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Effects of Circular Cylindrical Block On Heat Flow For MHD Free Convection In A Non-Uniformly Heated Trapezoidal Enclosure

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Abstract

MHD free convection in trapezoidal cavities has attracted the interest of many researchers due to its importance applications in several thermal engineering problems such as in the design of electronic devices, solar thermal receivers, and uncovered flat plate solar collectors having rows of vertical strips geothermal reservoirs, and so on. This analysis is largely concerned with the effects of circular cylindrical block on heat flow for MHD free convection in a non-uniformly heated trapezoidal enclosure. Free convection within a trapezoidal enclosure is considered as non-uniformly heated bottom wall and insulated top wall while side walls are isothermal with inclination angles (ϕ). Heat flows in the presence of free convection within trapezoidal enclosures have been analyzed with heatlines concept. A set of Similarity transformations are used to transform the momentum and energy equations under consideration into coupled non-dimensional governing equations. The non-dimensional governing equations are discretized by using Galerkin weighted residual method of finite element formulation. The numerical results are presented in terms of streamlines, isotherms, local Nusselt number along the bottom wall for non-uniform heating for different combinations of the governing parameters namely Prandtl number Pr, Hartmann number Ha and Rayleigh number Ra. The validity of the numerical results is checked graphically by comparing the results obtained for some specific cases with those available in the literature, and a comparatively excellent agreement is reached.

Keywords: Cylindrical block; free convection; MHD; non-uniform heat; trapezoidal enclosure.

1. Introduction

The analysis of free convection usually induced in enclosed cavities containing heating elements on one of its wall or on both walls are important both of theoretical and practical points of view. Most of the cavities as triangular, rectangular, cylindrical, and trapezoidal etc. are commonly used in industries. Trapezoidal cavities have received a considerable attention for its application in various fields. A comprehensive understanding of energy flow and entropy generation is needed for an optimal process design via reducing irreversibilities in terms of 'entropy generation'. This analysis on entropy generation during natural convection in a trapezoidal cavity with various inclination angles (φ = 45°, 60° and 90°) examined for an efficient thermal processing of various fluids of industrial importance (Pr = 0.015, 0.7 and 1000) in the range of Rayleigh number ($10^3 - 10^5$) by Basak [1]. Heat flow patterns within trapezoidal enclosures in the presence of natural convection with heatlines concept have been investigated by Basak [2]. In this investigation, natural convection within a trapezoidal enclosure for uniformly and non-uniformly heated bottom wall, insulated top wall but isothermal side walls with inclination angle are considered. The streamfunctions and heatfunctions have considered as momentum and energy transfer such that streamfunctions andheatfunctions satisfy the dimensionless forms of momentum and energy balance equations, respectively. The heatlines are found to be continuous lines connecting with the cold and hot walls, while the lines are perpendicular to the isothermal walls for the conduction dominant heat transfer.

The enhanced thermal mixing near the core for larger Rayleigh number is explained with dense heatlines and convective loop of heatlines. The boundary layer formation on the walls has a direct consequence based on heatlines. For side and bottom walls, the local Nusselt numbers have been shown. Moreover, variation of local Nusselt numbers with distance has been explained based on heatlines. The average heat transfer rate varies insignificantly with non-uniform heating of bottom wall. Boussaid [3], investigated the natural heat and mass transfer in a trapezoidal cavity heated from the bottom and cooled from the inclined upper wall. The obtained results show the flow configuration depends on the θ angle inclination of the upper wall. Baez [4], performed 2D natural convection flows in tilted cavities: Porous media and homogeneous fluids. Some numerical and experimental results of turbulent double-diffusive natural convection of a mixture of two gases in a trapezoidal enclosure with imposed unstable thermal stratification are reported by Eyden [5]. As in Nasir Uddin [6], the authors examine the effects of circular cylindrical block on flow field to explore the impact of the momentum and heat transfer characteristics with magnetic field with the absent of Hartmann number. They reported that he velocity boundary layer thickness increases with the increase of Rayleigh number in the middle of the trapezoidal but decreases both of the sides wall adjacent to the bottom walls with the absent of Hartmann number. As in Kuyper [7] examined laminar natural convection flow in trapezoidal enclosures to analysis the effects of the inclination angle on the flow and the dependence of the average Nusselt numbers on the Rayleigh number. A critical Rayleigh number *Ra* is presented depending on the tilting angle, where unicellular convection is observed. As in Kumar [8] investigated coupled non-linear partial differential equations, governing the natural convection from an isothermal wall of a trapezoidal porous enclosure have been solved numerically by finite element method (FEM). In view of the huge quantity of calculation, a similar numerical algorithm for unfinished LU-conjugate gradient (ILU-CG) solver on eight-noded ANUPAM cluster under MIMD paradigm based on ANULIB message passing library has been developed. Corresponding computations have been carried out for different values of flow and geometric parameters both under Darcian and non-Darcian assumptions on the porous model. Cumulative heat fluxes and Nusselt number (Nu) connected with convection process are accessible through computer generated plots. As in Kumar [9] the authors investigated straightforward thermal investigation to estimate the natural convective heat transfer coefficient, hc12 for a trapezoidal absorber plate-inner glass cover enclosure of a double-glazed box-type solar cooker. As in Natarajan [10] reported a numerical study of mutual natural convection and surface radiation heat transfer in a solar trapezoidal cavity absorber for Compact Linear Fresnel Reflector (CLFR). The numerical simulation results are reported in terms of Nusselt number correlation to show the outcome of these parameters on combined natural convection and surface radiation heat loss. The authors in Saleh [11] also investigated the effect of magnetic field in a trapezoidal enclosure filled with a fluid-saturated porous medium with steady convection by the finite difference method. The outcome point toward that the heat transfer performance decreases with the decreasing the angle of sloping wall. Optimum reducing of heat transfer rate was obtained for an acute trapezoidal enclosure, and large magnetic field in the parallel direction. However, overall heat defeat coefficients of trapezoidal cavity absorber with rectangular and round pipe were studied in the laboratory by Singh [12]. As there should be lowest heat defeat from the absorber to get better efficiency of the solar collector. Varol [13] studied the heat transfer and fluid flow inside two entrapped porous trapezoidal cavities involving cold inclined walls and hot horizontal walls. The numerical results are presented for different values of the governing parameters, such as Darcy-modified Rayleigh number, aspect ratio of two entrapped trapezoidal cavities and thermal conductivity ratio between the middle horizontal wall and fluid medium. The rates of heat transfer are estimated in terms of local and mean Nusselt numbers. The local Nusselt numbers with spatial distribution reveal monotonic tendency irrespective of all Rayleigh numbers for the upper trapezoidal while wavy distribution of local Nusselt number happen for the lower trapezoidal. Also a numerical work to determine the heat transfer and fluid flow due to buoyancy forces in divided trapezoidal enclosures filled with fluid saturated porous media is investigated by Varol [14]. In this investigation, bottom wall was non-uniformly heated whereas two vertical walls were insulated and the top wall was maintained at constant cold temperature. Therefore, in the light of above literatures, it is more reasonable to examine the effect of Hartmann number on heat flow for MHD free convection in a non-uniformly heated trapezoidal enclosure. The numerical results in terms of streamlines, isotherms, local Nusselt number along the bottom wall for non-uniform heating are presented for different combinations of the relevant governing parameters namely Prandtl number Pr, Hartmann number *Ha* and Rayleigh number *Ra*.

2. Physical Model and Mathematical Analysis

Figure 01 shows a schematic diagram and the coordinates with the significant geometric parameters. Heat transfers with the fluid flow within a two-dimensional trapezoidal cavity are considered. A trapezoidal cavity of height L with the left wall inclined at an angle $\phi = 25^0$ with y axis is considered subject to conditions that left wall and right wall (i.e. side walls) are subjected to the cold temperature T_c, and bottom wall is subjected to the hot temperature T_h whereas the top wall is kept insulated. The fluid which is considered as incompressible, Newtonian whereas the flow which is assumed to be laminar. The no-slip boundary conditions are considered for velocity on solid boundaries. The viscous dissipation effect in the energy equation is neglected. For the conduct of the buoyancy term in the momentum equation, Boussinesq approximation is employed to explanation for the variations of density as a function of temperature, and to couple in this way the temperature field to the flow field. Also for laminar incompressible thermal flow, the buoyancy force is included here as a body force in the momentum equation for y direction. Under the foregoing assumptions, the governing equations for steady free convection flow can be written as:

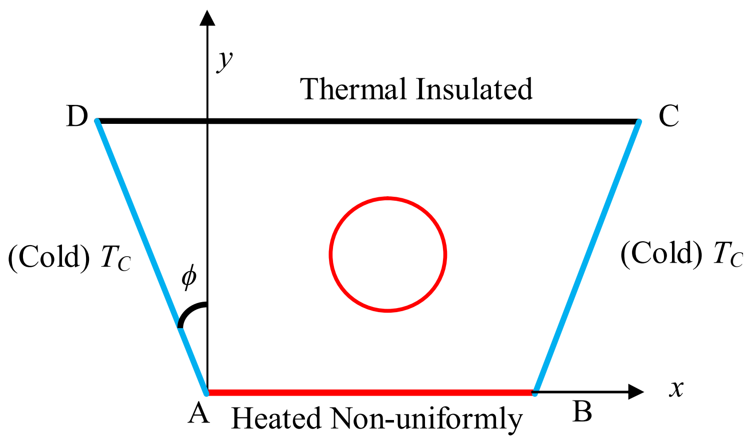


Fig. 1: Schematic diagram of the physical system for $\phi = 25^0$

Under the foregoing assumptions, the governing equations for steady free convection flow can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \quad (2)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})+g\beta(T-T_c)-\frac{\sigma B_0^2 v}{\rho} \quad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

where x and y are the horizontal and vertical directions respectively whereas u and v are the velocity components in the x and y directions respectively, T denote the fluid temperature whereas T_c denotes the reference temperature for which buoyant force vanishes, p is the pressure whereas ρ is the fluid density, g is the gravitational constant, β is the volumetric coefficient of thermal expansion, c_p is the fluid specific heat and k is the thermal conductivity of fluid. The appropriate boundary conditions for the flow field and relevant to this investigation are as follows for at the bottom wall, at the left wall, at the right wall and at the top wall:

$$u(x,0)=0,\ v(x,0)=0,\ T=T_h\ \forall\ y=0,\ 0\le x\le L$$

$$u(0,y)=0,\ v(0,y)=0,\ T=T_c,\ \forall\, x\cos\phi+y\sin\phi=0,\ 0\le y\le L$$

$$u(0,y)=0,\ v(0,y)=0,\ T=T_c,\ \forall\, x\cos\phi-y\sin\phi=L\cos\phi, 0\le y\le L$$

$$u(x,L)=0, v(x,L)=0, \frac{\partial}{\partial y}\left(\frac{T-T_c}{T_h-T_c}\right)=0,\ \forall\ y=L, -L\tan\phi\le x\le L\left(1+\tan\phi\right)$$

Where x and y are the horizontal and vertical directions, respectively whereas u and v are the velocity components in the x and y direction, respectively; L is the height of trapezoidal cavity with the left wall inclined at an angle $\phi = 25^0$ with y axis; T denotes the temperature; T_h and T_c are heated non-uniformly and colder uniformly temperatures respectively. The local Nusselt number which is defined by the following expression of the cavity at the heated surface:

$$Nu_l = Nu_r = Nu_s = Nu_b - \frac{h(x)L}{k}$$

Such local values have been further averaged over the entire heated surface to get the overall mean Nusselt number at the left, right, and bottom walls are:

$$Nu=\int_0^L Nu_l dx=\int_0^L Nu_r dx=\int_0^L Nu_s dx=\int_0^L Nu_b dx$$

where L is the length of the heated wall whereas *h(x)* is the local convective heat transfer coefficient of the heated wall. In order to reduce the number of independent variables and to make the governing differential equations (1- 4) dimensionless, the following dimensionless variables are applied:

$$X=\frac{x}{L}, Y=\frac{y}{L}, U=\frac{uL}{\alpha}, V=\frac{vL}{\alpha}, P=\frac{pL^2}{\rho\alpha^2}, \theta=\frac{T-T_c}{T_h-T_c}, \mathrm{Pr}=\frac{\nu}{\alpha}, Gr=\frac{g\beta L^3\left(T_h-T_c\right)}{\nu^2}, \alpha=\frac{k}{\rho c_p}$$

$$Ra=\frac{g\beta L^3\left(T_h-T_c\right)\mathrm{Pr}}{\nu^2}, Ha^2=\frac{\sigma B_0^2 L^2}{\rho\alpha}$$

where X and Y are the coordinates along horizontal and vertical directions, respectively while U and V are the velocity components in the X and Y directions, θ is the dimensionless temperature, P is the dimensionless pressure, $\Delta T = T_h - T_c$ is the temperature difference, and α is thermal diffusivity of the fluid. Also the dimensionless parameters are the Grashof number Gr, Prandtl number Pr, Hartmann number Ha and Rayleigh number Ra. Using dimensionless variables, the transformed continuity, momentum and energy equations together with the boundary conditions can be written as:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{5}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{6}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra\Pr\theta-Ha^2V \tag{7}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{8}$$

and the boundary conditions are as follows for at the bottom wall, at the left wall, at the right wall and at the top wall:

$$U=0,\quad V=0,\quad \theta=\sin(\pi X)\quad \forall\, Y=0,\quad 0\le X\le 1$$

$$U=0, V=0, \theta=0,\ \forall\, X\cos\phi+Y\sin\phi=0, 0\le Y\le 1$$

$$U=0, V=0, \theta=0, \forall\, X\cos\phi-Y\sin\phi=\cos\phi, 0\le Y\le 1$$

$$U=0, V=0, \frac{\partial\theta}{\partial Y}=0, \forall\, Y=1, -\tan\phi\le X\le(1+\tan\phi)$$

where X and Y are dimensionless coordinates along horizontal and vertical directions, respectively whereas U and V are dimensionless velocity components in X and Y directions and θ is the dimensionless temperature. The local Nusselt number which is defined by the following expression at the heated surface of the cavity:

$$Nu_l=Nu_r=Nu_b=Nu_s=-\frac{\partial\theta}{\partial n}$$

where n stands the normal direction on a plane. According to Singh and Sharif [15], the average Nusselt number on the non-dimensional variables at the heated bottom wall, cooled left and right walls, and insulated top wall of the cavity based can be expressed as:

$$Nu=\int_0^1 Nu_l\,dX=\int_0^1 Nu_r\,dX=\int_0^1 Nu_s\,dX=\int_0^1 Nu_b\,dX$$

3. Numerical Solution

The Galerkin weighted residual method of finite-element formulation procedure is used to solve the governing equations of the present work. In this method, a triangular mesh arrangement which is non-uniform is implemented in the present investigation particularly near the walls to capture the rapid changes in the dependent variables. Equations (5) - (8), results in a set of non-linear coupled equations for which an iterative method is adopted. For the development of the finite element equations, the six node triangular element is used in this work. All six nodes are related with velocities as well as temperature; only the corner nodes are associated with pressure. This means that a lower order polynomial is selected for pressure and which is satisfied through continuity equation.

4. Comparison

In order to verify the accuracy of the numerical results which are obtained throughout the present study are compared with the previously published results. The present results of streamlines and isotherms are compared with that of Hussein [16] while Pr = 0.07 and Ra = 10^3 and obtained good agreement which is shown in Fig. 2.

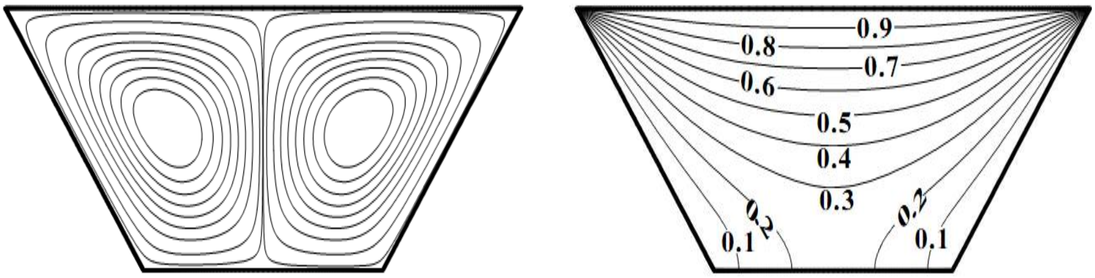


Fig. 2(a): Obtained results of streamlines and isotherms by Hussein [16]

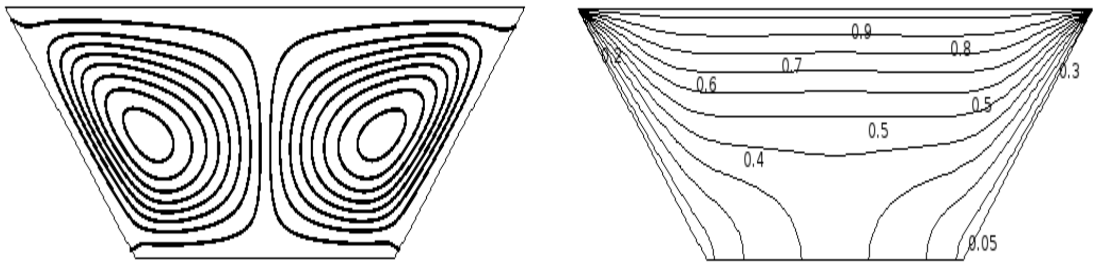


Fig. 2(b): Obtained results of streamlines and isotherms by present work

5. Results and Discussion

The effect of Hartmann number on the flow field are examined and discussed in this section. The numerical results have been carried out using different values of various physical parameters which are appeared transformed governing equations. Figure 3 and Fig. 4 illustrate the streamlines and isotherms for Hartmann number Ha (Ha = 100) while Ra = 10^3 - 10^6 and Pr = 0.71. From Fig. 3, it is seen that two small cell are formed adjacent the circular cylindrical block inside the square cavity in presence of magnetic field. On the other hand, increasing of the Rayleigh number, the cells which are formed in the cavity, changes its shapes which are observed in Fig. 3(b) - Fig. 3(d). The isotherms, in Fig. 6 are like as linear adjacent side walls as well as circular cylindrical block but bending at the near of circular cylindrical block increases with the increases of Rayleigh number Ra. Figure 5 represents the effects of Rayleigh number in presences of Hartmann number Ha on the flow field as velocity profiles and temperature profiles for varying Rayleigh number Ra = 10^3 - 10^6 along the bottom wall. As seen from the Fig. 5(a), the velocity increases with the increase of Rayleigh number in the middle of the cavity, but the velocity decreases with the increase of Rayleigh number above and below of the middle in the cavity because the magnetic field has slight effects on velocity field. The temperature fields versus the coordinate of X directions are plotted in Fig. 5(b) for different Rayleigh number. As seen from the Fig. 5(b), in presence of increasing Rayleigh number, the temperature field decreases middle of the trapezoidal but increasing near the top and bottom walls in presence of magnetic field. The local Nusselt number at the bottom wall for different Rayleigh number $Ra = 10^3$ - 10^6 is presented in Fig. 6 with Pr = 0.71. From the Fig. 6, it is seen that the local Nusselt number decreases with the increase of Rayleigh number in $0.10 \leq X \leq 0.2$ but the local Nusselt number increases with the increase of Rayleigh number in $0 \leq X \leq 0.1$.

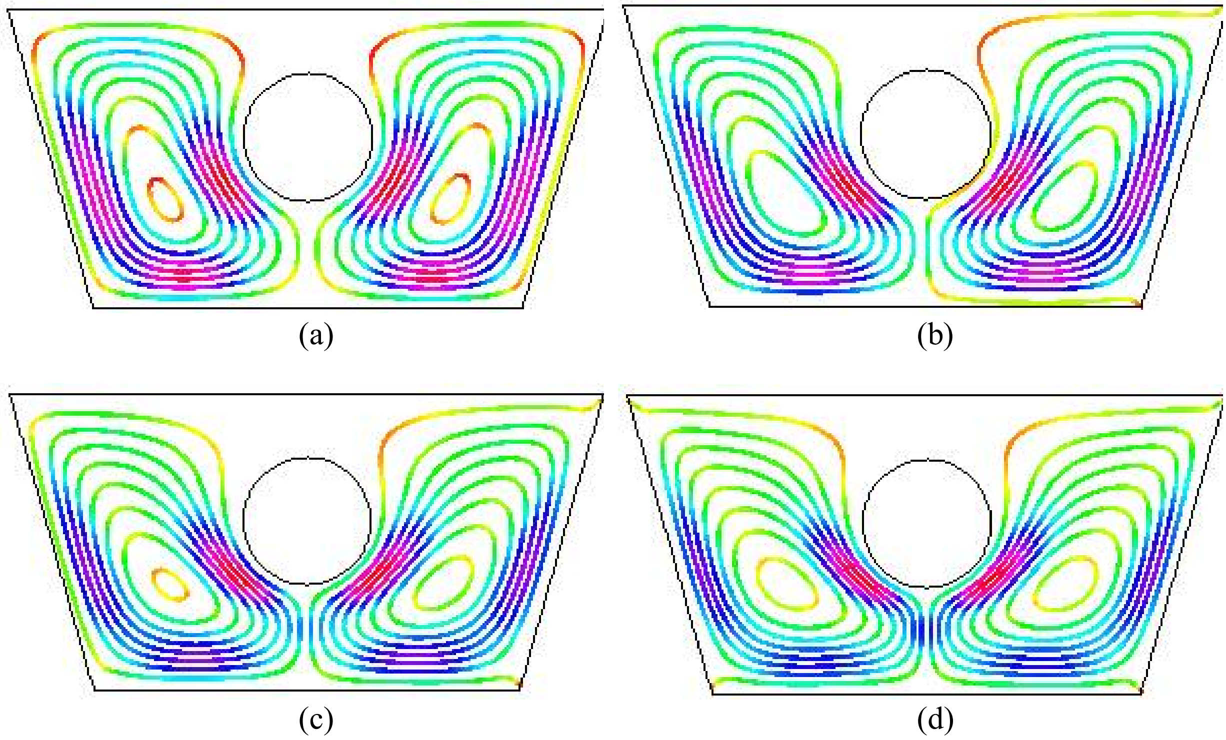


Fig. 3: Streamlines for (a) $Ra = 10^3$; (b) $Ra = 10^4$; (c) $Ra = 10^5$; and (d) $Ra = 10^6$ while Pr = 0.71 and Ha = 100

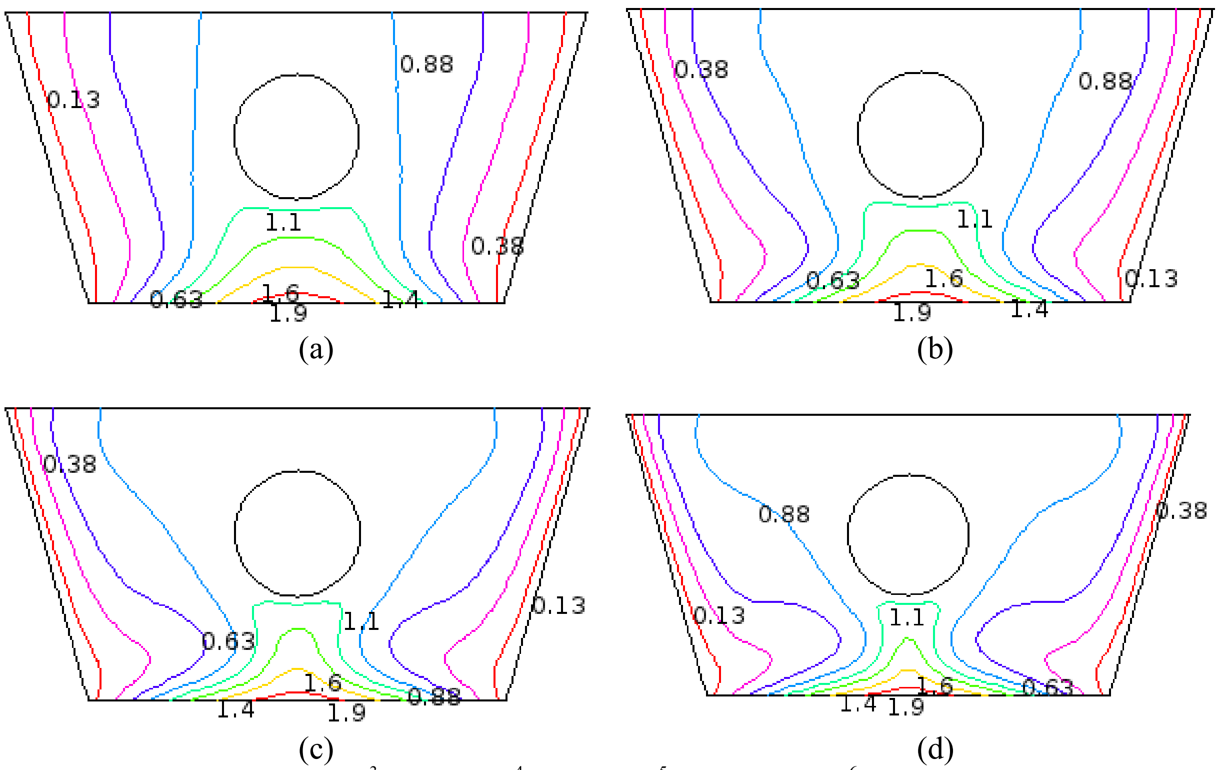
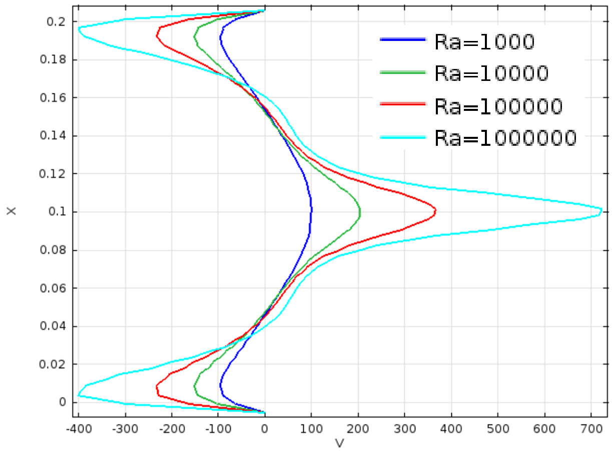
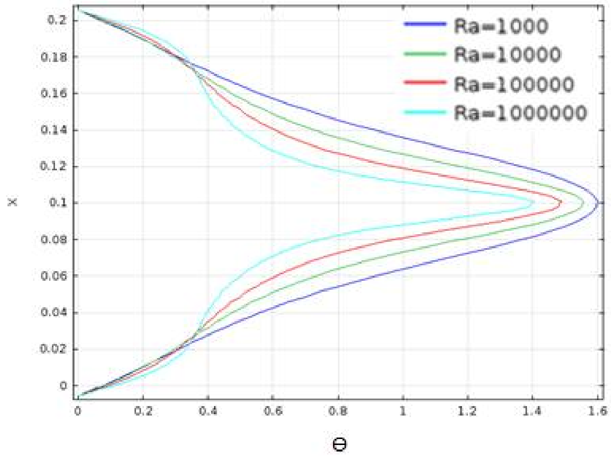


Fig. 4: Isotherms for (a) $Ra = 10^3$; (b) $Ra = 10^4$; (c) $Ra = 10^5$; and (d) $Ra = 10^6$ while Pr = 0.71 and Ha = 100



(a)



(b)

Fig. 5: (a) Velocity profiles and (b) Temperature profiles for Hartmann number *Ha* = 100 and Pr = 0.71 with varying Rayleigh number *Ra* = 10^3 - 10^6 along the bottom wall

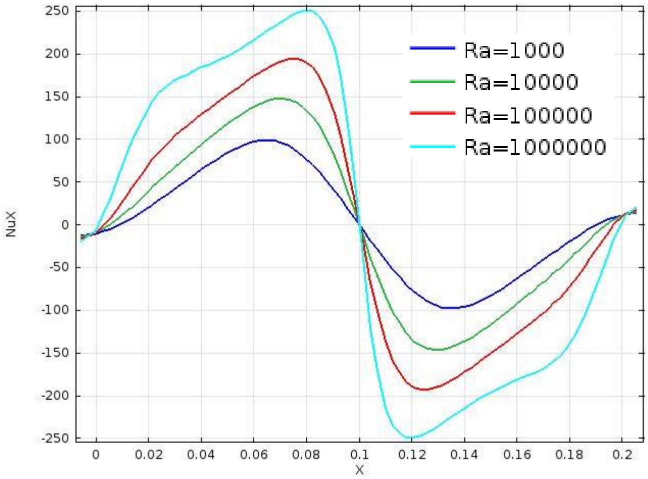


Fig. 6: Local Nusselt number for different Rayleigh number *Ra* = 10^3 - 10^6 with Pr = 0.71 and *Ha* = 100

6. Conclusions

MHD free convective flow in trapezoidal cavity for non-uniformly heated bottom wall with uniform magnetic field which is applied normal to the direction of the flow field has been analyzed with heatlines concept to find the behavior of the flow field. The conservation of mass, momentum, and energy equations were solved using Galerking weighted residual method of finite element formulation. The governing parameters were Prandtl number, Hartman number, and Rayleigh number. The numerical results which were presented in graphically show that if the Rayleigh number increases in the middle of the trapezoidal then the velocity boundary layer thickness increases but decreases both of the sides wall adjacent to the bottom walls due to the magnetic field has slight effects on velocity field. Because of upsurge of convective heat transfer rate the thermal boundary layer thickness is thinner for increasing of Rayleigh number. Local Nusselt number for non-uniform bottom heating is dominant at the bottom edge of the side wall, and thereafter that decreases sharply up to a point which is very adjacent to the bottom edge.

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