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Stability of A Non-Homogenous Porous Plate by Using Generalized Differantial Quadrature Method

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Abstract

This paper presents stability analysis of a non-homogeneous plate with porosity effect. Material properties of the plate vary in the thickness direction and depend on the porosity. In the solution of the problem, the Generalized Differential Quadrature method is used. In the porosity model, uniform porosity distribution is considered. The effects of the porosity and material distribution parameters on the critical buckling of the non-homogeneous plate are investigated.

Keywords: *Non-Homogeneous Plate; Porosity; Generalized Differential Quadrature Method.*

1. Introduction

Non-homogeneous structures, namely functionally graded structures are a type of composites where the volume fraction of the materials constituents vary gradually, giving a non-uniform microstructure with continuously graded macro properties such as elasticity modulus, density, heat conductivity, etc.. Typically, in non-homogeneous structures, one face of a structural component is ceramic that can resist severe thermal corrosion effects and the other face is metal which has excellent structural strength.

Non-homogeneous structures have been an area of intensive research over the last decade. Because of the wide material variations and applications, it is important to study the static and dynamic analysis of Non-homogeneous structures, such as plates. Therefore, an intensive study has been conducted recently on vibration of structures made of FGMs (i.e., [1–42]).

In the literature, some studies about the porosity effect in the Non-homogeneous structures are; Wattanasakulpong and Ungbhakorn [43] investigated vibration analysis of porous FG beams. Mechab et al. [44,45] examined free vibration analysis of a FG nano-plate resting on elastic foundations with the porosities effect. Şimşek and Aydın [46] examined forced vibration of FG microplates with porosity effects based on the modified couple stress theory. Jahwari and Naguib [47] investigated FG viscoelastic porous plates with a higher order plate theory and statistical based model of cellular distribution. Vibration characteristics of FG beams with porosity effect and various thermal loadings are investigated by [48-49]. Linear/ nonlinear analysis of buckling and vibration of FG beams reinforced porous nanocomposite are investigated by Chen et al. [50] and Kitipornchai et al. [51]. Akbaş [52] investigated static and vibration of FG porous plates by using Navier solution.

Stability analysis of a simply supported non-homogeneous plate is investigated with porosity effect by using Generalized Differential Quadrature Method based on the classical plate theory. The effects of the porosity and material distribution parameters on the critical buckling loads of the non-homogeneous plate are examined.

2. Formulations

A simply supported rectangular non-homogeneous porous plate with thickness h in X_3 *direction*, the lengths of L_X and L_Y the in X_1 and X_2 directions, respectively as shown in Figure 1. The non-homogeneous plate is subjected to biaxial plane compressive loads N in both X_1 and X_2 directions, respectively.

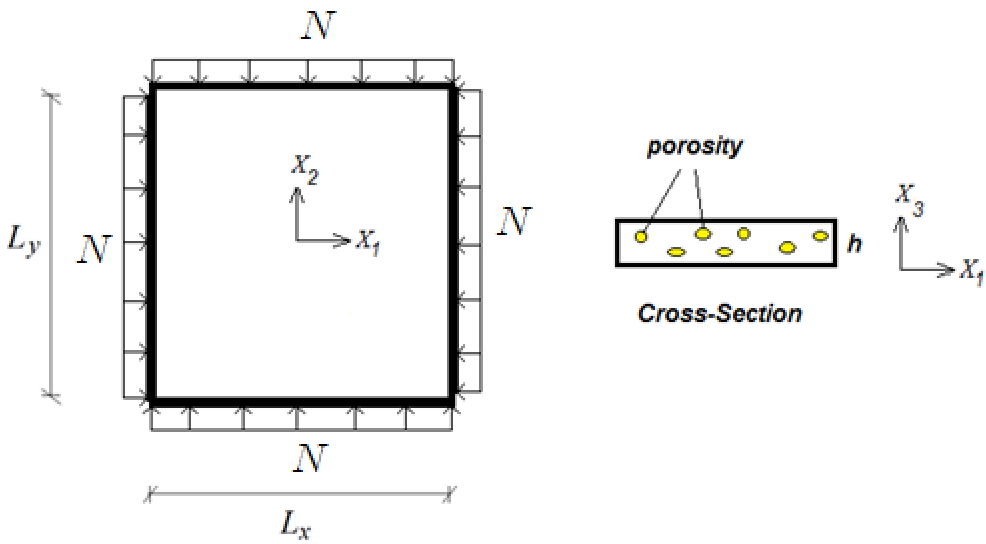

Fig. 1. A non-homogeneous plate subjected biaxial compressive loads with porosity.

The effective material properties of the non-homogeneous plate, P, such as, Young's modulus E, Poisson's ratio v, and shear modulus G vary continuously in the thickness direction (X_3 axis) according to a power-law function. In the porosity model, the porosity spread uniformly though height direction. According to the power law distribution, the effective material property with porosity can be expressed as follows:

$$P(X_3)=(P_T-P_B)\left(\frac{X_3}{h}+\frac{1}{2}\right)^k+P_B-(P_T+P_B)\frac{a}{2} \tag{1}$$

where a (a<<1) is the volume fraction of porosities. When a=0, the plate becomes perfect non-homogeneous plate.

According to classical plate theory, the strain- displacement relations are expressed as

$$\varepsilon_{X_1}=\frac{\partial u}{\partial X_1}=\varepsilon_{X_1}{}^0-X_3\frac{\partial^2 \mathrm{v}}{\partial {X_1}^2} \tag{2a}$$

$$\varepsilon_{X_2} = \frac{\partial v}{\partial X_2} = \varepsilon_{X_2}{}^0 - X_3 \frac{\partial^2 \mathrm{v}}{\partial {X_2}^2} \tag{2b}$$

$$\gamma_{X_1X_2} = \frac{1}{2}\left(\varepsilon_{X_1X_2}{}^0 - \frac{\partial^2 \mathrm{v}}{\partial X_1 \partial X_2}\right) \tag{2c}$$

where u, v, w are X_1, X_2 and X_3 components of the displacements respectively. The constitutive equations of the non-homogeneous plate are as follows:

$$\sigma_{\mathrm{ij}}(X_{3,}a) = \frac{E(X_3,a)}{(1-\nu^2)}\left[\nu\varepsilon_{kl}\delta_{ij} + (1-\nu)\varepsilon_{ij}\right] \tag{3}$$

The stress resultants of the non-homogeneous plate are given as follows;

$$N_{ij} = \int_{-0.5h}^{0.5h} \sigma_{\mathrm{ij}}\, dX_3 \quad i = j,\ M_{ij} = \int_{-0.5h}^{0.5h} \sigma_{\mathrm{ij}}\, X_3\, dX_3,\ Q_{ij} = \int_{-0.5h}^{0.5h} \sigma_{\mathrm{ij}}\, dX_3 \quad i \neq j \tag{4}$$

where N_{ij}, M_{ij} and Q_{ij} are normal force, moment and shear forces, respectively. The stability equation of the non-homogeneous plate is given as follows:

$$\nabla^4 \nu - \frac{A_1(1-\nu^2)}{A_1A_3 - {A_2}^2}\left({N_1}^0 \frac{\partial^2 \mathrm{v}}{\partial {X_1}^2} + {N_2}^0 \frac{\partial^2 \mathrm{v}}{\partial {X_2}^2}\right) = 0 \tag{5}$$

where ${N_1}^0$ and ${N_2}^0$ are the pre-buckling force resultants, A_1, A_2, A_3 are expressed as follows:

$$(A_1, A_2, A_3) = \int_{-0.5h}^{0.5h} E(X_3, a)(1, X_3, {X_3}^2)\, dX_3 \tag{6}$$

The boundary conditions at the simple supported plate ends are as follows;

$$v(X_1, 0) = v(L_X, 0) = w(0, X_2) = w(0, L_Y) = 0 \tag{7a}$$

$$M(X_1, 0) = M(L_X, X_2) = M(X_1, L_Y) = M(0, X_2) = 0 \tag{7b}$$

In the solution of the governing equations, the Generalized Differential Quadrature Method is used. In the differential quadrature method, the derivatives of a function are written as linear summation of the values at all points in the domain [53-56];

$$\frac{d^{(p)} w(x_j)}{dx^{(p)}} \approx \sum_{i=1}^{n} B_{ji}^{(p)}\, w(x_i) \tag{8}$$

where n is the number of the points in the domain, p is the order of derivative in the function, $B_{ji}^{(p)}$ is the weighting coefficient with pth derivative of the function with respect to x. The weight coefficients for first-order derivative (p=1) are as follows [53,54];

$$B_{ji}^{(1)} = \begin{cases} \dfrac{\prod_{j=1}^{n}(x_j - x_i)}{(x - x_j)\prod_{j=1}^{n}(x_i - x_j)} & i \neq j \\ -\sum_{j=1, i\neq j}^{n} B_{ji}^{(1)} & i = j \end{cases} \tag{9}$$

For the higher order derivatives, the weight coefficient is expressed as follows:

$$B_{ji}^{(p)} = \sum_{r=1}^{n} B_{jr}^{(1)} B_{ri}^{(p-1)} \quad (i,j=1,n) \tag{10}$$

For determined the sampling points in the domain, Chebyshev–Gauss–Lobatto grid points is employed[53,54];

$$x_j = \frac{1}{2}\left[1 - cos\left(\frac{j-1}{n-1}\pi\right)\right] \qquad (j=1,n_{x1}) \tag{11a}$$

$$x_i = \frac{1}{2}\left[1 - cos\left(\frac{i-1}{n-1}\pi\right)\right] \qquad (i=1,n_{x2}) \tag{11b}$$

where n_{x1} and n_{x2} are the number of the grid points in X_1 and X_2 direction, respectively.

Substituting eqs. (8-11) into eq. (5), and then using Generalized Differential Quadrature discretization, the governing equations of the problem can be obtained as follows;

$$\left(\sum_{k=1}^{n_{x1}} B_{jk}^{(4)}\, v_{kj} + 2\sum_{k=1}^{n_{x1}}\sum_{m=1}^{n_{x2}} B_{jk}^{(2)}\, B_{im}^{(2)} v_{km} + \sum_{k=1}^{n_{x2}} B_{ik}^{(4)}\, v_{ki}\right) - \frac{A_1(1-\nu^2)}{A_1A_3-{A_2}^2}\left({N_1}^0 \sum_{k=1}^{n_{x1}} B_{jk}^{(2)}\, v_{kj} + {N_2}^0 \sum_{k=1}^{n_{x2}} B_{ik}^{(2)}\, v_{ki}\right) = 0 \quad (j=1,n_{x1}), (i=1,n_{x2}), (k=1,\mathrm{p}+1) \tag{12}$$

The dimensionless critical buckling load can be expressed as follows;

$$\bar{N}_{\mathrm{cr}} = N_{cr}\, \frac{L_X^2}{E_B\, h^3} \tag{13}$$

3. Numerical Results

In the numerical results, the dimensionless critical buckling loads $\bar{N}_{\mathrm{cr}}$ are presented in figures for different porosity parameters and material distributions. The rectangular non-homogeneous porous plate considered in numerical examples is made of Zirconia (E=151*GPa*, *ν*=0.3) and Steel (E=210*GPa*, *ν*=0.3). The top surface material of the non-homogeneous plate is Zirconia, the bottom surface material of the non-homogeneous plate is Steel. When *k*=0 and *k*=∞, the material of the plate gets homogeneous Zirconia and homogeneous Steel, respectively, according to Eq. (1). The dimensions of the non-homogeneous plate are considered as follows: h = 0.2 *m*, L_X =3 *m*, L_Y=3 *m* in the numerical examples. In the numerical calculations, the numbers of the grid points are taken as n_{x1}=n_{x2}=20.

In figure 2, the effect of the material distribution parameter *k* on the dimensionless critical buckling loads of the porous non-homogeneous plate is presented for a=0. As seen from figure 2, the dimensionless critical buckling loads increase with increase in the power-law exponents *k*. With increase in the *k*, the plate gets to fully Steel. The Young's modulus of Steel is bigger than Zirconia’s. As it is expected, with increase the *k*, the Young's modulus and bending rigidity of the plate increase according to equation (1). So, the strength of material increases and the critical buckling loads increases naturally.

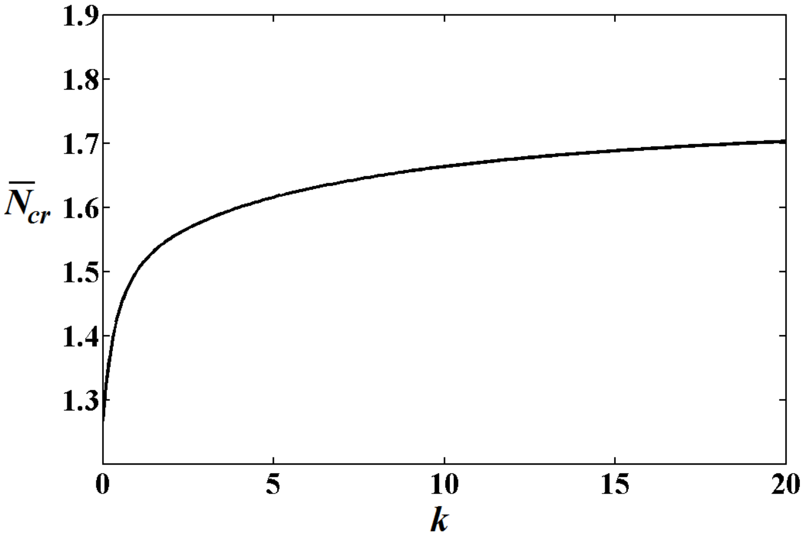

Fig. 2. The effect of the material distribution parameter *k* on the dimensionless critical buckling loads $\bar{N}_{cr}$.

Figure 3 displays the relationship between of porosity parameter *a* and the dimensionless critical buckling loads of the non-homogeneous porous plate for different the material distribution parameters. It is seen from figure 3 that the dimensionless critical buckling loads decrease with increase with increase porosity parameter *a*. This is because, with increase in the porosity, the strength of the material decreases. So, the critical buckling loads decreases naturally. It shows that Porosity parameters play an important role on the stability of the non-homogeneous porous plates.

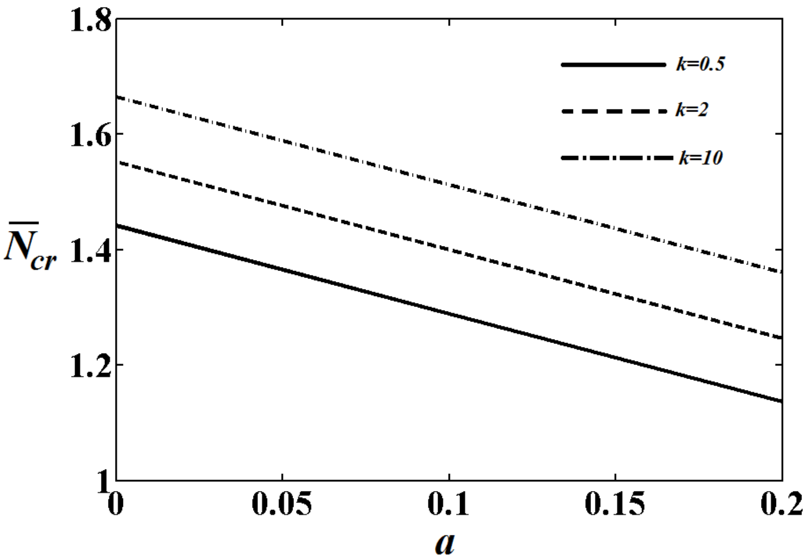

Fig. 3. The effect of the porosity parameter *a* on the dimensionless critical buckling loads $\bar{N}_{cr}$ for different the material distribution parameters.

4. Conclusions

In this paper, stability analysis of a simply supported porous non-homogeneous plate is studied by using Generalized Differential Quadrature Method. Material properties of the plate depend on both position and porosity. The Classical plate theory is used in the kinematic model of the plate. The effects of the porosity and material distribution parameters on the critical buckling loads of the non-homogeneous plate are presented in figures. Numerical results show that the porosity has important role on the stability of the non-homogeneous plate.

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