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Static analysis of functionally graded rectangular nanoplates based on nonlocal third order shear deformation theory

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Abstract

This paper presents size dependent formulations based on nonlocal elasticity theory and third order shear deformation theory. The formulations are then applied to the bending analysis of functionally graded rectangular nanoplates with simply supported boundary condition. Similar to functionally graded macro plates, it is assumed that the material properties of nanoplates are varied across the thickness direction by a power rule of the volume fraction of the constituents. Numerical results illustrate the influence of the power-law exponent and the nonlocality on the deflections of simply supported rectangular nanoplates.

Keywords: Nonlocal elasticity theory, Third order shear deformation theory, Functionally graded nanoplate, Bending analysis.

1. Introduction

In the past decade, carbon nanostructures have drawn substantial interest from the researches community for the future application of modern aerospace, micro electromechanical systems and nano electro-mechanical systems [1]. Graphene sheets as structural elements occupy an important position in carbon nanostructures. Usually graphene sheets as a nano-plate are subjected to the transverse loads at small scales. Thus the mechanical strength is influenced by small 'size-effect' [2]. Experimental results show that as length scales of a material are reduced, the influences of long-range interatomic and intermolecular cohesive forces on the mechanical properties become prominent and cannot be neglected [3]. Therefore, continuum models need to be extended to consider the scale effect in nanomaterial studies. Various size-dependent continuum theories which capture small scale parameter are reported [4]. One of these continuum theories is the nonlocal theory of Eringen which has the capability to predict behavior of the large nano-sized structures, while it avoids solving the large number of equations [5]. In this theory, the inter-atomic forces and atomic length scales directly come to the constitutive equations as material parameters [6].

A review of literature shows that there are only a few papers on the bending analysis of rectangular graphene sheets in comparison with the dynamic analysis of them. Aghababaei and Reddy [5] reformulated the third order shear deformation theory on the basis of nonlocal elasticity theory to study the bending and free vibration of isotropic rectangular nanoplates. Nami and Janghorban [7] proposed a new higher order shear deformation theory based on trigonometric shear deformation theory. In order to consider the size effects, the nonlocal elasticity theory was used. Nami and Janghorban [8] also presented the bending analysis of rectangular nanoplates subjected to mechanical loading. For this purpose, the strain gradient elasticity theory with one gradient parameter was used to study the nanoplates. Recently, Giunta et al [9] studied several higher-order atomistic-refined models for the static and free vibration analysis of nanoplates. Stemming from a two-dimensional approach, a general model derivation was used where the approximation order was a free parameter of the formulation.

The necessity of monitoring the local variations of the material properties in the whole component to meet the design requirements has led to creation of various functionally graded materials [10]. Recently, functionally graded micro/nano structures have attracted a great attention due to their unique material properties. Hosseini-Hashemi [11] studied the free vibration of FG nanobeams by considering surface effects. It was shown that making changes to voltage values and modifying mechanical properties of nanobeams are two main approaches to achieve desired natural frequencies. Jang et al [12] developed a model for sigmoid FG nanoplates on elastic medium based on a modified couple stress theory. Buckling response of rectangular S-FGM nanoplates was derived, and the obtained results were compared well with reference solutions. Nami and Janghorban [13] developed an analytical solution to study the free vibration analysis of functionally graded rectangular nanoplates. The governing equations of motion were derived based on second order shear deformation theory using nonlocal elasticity theory. Nami and Janghorban [14] also investigated the resonance behaviors of functionally graded micro/nano plates using Kirchhoff plate theory. To consider the small scale effects, the nonlocal elasticity theory and strain gradient theory with one gradient parameter were adopted.

In this study, for the first time, the influence of characteristic length parameter on the bending analysis of functionally graded rectangular nanoplate is investigated on the basis of the nonlocal theory of Eringen [15,16] and third order shear deformation theory presented by Reddy [17]. The results of the present methodology are compared with numerical results in the literature. This model is then used to study the effects of various design parameters such as the power law exponent and nonlocal parameter on the deflections of nanoplate.

2. Governing equations

Several modifications of the classical elasticity formulation have been proposed to address the small-scale effect. As mentioned above, one of the well-known models is the nonlocal continuum theory [18]. In the local elasticity theories, stress tensor at a point is assumed to be dependent on strain tensor at that point. But in the nonlocal theory, it is assumed that the stress tensor at a point depends on strain tensor at all the points of the continuum [19]. The nonlocal constitutive relation is expressed as,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} - \mu\nabla^2 \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \left(\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \right) \tag{1}$$

where

$$Q_{11} = Q_{22} = \frac{E(z)}{1-\nu^2}, \; Q_{12} = Q_{21} = \frac{\nu E(z)}{1-\nu^2}, \; Q_{44} = Q_{55} = Q_{66} = \frac{E(z)}{2(1+\nu)}$$

and μ is the nonlocal parameter. A graphene sheet which is described as a nano-sized rectangular plate is defined in Cartesian coordinate (x,y,z). The displacement field in terms of midplane displacements and rotations is defined as [20,17],

$$u = zQ_x - C_1 z^3 (Q_x + \frac{\partial w_0}{\partial x})$$

$$v = zQ_y - C_1 z^3 (Q_y + \frac{\partial w_0}{\partial y})$$

$$w = w_0 \tag{2}$$

where $C_1 = \frac{4}{3h^2}$ .The linear strains are given by

$$\varepsilon_x = z\frac{\partial Q_x}{\partial x} - C_1 z^3 (\frac{\partial Q_x}{\partial x} + \frac{\partial^2 w_0}{\partial x^2})$$
$$\varepsilon_y = z\frac{\partial Q_y}{\partial y} - C_1 z^3 (\frac{\partial Q_y}{\partial y} + \frac{\partial^2 w_0}{\partial y^2})$$
$$\gamma_{xy} = z(\frac{\partial Q_x}{\partial y} + \frac{\partial Q_y}{\partial x}) - C_1 z^3 (\frac{\partial Q_x}{\partial y} + \frac{\partial Q_y}{\partial x} + 2\frac{\partial^2 w_0}{\partial x \partial y})$$
$$\gamma_{xz} = (Q_x + \frac{\partial w_0}{\partial x}) - 3C_1 z^2 (Q_x + \frac{\partial w_0}{\partial x})$$
$$\gamma_{yz} = (Q_y + \frac{\partial w_0}{\partial y}) - 3C_1 z^2 (Q_y + \frac{\partial w_0}{\partial y})$$

(3)

The governing equations of the displacement model in equation (2) are derived using Hamilton's principle as follow [20,17],

$$\frac{\partial \emptyset_x}{\partial x} + \frac{\partial \emptyset_y}{\partial y} - 3c_1\left(\frac{\partial R_x}{\partial x} + \frac{\partial R_y}{\partial y}\right) + c_1\left(\frac{\partial^2 P_{xx}}{\partial x^2} + 2\frac{\partial^2 P_{xy}}{\partial x \partial y} + \frac{\partial^2 P_{yy}}{\partial y^2}\right) + q = 0$$
$$\frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - \emptyset_x + 3C_1 R_x - C_1\left(\frac{\partial P_{xx}}{\partial x} + \frac{\partial P_{xy}}{\partial y}\right) = 0$$
$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y} - \emptyset_y + 3C_1 R_y - C_1\left(\frac{\partial P_{xy}}{\partial x} + \frac{\partial P_{yy}}{\partial y}\right) = 0$$

(4)

Where

$$\begin{Bmatrix} N_{\alpha\beta} \\ M_{\alpha\beta} \\ P_{\alpha\beta} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\alpha\beta} \begin{Bmatrix} 1 \\ z \\ z^3 \end{Bmatrix} dz\ , \quad \begin{Bmatrix} \emptyset_\alpha \\ R_\alpha \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\alpha z} \begin{Bmatrix} 1 \\ z^2 \end{Bmatrix} dz$$

$$(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}) = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{ij}(z)(1, z, z^2, z^3, z^4, z^6) dz$$

(5)

By obtaining transverse displacement and rotation functions with considering equation (5), the force and moment resultants of the FG nanoplate can be computed by using the nonlocal constitutive relations in the following forms

$$\emptyset_x - \mu\nabla^2(\emptyset_x) = A_{55}\left(Q_x + \frac{\partial w_0}{\partial x}\right) - 3D_{55}C_1\left(Q_x + \frac{\partial w_0}{\partial x}\right)$$
$$\emptyset_y - \mu\nabla^2(\emptyset_y) = A_{66}\left(Q_y + \frac{\partial w_0}{\partial y}\right) - 3D_{66}C_1\left(Q_y + \frac{\partial w_0}{\partial y}\right)$$

(6)

(7)

$$P_{xx} - \mu\nabla^2(P_{xx}) = F_{11}\frac{\partial Q_x}{\partial x} - H_{11}C_1\left(\frac{\partial Q_x}{\partial x} + \frac{\partial^2 w_0}{\partial x^2}\right) + F_{12}\frac{\partial Q_y}{\partial y} - H_{12}C_1\left(\frac{\partial Q_y}{\partial y} + \frac{\partial^2 w_0}{\partial y^2}\right)$$

(8)

$$P_{yy} - \mu\nabla^2(P_{yy}) = F_{12}\frac{\partial Q_x}{\partial x} - H_{12}C_1\left(\frac{\partial Q_x}{\partial x} + \frac{\partial^2 w_0}{\partial x^2}\right) + F_{22}\frac{\partial Q_y}{\partial y} - H_{22}C_1\left(\frac{\partial Q_y}{\partial y} + \frac{\partial^2 w_0}{\partial y^2}\right)$$

(9)

$$P_{xy} - \mu\nabla^2(P_{xy}) = F_{44}\left(\frac{\partial Q_x}{\partial y} + \frac{\partial Q_y}{\partial x}\right) - H_{44}C_1\left(\frac{\partial Q_x}{\partial y} + \frac{\partial Q_y}{\partial x} + 2\frac{\partial^2 w_0}{\partial x\partial y}\right)$$

(10)

$$R_x - \mu\nabla^2(R_x) = D_{55}\left(Q_x + \frac{\partial w_0}{\partial x}\right) - 3F_{55}C_1\left(Q_x + \frac{\partial w_0}{\partial x}\right)$$

(11)

$$R_y - \mu\nabla^2(R_y) = D_{66}\left(Q_y + \frac{\partial w_0}{\partial y}\right) - 3F_{66}C_1\left(Q_y + \frac{\partial w_0}{\partial y}\right)$$

(12)

$$M_{xx} - \mu\nabla^2(M_{xx}) = D_{11}\frac{\partial Q_x}{\partial x} - E_{11}C_1\left(\frac{\partial Q_x}{\partial x} + \frac{\partial^2 w_0}{\partial x^2}\right) + D_{12}\frac{\partial Q_y}{\partial y} - E_{12}C_1\left(\frac{\partial Q_y}{\partial y} + \frac{\partial^2 w_0}{\partial y^2}\right)$$

(13)

$$M_{yy} - \mu\nabla^2(M_{yy}) = D_{12}\frac{\partial Q_x}{\partial x} - E_{12}C_1\left(\frac{\partial Q_x}{\partial x} + \frac{\partial^2 w_0}{\partial x^2}\right) + D_{22}\frac{\partial Q_y}{\partial y} - E_{22}C_1\left(\frac{\partial Q_y}{\partial y} + \frac{\partial^2 w_0}{\partial y^2}\right)$$

(14)

$$M_{xy} - \mu\nabla^2(M_{xy}) = D_{44}\left(\frac{\partial Q_x}{\partial y} + \frac{\partial Q_y}{\partial x}\right) - E_{44}C_1\left(\frac{\partial Q_x}{\partial y} + \frac{\partial Q_y}{\partial x} + 2\frac{\partial^2 w_0}{\partial x\partial y}\right)$$

(15)

The governing equations in terms of midplane displacements and rotations for bending analysis of FG nanoplates are obtained with considering the nonlocal force and moment resultants (equations (6-15)) and the equations of motion (Equation (4)) as follow,

$$A_{55}\left(\frac{\partial Q_x}{\partial x} + \frac{\partial^2 w_0}{\partial x^2}\right) - 3D_{55}C_1\left(\frac{\partial Q_x}{\partial x} + \frac{\partial^2 w_0}{\partial x^2}\right) + A_{66}\left(\frac{\partial Q_y}{\partial y} + \frac{\partial^2 w_0}{\partial y^2}\right) - 3D_{66}C_1\left(\frac{\partial Q_y}{\partial y} + \frac{\partial^2 w_0}{\partial y^2}\right) +$$
$$C_1\Bigg(F_{11}\left(\frac{\partial^3 Q_x}{\partial x^3}\right) - H_{11}C_1\left(\frac{\partial^3 Q_x}{\partial x^3} + \frac{\partial^4 w_0}{\partial x^4}\right) + F_{12}\left(\frac{\partial^3 Q_y}{\partial y\partial x^2}\right) - H_{12}C_1\left(\frac{\partial^3 Q_y}{\partial y\partial x^2} + \frac{\partial^4 w_0}{\partial x^2\partial y^2}\right) +$$
$$2F_{44}\left(\frac{\partial^3 Q_x}{\partial x\partial y^2} + \frac{\partial^3 Q_y}{\partial y\partial x^2}\right) - 2C_1H_{44}\left(\frac{\partial^3 Q_x}{\partial x\partial y^2} + \frac{\partial^3 Q_y}{\partial y\partial x^2} + 2\frac{\partial^4 w_0}{\partial x^2\partial y^2}\right) + F_{12}\left(\frac{\partial^3 Q_x}{\partial x\partial y^2}\right) - H_{12}C_1\left(\frac{\partial^3 Q_x}{\partial x\partial y^2} +\right.$$
$$\left.\frac{\partial^4 w_0}{\partial x^2\partial y^2}\right) + F_{22}\left(\frac{\partial^3 Q_y}{\partial y^3}\right) - H_{22}C_1\left(\frac{\partial^3 Q_y}{\partial y^3} + \frac{\partial^4 w_0}{\partial y^4}\right)\Bigg) - 3C_1\Bigg(D_{55}\left(\frac{\partial Q_x}{\partial x} + \frac{\partial^2 w_0}{\partial x^2}\right) - 3F_{55}C_1\left(\frac{\partial Q_x}{\partial x} +\right.$$
$$\left.\frac{\partial^2 w_0}{\partial x^2}\right) + D_{66}\left(\frac{\partial Q_y}{\partial y} + \frac{\partial^2 w_0}{\partial y^2}\right) - 3F_{66}C_1\left(\frac{\partial Q_y}{\partial y} + \frac{\partial^2 w_0}{\partial y^2}\right)\Bigg) = -q - \mu\nabla^2(-q)$$

(16)

$$D_{22}\left(\frac{\partial^2 Q_y}{\partial y^2}\right)-F_{22}C_1\left(\frac{\partial^2 Q_y}{\partial y^2}+\frac{\partial^3 w_0}{\partial y^3}\right)+D_{12}\left(\frac{\partial^2 Q_x}{\partial y\partial x}\right)-F_{12}C_1\left(\frac{\partial^2 Q_x}{\partial y\partial x}+\frac{\partial^3 w_0}{\partial y\partial x^2}\right)+D_{44}\left(\frac{\partial^2 Q_y}{\partial x^2}+\frac{\partial^2 Q_x}{\partial y\partial x}\right)-F_{44}C_1\left(\frac{\partial^2 Q_y}{\partial x^2}+\frac{\partial^2 Q_x}{\partial y\partial x}+2\frac{\partial^3 w_0}{\partial y\partial x^2}\right)-A_{66}\left(Q_y+\frac{\partial w_0}{\partial y}\right)+3D_{66}C_1\left(Q_y+\frac{\partial w_0}{\partial y}\right)+3D_{66}C_1\left(Q_y+\frac{\partial w_0}{\partial y}\right)-9F_{66}C_1^2\left(Q_y+\frac{\partial w_0}{\partial y}\right)-C_1F_{22}\left(\frac{\partial^2 Q_y}{\partial y^2}\right)+H_{22}C_1^2\left(\frac{\partial^2 Q_y}{\partial y^2}+\frac{\partial^3 w_0}{\partial y^3}\right)-C_1F_{12}\left(\frac{\partial^2 Q_x}{\partial y\partial x}\right)+H_{12}C_1^2\left(\frac{\partial^2 Q_x}{\partial y\partial x}+\frac{\partial^3 w_0}{\partial y\partial x^2}\right)-C_1F_{44}\left(\frac{\partial^2 Q_x}{\partial y^2}+\frac{\partial^2 Q_y}{\partial y\partial x}\right)+H_{44}C_1^2\left(\frac{\partial^2 Q_x}{\partial y^2}+\frac{\partial^2 Q_y}{\partial y\partial x}+2\frac{\partial^3 w_0}{\partial x\partial y^2}\right)=0$$

(17)

$$D_{11}\left(\frac{\partial^2 Q_x}{\partial x^2}\right)-F_{11}C_1\left(\frac{\partial^2 Q_x}{\partial x^2}+\frac{\partial^3 w_0}{\partial x^3}\right)+D_{12}\left(\frac{\partial^2 Q_y}{\partial y\partial x}\right)-F_{12}C_1\left(\frac{\partial^2 Q_y}{\partial y\partial x}+\frac{\partial^3 w_0}{\partial x\partial y^2}\right)+D_{44}\left(\frac{\partial^2 Q_x}{\partial y^2}+\frac{\partial^2 Q_y}{\partial y\partial x}\right)-F_{44}C_1\left(\frac{\partial^2 Q_x}{\partial y^2}+\frac{\partial^2 Q_y}{\partial y\partial x}+2\frac{\partial^3 w_0}{\partial x\partial y^2}\right)-A_{55}\left(Q_x+\frac{\partial w_0}{\partial x}\right)+3D_{55}C_1\left(Q_x+\frac{\partial w_0}{\partial x}\right)+3D_{55}C_1\left(Q_x+\frac{\partial w_0}{\partial x}\right)-9F_{55}C_1^2\left(Q_x+\frac{\partial w_0}{\partial x}\right)-C_1F_{11}\left(\frac{\partial^2 Q_x}{\partial x^2}\right)+H_{11}C_1^2\left(\frac{\partial^2 Q_x}{\partial x^2}+\frac{\partial^3 w_0}{\partial x^3}\right)-C_1F_{12}\left(\frac{\partial^2 Q_y}{\partial y\partial x}\right)+H_{12}C_1^2\left(\frac{\partial^2 Q_y}{\partial y\partial x}+\frac{\partial^3 w_0}{\partial x\partial y^2}\right)-C_1F_{44}\left(\frac{\partial^2 Q_x}{\partial y^2}+\frac{\partial^2 Q_y}{\partial y\partial x}\right)+H_{44}C_1^2\left(\frac{\partial^2 Q_x}{\partial y^2}+\frac{\partial^2 Q_y}{\partial y\partial x}+2\frac{\partial^3 w_0}{\partial x\partial y^2}\right)=0$$

(18)

Obviously, only one length scale parameter is involved in the above governing equations. By setting this parameter equal to zero, one can have the governing equations based on third order shear deformation theory for FG macro plates. The transverse and in-plane displacements and in-plane rotations equations are considered as [20,17],

$$u=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} U\cos\alpha x\,\sin\beta y\,e^{i\omega t}$$
$$v=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} V\sin\alpha x\,\cos\beta y\,e^{i\omega t}$$
$$w=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} W\sin\alpha x\,\sin\beta y\,e^{i\omega t}$$
$$Q_x=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} X\cos\alpha x\,\sin\beta y\,e^{i\omega t}$$
$$Q_y=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} Y\sin\alpha x\,\cos\beta y\,e^{i\omega t}$$

(19)

As the last step, by inserting above approximation in equations (16-18), one can easily study the bending analysis of FG nanoplates with simply supported edge condition.

3. Numerical results

The study, here, has been focused on the free vibration behavior of functionally graded rectangular nanoplates based on the nonlocal third order shear deformation theory. In this analysis, the material properties are assumed to vary with the power law distribution along the thickness directions, that is [14],

$$P(z) = P_m + P_{cm}\left(\frac{2z+h}{2h}\right)^p,\ P_{cm} = P_c - P_m \tag{20}$$

where P can be defined as Young modulus and p and h are the power law exponent and thickness of nanoplate. It is assumed that the top surface of nanoplate is subjected to mechanical loading and the bottom surface is traction free. In order to justify the validity of the suggested model, consider a convectional isotropic nanoplate subjected to a uniformly distributed loading. In figure 1, the results given by the present model are compared with the known data for nonlocal classical plate theory for different values of nonlocal parameter. From the data given in this figure the accuracy of our method is demonstrated. Moreover, it is evident that for nonlocal CPT and nonlocal TSDT, increasing the nonlocal parameter, the deflections increase linearly.

Figure 2 depicts the influences of power law exponent and nonlocality on the deflections of simply supported functionally graded nanoplates. It can be seen that with the increase of power law exponent the deflections of nanoplate under uniform loading will increase. From this figure it is also found that increasing the nonlocal parameter will cause the deflections to increase because of the reduction in the rigidity of the functionally graded nanoplate. A similar trend may be observed for the isotropic simply supported rectangular nanoplates in the work of Nami and Janghorban [7]. It is worth to note that the power law index has more effect for higher nonlocal parameters. From this figure one can also understand that in some cases, increasing the power law exponent has no significant effect on the results. Similar conclusion was reported for nonlocal parameters more than 4 nm^2 in the open literature.

Figure 3 shows the influences of the variations of aspect ratio and nonlocal parameter on the deflections of functionally graded nanoplate. It is obtained that with the increase of aspect ratio the deflection increases. It is also concluded that the aspect ratios have more effect on the results in comparison with nonlocal parameter although the small scale effects cannot be ignored.

Now we examine the effects of the power law exponent and nonlocality on the deflections of functionally graded single-layered graphene sheet subjected to sinusoidal loading. Figure 4 shows the variation of deflections versus power law exponent for different nonlocal parameter. It is shown that the trend of the results is similar to the graphene sheets under uniformly distributed loading. Moreover, for each nonlocal parameter, it can be seen that increasing the power law indexes, the deflection approaches to a limit value although increasing the nonlocal parameter will cause a delay to reach this limit value.

In figure 5, the effects of length to thickness ratio and power law index for different nonlocal parameter on the transverse deflections of FG nanoplate is figured. It is shown that with the increase of length to thickness ratio the deflections of FG nanoplate will increase. From this figure it is concluded that the influences of length to thickness ratio can be important especially for thick nanoplates. It is noted that although present methodology is complex in comparison with nonlocal classical plate theory but it has the ability to study both thin and thick nanoplates.

The methodology proposed in present article may provide useful guidance for design of nanodevices that make use of the bending properties of functionally graded nanoplates in the near future.

4. Conclusion

An analytical solution was obtained for the bending analysis of simply supported functionally graded rectangular nanoplate. The formulation was based on third order shear deformation theory and included the nonlocality effects. The material properties of functionally graded rectangular nanoplates were assumed to be varied through the thickness direction on the basis of power law distribution. According to the best of the author's knowledge, the problem described in this article has not been addressed yet so our results can be used as a reference for future works. From above numerical results, it was obtained that,

- It seems that the nonlocal effects for the bending of nanoscale FG plate are more noticeable in higher power law exponent.
- It is shown that increasing the power law indexes, the deflection approaches to a limit value.

- With the increase of length to thickness ratio the deflections of FG nanoplate will increase.

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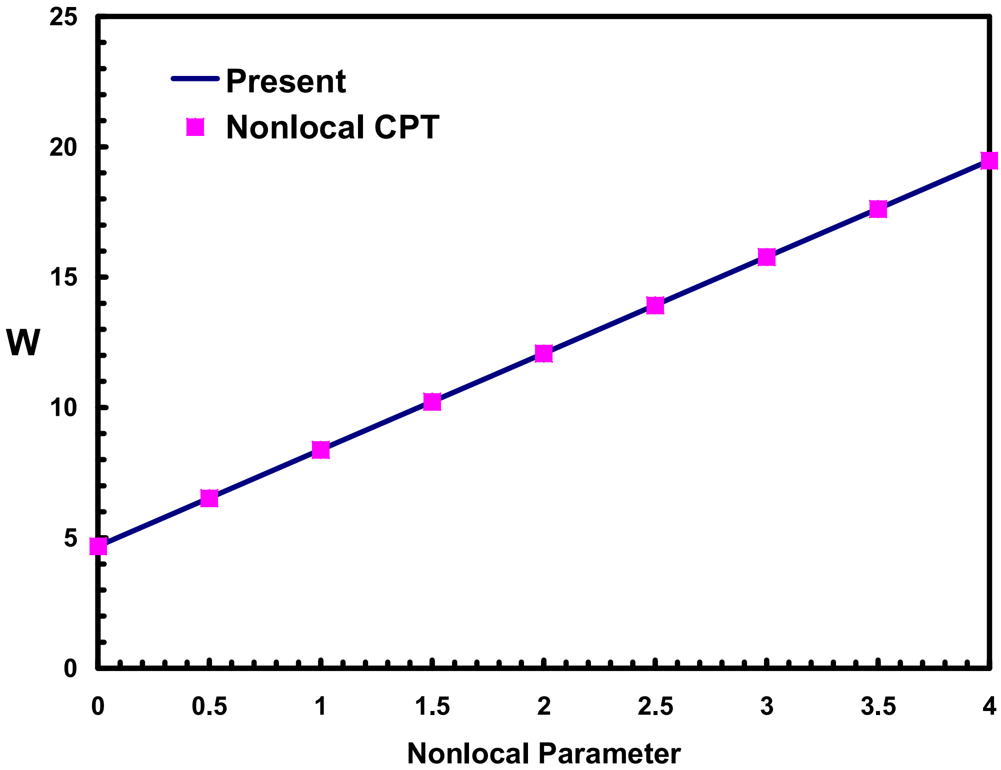


Fig 1. Comparing present results for thin nanoplates with nonlocal CPT

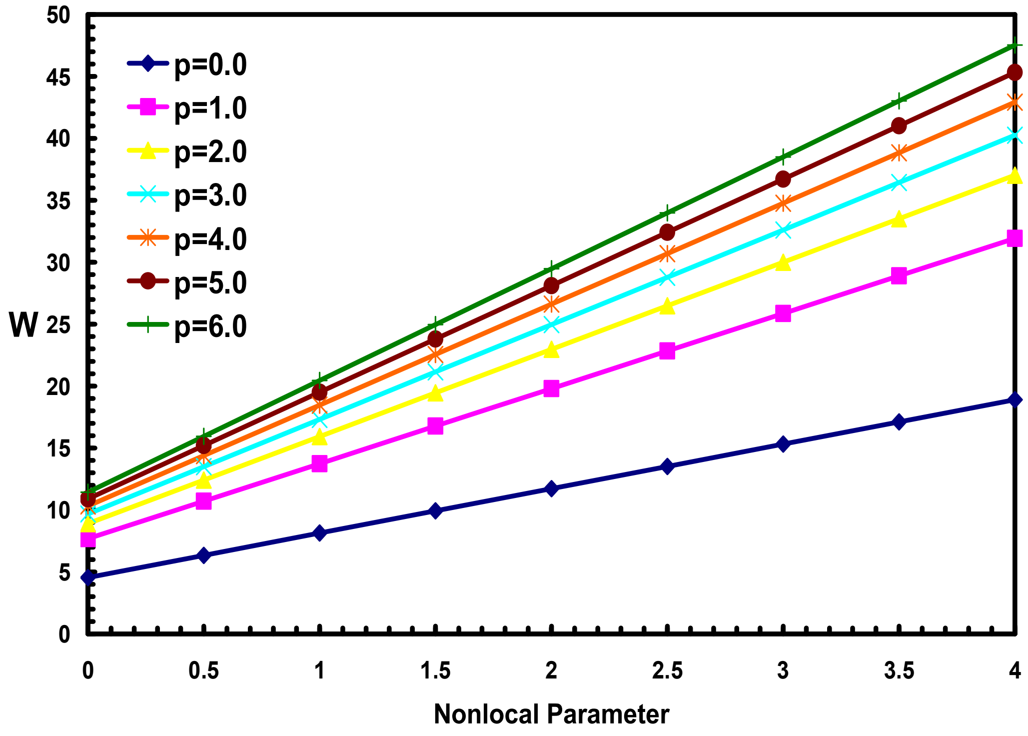


Fig 2. The effects of nonlocal parameter and power index on the deflections

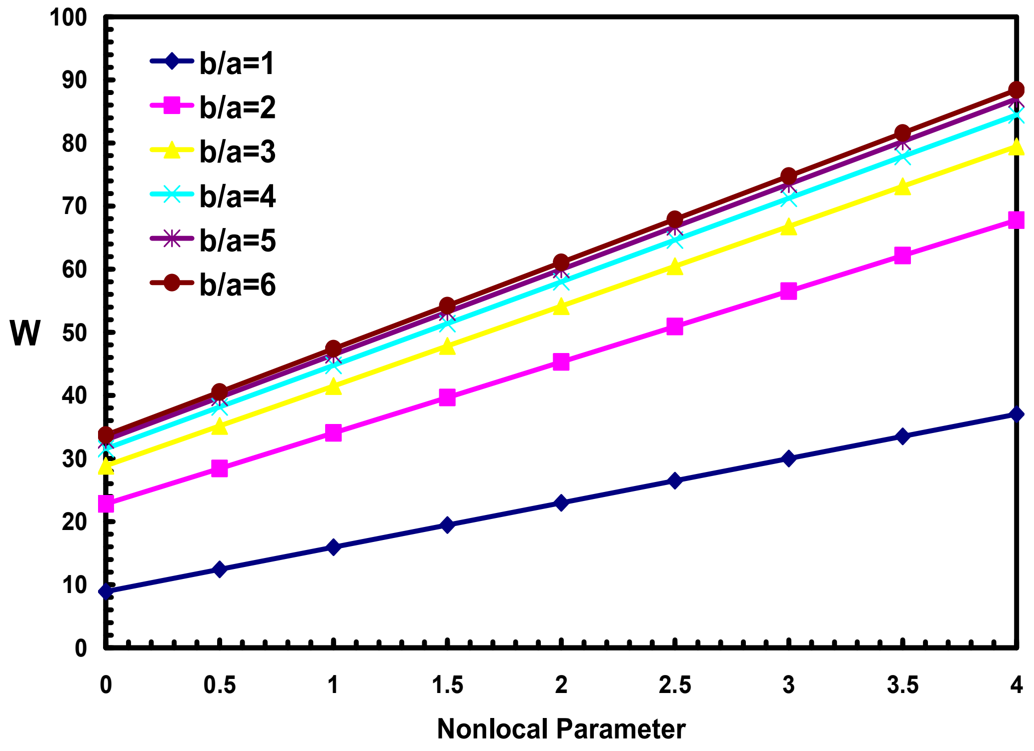


Fig 3. The effects of aspect ratio and nonlocal parameter on the deflections

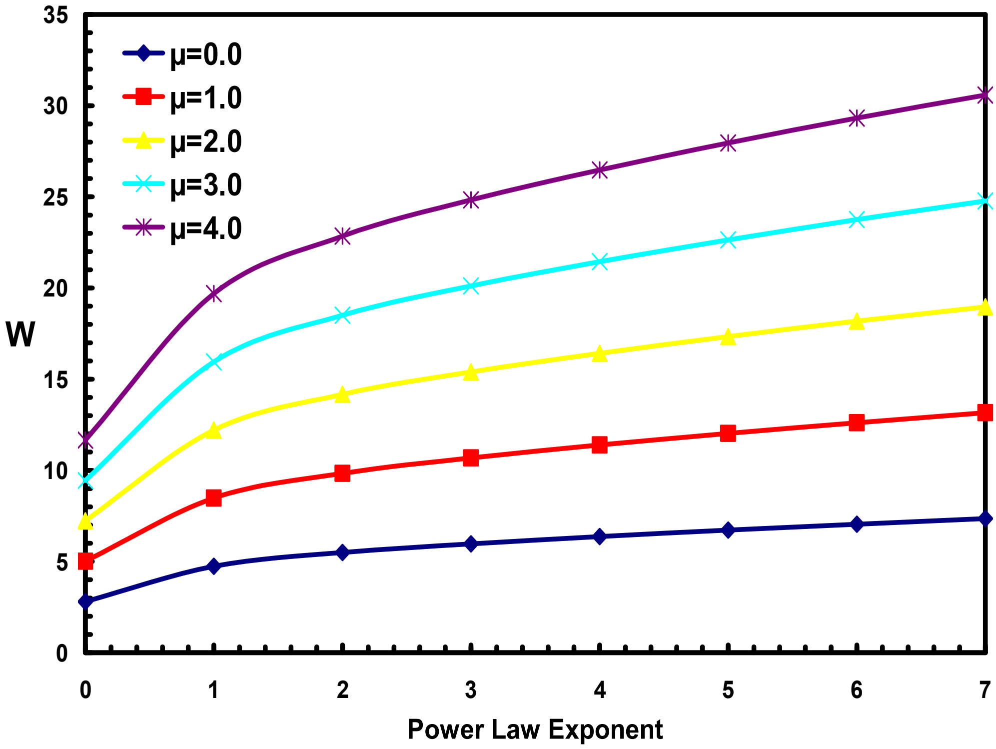
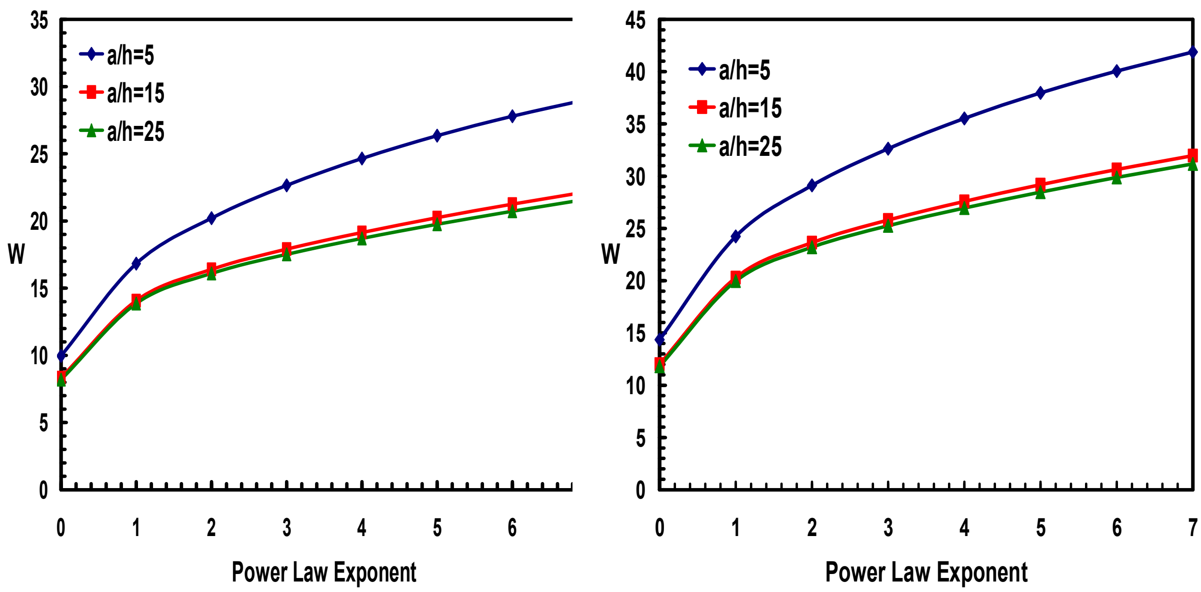
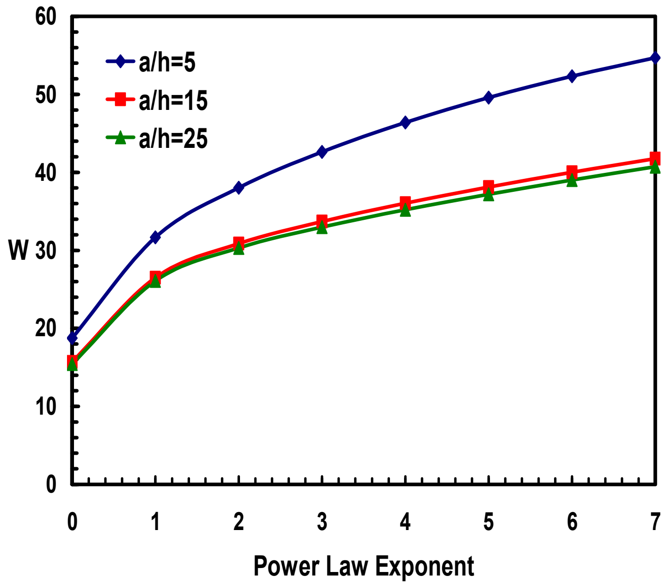


Fig 4. The effects of nonlocal parameter and power index on the deflections under sinusoidal loading



$\mu = 1.0$ $\mu = 2.0$



$\mu = 3.0$

Fig 5. The effects of length to thickness ratio and nonlocality on the deflections