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Nonlocal Vibration Analysis for Micro/Nano Beam on Winkler Foundation via DTM

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Abstract

In the present study, vibration of micro/nano beams on Winkler foundation is studied using Eringen's nonlocal elasticity theoy. Hamilton's principle is employed to derive the governing equations. Differential transform method is used to obtain result. Simply supported and clamped–clamped boundary conditions are used to study natural frequencies. The effect of nonlocal parameter and Winkler elastic foundation modulus on the natural frequencies of the nonlocal Euler-Bernoulli beam is investigated and tabulated. The differential transform method is applicable for micro/nano beams and gives high accuracy results.

Keywords: Natural frequency, differential transforms method, nonlocal elasticity theory, Winkler elastic foundation

1. Introduction

Nanoscience and nanotechnology have made a major contribution to the introduction of small-scale structures and devices. Some of the potential applications of nanorods and nanobeams are image technology, microelectromechanical systems (MEMS) and nanoelectromechanical systems (NEMS). Nanorod and nanobeam, together with other noble metal nanoparticles function as teranostatic agents. Nanorod and nanobeam absorb infrared rays. They also generate heat when the infrared rays are passing through them. This feature allows the use of nanorod and nanobeam in cancer treatment. When a patient is exposed to infrared light, nanorods selectively pick up tumor cells which are heated locally and only destroy cancerous tissue, but healthy cells are left intact. Nanorods and beams which are produced as semiconductor materials can be used as nanosensors and nanoactuators as energy collection, sensing and light emission applications.

In many engineering applications, mechanical behavior must be investigated and well defined to increase the use of nanoscale systems with such a wide range of applications and to propose new designs. This problem can be solved by molecular dynamic simulations, but it requires too much computational effort and therefore a lot of time is required. For this reason, researchers have been directed to continuum mechanics and nano systems have been modeled as rods, beams, plates, shells. Classical theories can interpret behavior of structures up to a certain size [1-16]. To incorporate the small-scale effect into account, nonlocal elasticity theories are proposed. The most widely known of these is the nonlocal elasticity theory of Eringen[17]. Extensive studies have been conducted on the mechanical properties of micro/nano beam such as static bending [18-29], free vibration [20, 30-41], and buckling [42-52].

In this present paper the vibration of nano / micro beams resting on elastic foundation with simply supported and clamped-clamped boundary conditions is investigated. Euler Bernoulli beam theory and nonlocal elasticity theory is used. The interaction of the elastic medium with the micro/nano beam is considered as the Winkler type foundation model. Numerical results were obtained for the vibration with the differential transform method. The effect of the nonlocal parameter, the Winkler foundation parameter and modes for micro/nano beam of frequency is discussed and tabulated.

2. Nonlocal Euler-Bernoulli Beam Model

2.1. Nonlocal Elasticity

According to the nonlocal elasticity theory of Eringen [1], the stress at any reference point is effecting the whole body which not depends only on the strains at this point but also on strains at all points of the body. This definition of the Eringen's nonlocal elasticity is based on the atomic theory of lattice dynamics, and some experimental observations on phonon dispersion. The simplified version of the Eringen nonlocal elasticity theory is as followed,

$$[1-(e_0 a)^2\nabla^2]\sigma_{kl} = \tau_{kl} \tag{1}$$

where e_0 is a material constant, and a is the internal characteristic lengths, respectively. The specific form of the Eq. (1) for Euler-Bernoulli beams, [17]

$$\tau_{xy} - (e_0 a)^2 \frac{\partial^2 \tau_{xy}}{\partial x^2} = 0 \;\; , \; \sigma_{xx} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E\varepsilon_{xx} \tag{2}$$

The nonlocal moment resultants for Euler-Bernoulli beam can be obtained via Eq. (2) as

$$M - (e_0 a)^2 \frac{\partial^2 M}{\partial x^2} = -EI\frac{\partial^2 w}{\partial x^2} \tag{3}$$

2.2. Governing equations of beam based on nonlocal elasticity

The displacement field based on the classical Euler-Bernoulli beam theory can be written

$$u = -z\frac{\partial w}{\partial x}, \;\; v = 0, \;\; w = w(x,t) \tag{4}$$

where 'w' is the transverse displacement of the beam. The strain-displacement, stress-strain equations and general expression of bending moment according to Euler-Bernoulli beam theory can be written as

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2}(x,t), \;\; \sigma_{xx} = -Ez\frac{\partial^2 w}{\partial x^2}(x,t) \;\; , \; M = \int_A z\sigma_{xx} dA \tag{5}$$

The generalized Hamilton's principle is as it shown below

$$\delta\int_0^t [T-(U-W)]dt = 0 \tag{6}$$

The strain and kinetic energies and work of the classical Euler-Bernoulli beam can be stated as

$$T=\frac{1}{2}\rho\int_V\left[\left(\frac{\partial w}{\partial t}\right)^2\right]dV\ ,\ U=\frac{1}{2}\int_V\sigma_{xx}\varepsilon_{xx}dV\ ,\ W=\frac{1}{2}\int_0^L\left[-k_w(w)^2\right]dx \tag{7}$$

Substitution of Eq. (7) into Eq. (6) and when the necessary arrangements are made according to Eq. (5) leads to

$$\int_0^t\left[\int_0^L\rho A\frac{\partial w}{\partial t}\delta\frac{\partial w}{\partial t}dx-\left(-M\delta\left(\frac{\partial^2 w}{\partial x^2}\right)dx\right)+\left(-k_w w\delta w\right)dx\right]dt=0 \tag{8}$$

When Eq. (8) is equal to zero under double integral, differential equations of motion becomes

$$\frac{\partial^2 M}{\partial x^2}=\rho A\frac{\partial^2 w}{\partial t^2}+k_w w \tag{9}$$

Substitution of Eq. (9) into Eq. (3) leads to

$$M=(e_0a)^2\left(\rho A\frac{\partial^2 w}{\partial t^2}+k_w w\right)-EI\frac{d^2 w}{dx^2} \tag{10}$$

Finally, by substituting Eq. (18) into Eq. (15), we obtain the governing equations for nonlocal Euler Bernoulli beam [23, 45, 53, 54]

$$-\rho A\frac{\partial^2 w}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left[(e_0a)^2\left(\rho A\frac{\partial^2 w}{\partial t^2}+k_w w\right)-EI\frac{\partial^2 w}{\partial x^2}\right]-k_w w=0 \tag{11}$$

The essential boundary conditions

$$\delta\left[w\right]_0^L=0\ \text{ and }\ \delta\left[\frac{dw}{dx}\right]_0^L=0 \tag{12}$$

The natural boundary conditions

$$\left[EI\frac{\partial^2 w}{\partial x^2}-e_0a^2(-\rho A\frac{\partial^2 w}{\partial t^2}-k_w)\right]_0^L=0\ \text{ and }\ \left[EI\frac{\partial^3 w}{\partial x^3}-e_0a^2\frac{\partial}{\partial x}(-\rho A\frac{\partial^2 w}{\partial t^2}-k_w)\right]_0^L=0 \tag{13}$$

In the equation; k_w is the Winkler spring constant, w is deflection, ρ is density, A is cross-sectional area, E is young modulus, I is moment of inertia and t is time. When analyzing the vibration of the Euler-Bernoulli beam resting on Winkler elastic foundation,

$$w(x,t)=W(x)e^{i\omega t} \tag{14}$$

If Eq.(14) is substituted in Eq.(11) the equation of motion becomes

$$-\rho A\omega^2 W-\left((e_0 a)^2\frac{d^2W}{dx^2}\left(-\rho A\omega^2+k_w\right)-EI\frac{d^4W}{dx^4}\right)+k_w W=0 \tag{15}$$

2.3. Nondimensional Form of the Equation

The nondimensional parameters of the Euler-Bernoulli beam resting on the Winkler elastic foundation can be expressed as

$$k=\frac{k_w L^4}{EI},\quad \overline{\omega}=\omega\sqrt{\frac{\rho A L^4}{EI}},\quad \mu^2=\left(\frac{e_0 a}{L}\right)^2,\quad \xi=\frac{x}{L} \tag{16}$$

When these parameters are used, Eq. (15) becomes

$$\frac{d^4W}{dx^4}+\mu^2\left(\overline{\omega}^2-k\right)\frac{d^2W}{dx^2}-\left(\overline{\omega}^2-k\right)W=0 \tag{17}$$

and nondimensional boundary conditions

$$\delta[W]_0^L=0\ ,\ \delta[W']_0^L=0\ ,\ \left[\frac{d^2W}{dx^2}-\mu^2\left(\overline{\omega}^2-k\right)\right]_0^L=0\ ,\ \left[\frac{d^3W}{dx^3}-\mu^2\frac{d}{dx}\left(\overline{\omega}^2-k\right)\right]_0^L=0 \tag{18}$$

2.4. The Differential Transform Method (DTM)

The differential transformation method is a transformation method based on Taylor series expansion. In this method, certain conversion rules are applied. The differential equations and boundary conditions are transformed into a set of equations which is the differential transformation of the main function. The solution of the obtained equations gives the result of the problem. Theorems used in DTM solutions are given in Table 1-2 [53,55]

Table 1. DTM theorems used for equations of motion.

Original function	Transformed function
$f(x)=g(x)\pm h(x)$	$F(k)=G(k)\pm H(k)$
$f(x)=\lambda g(x)$	$F(k)=\lambda G(k)$
$f(x)=g(x)h(x)$	$F(k)\sum_{l=0}^{k}G(l)H(k-l)$
$f(x)=\frac{d^n g(x)}{dx^n}$	$F(k)=\frac{(k+n)!}{k!}G(k+n)$
$f(x)=x^n$	$F(k)=\delta(k-n)=\begin{cases}0 & if\quad k\neq n\\ 1 & if\quad k=n\end{cases}$

Table 2. DTM theorems used for boundary conditions

x=0		x=1	
Original	Transformed	Original	Transformed
$f(0)=0$	$F(0)=0$	$f(1)=0$	$\sum_{k=0}^{\infty} F(k)=0$
$\frac{df(0)}{dx}=0$	$F(1)=0$	$\frac{df(1)}{dx}=0$	$\sum_{k=0}^{\infty} kF(k)=0$
$\frac{d^2 f(0)}{dx^2}=0$	$F(2)=0$	$\frac{d^2 f(1)}{dx^2}=0$	$\sum_{k=0}^{\infty} k(k-1)F(k)=0$
$\frac{d^3 f(0)}{dx^3}=0$	$F(3)=0$	$\frac{d^3 f(1)}{dx^3}=0$	$\sum_{k=0}^{\infty} k(k-1)(k-2)F(k)=0$

Firstly, the DTM form of Eq. (17), which models the Euler-Bernoulli beam resting on Winkler foundation, needs to be written. Applying the rules given in Table 1, the equation becomes:

$$W(k+4)=\frac{-\mu^2\left(\overline{\omega}^2-k\right)(k+1)(k+2)W(k+2)+\left(\overline{\omega}^2-k\right)W(k)}{(k+1)(k+2)(k+3)(k+4)} \tag{19}$$

2.5. Case study for boundary conditions

- Clamped-Clamped

The nondimensional boundary conditions for this case can be defined as follows

$$w(0)=0,\ \ w'(0)=0,\ \ w(1)=0,\ \ w'(1)=0 \tag{20}$$

Using Table 2, the transformed boundary conditions can be written as:

$$W(0)=0,\ \ W(1)=0,\ \ \sum_{k=0}^{\infty} W(k)=0,\ \ \sum_{k=0}^{\infty} kW(k)=0 \tag{21}$$

It is assumed that W (2) = c and W (3) = d, and frequencies can be obtained if the boundary conditions apply to Eq. (19). The equation is calculated for n terms. The more the number of terms, the more accurate the result will be.

- Simple-simple

The boundary conditions for this case are defined as

$$w(0)=0,\ \ M(0)=0,\ \ w(1)=0,\ \ M(1)=0 \tag{22}$$

Using Table 2, the transformed boundary conditions can be written as:

$$W(0)=0\,,\quad W(2)=0\,,\quad \sum_{k=0}^{\infty}W(k)=0\,,\ \sum_{k=0}^{\infty}\left[k(k-1)-\mu^{2}\left(\overline{\omega}^{2}-k\right)\right]W(k)=0 \qquad (22)$$

It is assumed that W (1) = c and W (3) = d and the same method with clamped-clamped boundary conditions applied for the solution.

3. Numerical Result

In this section, numerical results will be obtained by the DTM described in the previous section. Since it is working in nondimensional form, the nondimensional Winkler spring constant and nondimensional small scale effect are sufficient to calculate the results. In this study, simple-simple and clamped-clamped boundary conditions are applied. Firstly, the results are compared for the Euler-Bernoulli nonlocal beam resting on Winkler elastic foundation with current literature. Togun [54] has studied the nonlinear vibrations of an Euler-Bernoulli nanobeam resting on an elastic foundation using nonlocal elasticity theory. It is seen that in Table 3, there is a great harmony when the results are compared with Togun[54]. The effect of the nonlocal parameter (μ) and the Winkler foundation parameter *(k)* on the natural frequency is presented in Table 4 for various boundary conditions (simply supported and clamped-clamped, respectively). Nondimensional nonlocal parameter with μ =0, 0.05, 0.1, 0.15, 0.2 and nondimensional Winkler foundation parameters with *k*=0, 1, 10, 100, 1000, 10000, respectively. It can be said that for both support conditions, the Winkler foundation parameter increases the natural frequency and the nonlocal parameter decreases the natural frequency. Because increasing Winkler foundation parameters increases the stiffness of the beam. It can be clearly seen that the results obtained from nonlocal elasticity theory for boundary conditions are always smaller from the classical results. Also the frequency of clamped-clamped boundary condition is always higher than simply supported. The results are calculated for 30 terms for DTM. As the number of terms increases, it is clear that the solution will be more accurate.

Table 3. Comparative result for simple-simple boundary condition

k	μ	Ref [54]	Present
10	0	10.3638	10.3638
	0.1	9.93271	9.93271
	0.2	8.93522	8.93522
	0.3	7.84771	7.84771
	0.4	6.91145	6.91145
	0.5	6.17194	6.17194

Table 4. Nondimensional natural frequency resting on Winkler foundation for simply supported and clamped-clamped boundary condition

		Simply Supported			Clamped-Clamped		
k	μ	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
0	0.0	3.14159	6.28319	9.42394	4.73004	7.8532	11.0856
	0.05	3.12251	6.13706	8.96310	4.69433	7.64178	10.4625
	0.10	3.06853	5.78167	8.03924	4.59446	7.14024	9.43622
	0.15	2.98797	5.35999	7.16144	4.44836	6.56567	8.19880
	0.20	2.89083	4.95805	6.45140	4.27661	6.03520	7.28636
1	0.0	3.14962	6.28419	9.42424	4.73240	7.85372	11.0858
	0.05	3.13069	6.13814	8.96345	4.69674	7.64234	10.4628
	0.10	3.07715	5.78296	8.03972	4.59703	7.14093	9.43652
	0.15	2.9973	5.36162	7.16212	4.45120	6.56655	8.19926
	0.20	2.90113	4.96010	6.45233	4.27981	6.03634	7.28701
10	0.0	3.21929	6.29324	9.42693	4.75349	7.85836	11.0875
	0.05	3.20157	6.14785	8.96657	4.71831	7.64738	10.4647
	0.10	3.15162	5.79456	8.04405	4.62002	7.14710	9.43919
	0.15	3.07757	5.37615	7.16824	4.47650	6.57448	8.20334
	0.20	2.98918	4.97844	6.46069	4.30822	6.04654	7.29281
10^2	0.0	3.74836	6.38163	9.45367	4.95039	7.90432	11.10390
	0.05	3.73718	6.24247	8.99792	4.91930	7.69720	10.48430
	0.10	3.70612	5.90689	8.08693	4.83300	7.20795	9.46583
	0.15	3.66136	5.51545	7.22856	4.70863	6.65227	8.24379
	0.20	3.61001	5.15155	6.54255	4.56560	6.14585	7.44536
10^3	0.0	5.75562	7.11211	9.70941	6.22391	8.32511	11.26470
	0.05	5.75254	7.01275	9.29177	6.20836	8.14919	10.67430
	0.10	5.74411	6.78346	8.48240	6.16611	7.74558	9.72064
	0.15	5.73227	6.53640	7.76220	6.10767	7.31184	8.62865
	0.20	5.71912	6.32879	7.22988	6.04408	6.94519	7.86100
10^4	0.0	10.02430	10.36873	11.56476	10.12290	10.83920	12.58720
	0.05	10.02368	10.33719	11.32578	10.11930	10.76120	12.17640
	0.10	10.02209	10.26836	10.91178	10.10960	10.59460	11.57140
	0.15	10.01987	10.20025	10.60116	10.09649	10.43532	10.98161
	0.20	10.01741	10.14776	10.40748	10.08260	10.31634	10.64047

4. Concluding remarks

The vibration of an Euler-Bernoulli nanobeam resting on an elastic foundation is investigated for simply supported and clamped-clamped boundary conditions. Results for natural frequencies are obtained with Differential Transform Method. The effects of the nondimensional nonlocal parameter (μ), nondimensional Winkler foundation parameter *(k),* and boundary conditions (Simply supported and clamped-clamped) are tabulated. The numerical results show that the natural frequency of the nanobeam decreases with increasing the nondimensional nonlocal parameters and increasing with increasing nondimensional Winkler foundation parameters.

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