}'^{\mathrm{T}}\mathbf{B}'\right)\mathrm{d}x=\int_0^l EI\begin{Bmatrix}\phi_1''\\ \phi_2''\\ \phi_3''\\ \phi_4''\end{Bmatrix}\left[\phi_1''\quad \phi_2''\quad \phi_3''\quad \phi_4''\right]\mathrm{d}x=\frac{EI}{l^3}\begin{bmatrix}12 & 6l & -12 & 6l\\ 6l & 4l^2 & -6l & 2l^2\\ -12 & -6l & 12 & -6l\\ 6l & 2l^2 & -6l & 4l^2\end{bmatrix} \tag{10}$$

$$K_{T,c}=\int_0^l EA\alpha\Delta T\left(\mathbf{B}^{\mathrm{T}}\mathbf{B}\right)\mathrm{d}x=\int_0^l EA\alpha\Delta T\begin{Bmatrix}\phi_1'\\ \phi_2'\\ \phi_3'\\ \phi_4'\end{Bmatrix}\begin{bmatrix}\phi_1' & \phi_2' & \phi_3' & \phi_4'\end{bmatrix}\mathrm{d}x$$

$$=\frac{EA\alpha\Delta T}{30L}\begin{bmatrix}36 & 3l & -36 & 3l\\ 3l & 4l^2 & -3l & -l^2\\ -36 & -3l & 36 & -3l\\ 3l & -l^2 & -3l & 4l^2\end{bmatrix} \tag{11}$$

$$K_{T,nl}=\int_0^l (e_0a)^2 EA\alpha\Delta T\left(\mathbf{B}'^{\mathrm{T}}\mathbf{B}'\right)\mathrm{d}x=\int_0^l (e_0a)^2 EA\alpha_L\Delta T\begin{Bmatrix}\phi_1''\\ \phi_2''\\ \phi_3''\\ \phi_4''\end{Bmatrix}\begin{bmatrix}\phi_1'' & \phi_2'' & \phi_3'' & \phi_4''\end{bmatrix}\mathrm{d}x$$

$$=\frac{(e_0a)^2 EA\alpha_L\Delta T}{l^3}\begin{bmatrix}12 & 6l & -12 & 6l\\ 6L & 4l^2 & -6l & 2l^2\\ -12 & -6l & 12 & -6l\\ 6L & 2l^2 & -6l & 4l^2\end{bmatrix} \tag{12}$$

$$M_c=\int_0^l \rho A\left(\phi^{\mathrm{T}}\phi\right)\mathrm{d}x=\int_0^l \rho A\begin{Bmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\end{Bmatrix}\begin{bmatrix}\phi_1 & \phi_2 & \phi_3 & \phi_4\end{bmatrix}\mathrm{d}x=\frac{\rho Al}{420}\begin{bmatrix}156l & 22l^2 & 54l & -13l^2\\ 22l^2 & 4l^3 & 13l^3 & -3l^3\\ 54l & 13l^2 & 156l & -22l^2\\ -13l & -3l^3 & -22l^2 & 4l^3\end{bmatrix} \tag{13}$$

$$M_{nl}=\int_0^l (e_0a)^2\rho A\left(\mathbf{B}^{\mathrm{T}}\mathbf{B}\right)\mathrm{d}x=\int_0^l (e_0a)^2\rho A\begin{Bmatrix}\phi_1'\\ \phi_2'\\ \phi_3'\\ \phi_4'\end{Bmatrix}\begin{bmatrix}\phi_1' & \phi_2' & \phi_3' & \phi_4'\end{bmatrix}\mathrm{d}x$$

$$=\frac{(e_0a)^2\rho A}{30l}\begin{bmatrix}36 & 3l & -36 & 3l\\ 3l & 4l^2 & -3l & -l^2\\ -36 & -3l & 36 & -3l\\ 3l & -l^2 & -3l & 4l^2\end{bmatrix} \tag{14}$$

$$\mathbf{f}_c=\int_0^l f\phi^{\mathrm{T}}\mathrm{d}x=\int_0^l f\begin{Bmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\end{Bmatrix}\mathrm{d}x=\frac{l}{12}f\begin{Bmatrix}6\\ l\\ 6\\ -l\end{Bmatrix} \tag{15}$$

$$\mathbf{f}_{nl} = \int_0^l (e_0a)^2 f' \mathbf{B}^{\mathrm{T}} \mathrm{d}x = \int_0^l (e_0a)^2 f' \begin{Bmatrix} \phi_1' \\ \phi_2' \\ \phi_3' \\ \phi_4' \end{Bmatrix} \mathrm{d}x = (e_0a)^2 f' \begin{Bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{Bmatrix} \tag{16}$$

where K is bending stiffness matrix. $K_{T,c}$ and $K_{T,nl}$ state the local and nonlocal negative stiffness matrices originating from temperature change, respectively. On the other hand, M_c and M_{nl} are local and nonlocal mass matrices, respectively. $\mathbf{f}_c$ and $\mathbf{f}_{nl}$ express local and nonlocal external force vectors, respectively.

If the $f = 0$ is taken for free vibration and $w(x,t) = W(x)\sin(\omega t - \alpha)$ expression is utilized into Eq. (9), the eigenvalue formulation of finite element analysis is obtained as follows:

$$\det\left(\sum[K] - \omega^2 \sum[M]\right) = 0 \tag{17}$$

where $\sum[K]$ and $\sum[M]$ are total stiffness and mass matrices. ω is natural frequency of nanobeam.

Also, the frequency equation of simply supported beams can be solved analytically. According to this, the series expansion as follows, ensures geometric and mechanical boundary conditions of simply supported beams:

$$w(x,t) = \sum_{n=1}^{\infty} W_n \left(\frac{n\pi x}{L}\right) \sin(\omega t - \alpha) \tag{18}$$

where W_n is unknown series coefficient, n is mode number, L is length of nanobeam. ω explains the natural frequency of nanobeam. Additionally, t is time and α is phase angle. Using Eq. (18) into Eq. (1), the following expression can be obtained

$$m_1\left(\frac{n\pi}{L}\right)^4 + m_2\left(\frac{n\pi}{L}\right)^2 + m_3 = 0 \tag{19}$$

Where

$$m_1 = EI - (e_0a)^2 EA\alpha\Delta T, \quad m_2 = -EA\alpha\Delta T - \omega^2 (e_0a)^2 \rho A, \quad m_3 = -\omega^2 \rho A \tag{20}$$

Substituting of Eq. (20) into Eq. (19) yields the natural frequency equation of simply supported nano beams for nonlocal parameter and temperature change:

$$\omega^2 = \frac{\left[EI - (e_0a)^2 EA\alpha\Delta T\right]\left(\frac{n\pi}{L}\right)^4 - EA\alpha\Delta T\left(\frac{n\pi}{L}\right)^2}{\rho A\left[(e_0a)^2\left(\frac{n\pi}{L}\right)^2 + 1\right]} \tag{21}$$

3. Numerical Examples

In this section, vibration frequencies are calculated for thermal vibration analysis of ZnO NWs. The numerical results are given for simply supported (S-S), cantilever (C-F), propped cantilever (C-S) and clamped supported (C-C) boundary conditions. In order to include the nano scale effect in the analysis, nonlocal elasticity theory is considered. Mechanical properties are taken as follows in the thermal vibration analysis: modulus of elasticity $E = 58\,\text{GPa}$ [61], unit volume mass $\rho = 5600\,\text{kg/m}^3$ [62] and thermal expansion coefficient $\alpha = 2.9\times10^{-6}\,\text{K}^{-1}$ [63]. Additionally, the geometric features are chosen as follows: beam length $L = 20\,\text{nm}$ and circular cross-section diameter $d = 2\,\text{nm}$. On the other hand, 20 finite elements are used for nonlocal finite element analyses.

In Table 1, the first three mode vibration frequencies of simply supported beams modeled with ZnO NWs are calculated and compared with analytical and finite elements for different nondimensional nonlocal parameter values. In addition, the frequencies of the beams not under temperature change were compared with frequencies of the beams under temperature change. First of all, nonlocal expression is a parameter that reduces classical vibration frequencies. By the increase of this value reveals, the frequencies of nanoscaled beams more decrease. Also, temperature change decreases the frequencies of ZnO NWs. In the case that the nonlocal parameter is higher, the temperature factor is more influential. On the other hand, it is seen that the values obtained by the finite element method are very close to the analytically calculated frequencies. In general, while the increase in the mode number raises the difference between calculated values by using the analytical method and NL-FEM, the increase of nonlocal parameter decreases this difference.

In Table 2, the first three mode frequencies of ZnO NWs are tabulated for three different boundary conditions and temperature change. Analytical vibration analysis for boundary conditions except S-S is not possible in case of nonlocal elasticity. Also, when it is considered that the temperature parameter is included in the analysis, an alternative to the analytical method has to be used and therefore the analyses are given only with the finite element formulation. When the stiffness states between the boundary conditions are compared, it can be said that the results obtained are reasonable. The frequencies of the clamped supported beams are the highest, while the frequencies of the cantilever beams are the lowest. Additionally, the boundary condition in which the nonlocal parameter has the highest effect is C-C.

Table 1. Comparison of the first three modes flexural frequencies (GHz) of simply supported Zinc Oxide nanowires.

Nonlocal parameter	Mode Number	$\Delta T = 0\,\text{K}$		$\Delta T = 300\,\text{K}$	
		Analytical	NL-FEM	Analytical	NL-FEM
$e_0a/L = 0$	1	6.3190	6.3190	5.8565	5.8565
	2	25.2761	25.2763	24.8265	24.8266
	3	56.8712	56.8731	56.4238	56.4258
$e_0a/L = 0.15$	1	5.7161	5.7161	5.2002	5.2002
	2	18.3941	18.3942	17.7712	17.7713
	3	32.8423	32.8434	32.0614	32.0625
$e_0a/L = 0.35$	1	4.2516	4.2516	3.5276	3.5276
	2	10.4628	10.4629	9.3244	9.3244
	3	16.4991	16.4997	14.8841	14.8846

Table 2. The first three modes flexural frequencies (GHz) of Zinc Oxide nanowires with different boundary conditions under temperature change.

Nonlocal parameter	Mode Number	Boundary Conditions ($\Delta T = 300\ \mathrm{K}$)		
		C-F	C-S	C-C
$e_0 a/L = 0$	1	1.5191	9.5326	14.0772
	2	13.4342	31.6055	39.1525
	3	38.9400	66.3446	77.0480
$e_0 a/L = 0.15$	1	1.3875	8.3462	12.2447
	2	10.0411	22.1828	26.9237
	3	23.5060	37.0263	42.2733
$e_0 a/L = 0.35$	1	0.9311	5.5268	8.0226
	2	5.6495	11.5262	13.7018
	3	11.6736	17.1731	19.6303

4. Conclusions

In this study, a vibration analysis is performed for elastic beam models of ZnO NWs based on the nonlocal elasticity theory. It is also thought that the beams are under the influence of temperature change. Finite element formulation is used to solve the equation of motion. With this formulation, frequencies of different vibration modes of ZnO NW beams with different boundary conditions are calculated under nondimensional nonlocal parameter and temperature change values and the results are discussed.

In general, it is understood that the atomic scale effect and ambient temperature are definitely factors to be taken into account in the dynamic analysis of continuous models of nanoscale structures. In addition, it is concluded that the use of finite element formulation based on the size effect is an important way for the cases where dynamic analysis cannot be performed by analytical methods. It is thought that these results will guide the proper and optimum structural designs of NEMS using ZnO NWs.

References

[1] Feynman, R.P., There’s plenty of room at the bottom. *Engineering and Science*, 23, 22-36, 1960.

[2] Iijima, S., Helical microtubules of graphitic carbon. *Nature*, 354, 56-58, 1991.

[3] Iijima, S., Ichihashi, T., Single-shell carbon nanotubes of 1-nm diameter. *Nature*, 363, 603-605, 1993.

[4] Chopra, N.G., Zettl, A., Measurement of the elastic modulus of a multi-wall boron nitride nanotube. *Solid State Communications*, 105, 297-300, 1997.

[5] Zhu, Y., Murali, S., Cai, W., Li, Suk, J.W., Potts, J.R., Ruoff, R.S. Graphene and Graphene Oxide: Synthesis, Properties, and Applications, *Advanced Materials*, 22, 2010.

[6] Chen, K.I., Li, B.R., Chen, Y.T., Silicon nanowire field-effect transistor-based biosensors for biomedical diagnosis and cellular recording investigation. *Nano Today*, 6, 131-154, 2011.

[7] Liu, Y.Y., Wang, X.Y., Cao, Y., Chen, X.D., Xie, S.F., Zheng, X.J., Zeng, H.D., A flexible blue light-emitting diode based on ZnO nanowire/polyaniline heterojunctions. *Journal of Nanomaterials*, 870254, 2013.

[8] Zhang, P., Wyman, I., Hu, J., Lin, S., Zhong, Z., Tu, Y., Huang, Z., Wei, Y., Silver nanowires: Synthesis Technologies, growth mechanism and multifunctional applications, *Materials Science and Engineering B*, 223, 1–23, 2017.

[9] Eringen, A.C., Edelen, D.G.B., On nonlocal elasticity. *International Journal of Engineering Science*, 10, 233-248, 1972.

[10] Eringen, A.C., On differential equations of non local elasticity and solutions of screw dislocation and surface waves. *Journal of Applied Physics*, 54, 4703, 1983.

[11] Toupin, R.A., Elastic materials with couple-stresses. *Archive for Rational Mechanics and Analysis*, 11, 385-414. 1962.

[12] Koiter, W.T., Couple stresses in the theory of elasticity. *I & II. Philosophical Transactions of the Royal Society of London B*, 67, 17-44, 1964.

[13] Yang, F., Chong, A.C.M., Lam, D.C.C., Tong, P., Couple stress based strain gradient theory for elasticity. *International Journal of Solids and Structures*, 39, 2731-2743, 2002.

[14] Akgöz, B., Civalek, Ö., A size-dependent beam model for stability of axially loaded carbon nanotubes surrounded by Pasternak elastic foundation. *Composite Structures*, 176, 1028-1038, 2017.

[15] Gurtin, M.E., Murdoch, A.I., A continuum theory of elastic material surfaces. *Archive for Rational Mechanics and Analysis*, 57, 291-323, 1975.

[16] Gurtin, M.E., Murdoch, A.I., Surface stress in solids. *International Journal of Solids and Structures*, 14, 431-440. 1978.

[17] Granik, V.T., Ferrari, J.W., Microstructural mechanics of granular media. *Mechanics of Materials*, 15.301-322, 1993.

[18] Sudak, L.J., Column buckling of multiwalled carbon nanotubes using nonlocal continuum mechanics. Journal of Applied Physics, 94, 7281-7287, 2003.

[19] Wang, Q., Liew, K.M., Application of nonlocal continuum mechanics to static analysis of micro- and nano-structures. *Physics Letters A*, 363, 236-242, 2007.

[20] Wang, Q., Varadan, V.K., Vibration of carbon nanotubes studied using nonlocal continuum mechanics. *Smart Materials and Structures*, 15, 659, 2006.

[21] Reddy, J.N., Nonlocal theories for bending, buckling and vibration of beams. *International Journal of Engineering Science*, 45, 288-307, 2007.

[22] Reddy, J.N., Pang, S.D., Nonlocal continuum theories of beams for the analysis of carbon nanotubes. *Journal of Applied Physics*, 103, 023511, 2008.

[23] Ghannadpour, S.A.M., Mohammadi, B., Fazilati, J., Bending, buckling and vibration problems of nonlocal Euler beams using Ritz method. *Composite Structures*, 96, 584-589, 2013.

[24] Lu, P., Lee, H.P., Lu, C., Zhang, P.Q., Application of nonlocal beam models for carbon nanotubes. *International Journal of Solids and Structures*, 44, 5289-5300, 2007.

[25] Numanoğlu, H.M., Vibration analysis of beam and rod models of nanostructures based on nonlocal elasticity theory (In Turkish). BSc. Thesis, Akdeniz University, Antalya, 2017.

[26] Aydogdu, M., Axial vibration of the nanorods with the nonlocal continuum rod model. *Physica E: Low-dimensional Systems and Nanostructures*, 41, 861-864, 2009.

[27] Demir, Ç., Civalek, Ö., Torsional and longitudinal frequency and wave response of microtubules based on the nonlocal continuum and nonlocal discrete models. *Applied Mathematical Modelling*, 37, 9355-9367, 2013.

[28] Lim, C.W., Islam, M.Z., Zhang, G., A nonlocal finite element method for torsional statics and dynamics of circular nanostructures. *International Journal of Mechanical Sciences*, 94-95, 232-243, 2015.

[29] Li, X.-F., Shen, Z.B., Lee, K.Y., Axial wave propagation and vibration of nonlocal nanorods with radial deformation and inertia. *ZAMM Journal of Applied Mathematics and Mechanics: Zeitschrift für Angewandte Mathematik und Mechanik*, 97, 602-616, 2017.

[30] Yayli, M.Ö., On the torsional vibrations of restrained nanotubes embedded in an elastic medium. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 40, 419, 2018.

[31] Numanoğlu, H.M., Akgöz, B., Civalek, Ö., On dynamic analysis of nanorods. *International Journal of Engineering Science*, 130, 33-50, 2018.

[32] Karlicic, D.Z., Ayed, S., Flaieh, E., Nonlocal axial vibration of the multiple Bishop nanorod system. *Mathematics and Mechanics of Solids*, 24, 1668-1691, 2018.

[33] Jalaei, M., Civalek, Ö., On dynamic instability of magnetically embedded viscoelastic porous FG nanobeam. *International Journal of Engineering Science*, 143, 14-32, 2019.

[34] Numanoğlu, H.M., Mercan, K., Civalek, Ö., Frequency and mode shapes of Au nanowires using the continuous beam models. *International Journal of Engineering and Applied Sciences*, 9, 55-61, 2017.

[35] Uzun, B., Civalek, Ö., Carbon nanotube beam model and free vibration analysis. *International Journal of Engineering & Applied Sciences*, 10, 1-4, 2018.

[36] Numanoğlu, H.M., Civalek, Ö., Elastic beam model and bending analysis of silver nanowires. *International Journal of Engineering and Applied Sciences*, 10, 13-20, 2018.

[37] Civalek, Ö., Finite Element analysis of plates and shells. Fırat University, Elazığ, 1998.

[38] Adhikari, S., Murmu, T., McCarthy, M.A., Dynamic finite element analysis of axially vibrating nonlocal rods. *Finite Elements in Analysis and Design*, 630, 42-50, 2013.

[39] Adhikari, S., Murmu, T., McCarthy, M.A., Frequency domain analysis of Nonlocal rods embedded in an elastic medium. *Physica E: Low-dimensional Systems and Nanostructures*, 59, 33-40, 2014.

[40] Eltaher, M.A., Alshorbagy, A.E., Mahmoud, F.F., Vibration analysis of Euler–Bernoulli nanobeams by using finite element method. *Applied Mathematical Modelling*, 37, 4787-4797, 2013.

[41] Pradhan, S.C., Mandal, U., Finite element analysis of CNTs based on nonlocal elasticity and Timoshenko beam theory including thermal effect. *Physica E: Low-dimensional Systems and Nanostructures*, 53, 223-232, 2013

[42] Civalek, Ö., Demir, C., A simple mathematical model of microtubules surrounded by an elastic matrix by nonlocal finite element method. *Applied Mathematics and Computation*, 289, 335-352, 2016.

[43] Demir, Ç., Civalek, Ö., A new nonlocal FEM via Hermitian cubic shape functions for thermal vibration of nano beams surrounded by an elastic matrix. Composite Structures, 168, 872-884, 2017.

[44] Işik Ç., Mercan K., Numanoğlu H.M., Civalek Ö., Bending response of nanobeams resting on elastic foundation. *Journal of Applied and Computational Mechanics*, 4, 105-114, 2017.

[45] Uzun B., Numanoğlu H.M., Civalek Ö., Free vibration analysis of BNNT with different cross-sections via nonlocal FEM. *Journal of Computational Applied Mechanics*, 49, 252-260, 2018.

[46] Numanoğlu H.M., Uzun, B., Civalek, Ö., Derivation of nonlocal finite element formulation for nano beams. *International Journal of Engineering and Applied Sciences*, 10, 131-139, 2018.

[47] Numanoğlu H.M., Civalek Ö., On the dynamics of small-sized structures. *International Journal of Engineering Science*, 145, 103164, 2019.

[48] Numanoğlu H.M., Civalek Ö., On the torsional vibration of nanorods surrounded by elastic matrix via nonlocal FEM. *International Journal of Mechanical Sciences*, 161-162, 105076, 2019.

[49] Numanoğlu, H.M., Dynamic analysis of nano continuous and discrete structures based on nonlocal finite element formulation (NL-FEM) (In Turkish). MSc. Thesis, Akdeniz University, Antalya, 2019.

[50] Civalek, Ö., Numanoğlu H.M., Nonlocal finite element analysis for axial vibration of embedded Love–Bishop nanorods. *International Journal of Mechanical Sciences*, 188, 105939, 2020.

[51] Uzun, B., Civalek, O., Nonlocal FEM formulation for vibration analysis of nanowires on elastic matrix with different materials. *Mathematical and Computational Applications*. 24, 38, 2019.

[52] Civalek, O., Uzun, B., Yaylı, M.O., Akgöz, B., Size-dependent transverse and longitudinal vibrations of embedded carbon and silica carbide nanotubes by nonlocal finite element method. *European Physical Journal Plus*, 135, 381, 2020.

[53] AlSaid-Alwan, H.H.S., Avcar, M., Analytical solution of free vibration of FG beam utilizing different types of beam theories: A comparative study. *Computers and Concrete*, 26, 285-292, 2020.

[54] Civalek, Ö., Kiracioglu, O., Free vibration analysis of Timoshenko beams by DSC method. *International Journal for Numerical Methods in Biomedical Engineering*, 26, 1890-1898, 2010.

[55] Civalek, O., Yavas, A., Large deflection static analysis of rectangular plates on two parameter elastic foundations. *International Journal of Science and Technology*, 1, 43-50, 2006.

[56] Civalek, Ö., Geometrically non-linear static and dynamic analysis of plates and shells resting on elastic foundation by the method of polynomial differential quadrature (PDQ) (In Turkish). PhD Thesis, Fırat University, Elazığ, 2004.

[57] Mercan, K., Demir, Ç., Civalek, Ö., Vibration analysis of FG cylindrical shells with power-law index using discrete singular convolution technique. *Curved and Layered Structures* 3, 82-90, 2016.

[58] Civalek, Ö., Geometrically nonlinear dynamic and static analysis of shallow spherical shell resting on two-parameters elastic foundations. *International Journal of Pressure Vessels and Piping*, 113, 1-9, 2014.

[59] Gurses, M., Akgoz, B., Civalek, O., Mathematical modeling of vibration problem of nano-sized annular sector plates using the nonlocal continuum theory via eight-node discrete singular convolution transformation. *Applied Mathematics and Computation*, 219, 3226-3240, 2012.

[60] Civalek, Ö., Avcar, M., Free vibration and buckling analyses of CNT reinforced laminated non-rectangular plates by discrete singular convolution method. *Engineering with Computers*, 2020.

[61] Huang, Y., Bai, X., Zhang, Y., In situ mechanical properties of individual ZnO nanowires and the mass measurement of nanoparticles. *Journal of Physics: Condensed Matter*, 18, L179, 2006.

[62] Zinc Oxide Nanowires. (05.08.2020) https://www.americanelements.com/zinc-oxide-nanowires-1314-13-2. 2019.

[63] Shrama, S.K, Saurakhiya, N., Barthwal, S., Kumar, R., Sharma, A., Tuning of structural, optical, and magnetic properties of ultrathin and thin ZnO nanowire arrays for nano device applications. *Nanoscale Research Letters*, 9, 122, 2014.