International Journal of Engineering & Applied Sciences (IJEAS)
Vol.10, Issue 1 (2018)21-34
http://dx.doi.org/10.24107/ijeas.420838
Int J Eng Appl Sci 10(1) (2018) 21-34

Analytical Buckling of FG Nanobeams on The Basis of A New One Variable First-Order Shear Deformation Beam Theory

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Received date: 03.05.2018
Accepted date: 24.05.2018

Abstract

In this work, buckling analysis of functionally graded (FG) nanobeams based on a new refined beam theory has been analyzed. The beam is modeled as an elastic beam subjected to unidirectional compressive loads. To achieve this aim, the new obtained beam theory has only one variable which lead to one equation similar to Euler beam theory and also is free of any shear correction factor. The equilibrium equation has been formulated by the nonlocal theory of Eringen to predict small-scale effects. The equation has been solved by Navier's approach by which critical buckling loads have been obtained for simple boundaries. Finally, to approve the results of the new beam theory, various beam theories have been compared.

Keywords: Buckling analysis, FG nanobeams, A new refined beam theory, Nonlocal elasticity theory, Navier's approach

1. Introduction

Carbon nanotubes (CNTs) are seamless cylinders included one to multi-graphene layers with open or close ending that they are called single-walled (SWCNT) or multi-walled carbon nanotubes (MWCNT) [1]. Todays, the most manufactured CNTs are used in composite materials and thin films [1]. The SWCNT is remarkably strong and hard [2], conducting electric current and directing heat [3-5], which has led to the use of these materials in the electronics industry [6-7]. The carbon nanotube promises a bright future in cellular experiments because they can be used as nano-pipes to distribute very small volumes of fluid or gas into living cells or on surfaces [8-10].
To exploit the industrial amazing properties of nanostructures, it can be highly recommended that the mechanical behavior of them should be analyzed. In last years, this issue has been taken into consideration by researchers around the world in order to identify the behavior of them under various mechanical conditions. Among these researchers, Reddy [11] reformulated beam theories by using nonlocal elasticity theory for vibrations, buckling and bending analyses. Civalek et al. [12] analyzed natural frequencies of a skew symmetric composite plate using discrete convolution method (DSC). Malikan et al. [13] published stability of bi-layer graphene nanoplates subjected to shear and thermal forces on the basis of a medium using numerical solutions. Malikan investigated stability analysis of a micro

sandwich plate with graphene coating using the refined couple stress theory [14] and buckling of graphene sheets subjected to nonuniform compression based on the four-variable plate theory using an analytical approach [15]. Yao and Han [16] presented buckling of double-walled carbon nanotubes with considering thermal influences. They obtained critical buckling loads on the basis of Donnell's equilibrium equation and solved the equation for simply-supported boundary condition. Ansari et al. [17] studied coupled natural frequency analysis of post stability functionally graded micro/nanobeams on the basis of the strain gradient theory. Wang et al. [18] presented exact modes for post stability characteristics of nonlocal nanobeams in a longitudinal magnetic field. Wang et al. [19] utilized both stress and strain gradient continuum theories to consider buckling of nanotube which was embedded in an elastic foundation. Timoshenko beam theory and Navier solution method were employed in their study. They proved that both stress gradient and strain gradient predict the same results if the nonlocal effect is not taken into account. Xiang et al. [20] used nonlocal elasticity theory for studying nonlinear free vibration of double-walled carbon nanotubes based on Timoshenko beam theory. Ansari et al. [21] developed Rayleigh–Ritz method for buckling of carbon nanotubes considering thermal effects. They classical Donnell shell theory was incorporated in conjunction with nonlocal elasticity theory of Eringen. Ansari et al. [22] employed Timoshenko beam model to consider buckling and postbuckling of nanotubes using nonlocal elasticity theory. The equations were solved with generalized differential quadrature method and the pseudo arc-length technique for several boundary conditions. Ansari and Arjangpay [23] presented using the meshless local Petrov–Galerkin method for various boundary conditions to analyze carbon nanotubes under buckling and vibrations. The vibration of thermally post-buckled carbon nanotube-reinforced composite beams resting on elastic foundations has been examined by Shen et al. [24]. Beni et al. [25] studied vibration of shell nanotubes using nonlocal strain gradient theory and molecular dynamics simulation. Wang et al. [26] presented nonlinear vibration of nonlocal carbon nanotubes placed on the visco-Pasternak foundation under excitation frequency. Civalek et al. [27] investigated laminated composites in static conditions on the basis of nonlinear first-order shear deformation theory. The equations were discretized and solved with the singular convolution method (DSC). Reddy [28] developed couple stress theories for functionally graded Euler-Bernoulli and Timoshenko microbeams. Reddy and Arbind [29] derived a couple stress theory for bending analysis of Euler and Timoshenko functionally graded beams. Stability analysis of nanotubes made of boron nitride embedded on the elastic matrix using DSC has been presented by Mercan and Civalek [30]. Akgöz and Civalek [31] studied nonlocal buckling of carbon nanotubes subjected to an axial compressive load sorrounded by Pasternak matrix. In their study various beam theories were applied and governing equations were analytically solved by Navier solution method. Civalek et al. [32] developed the modified couple stress, the strain gradient and nonlocal elasticity theories for buckling of silicon carbide nanowires-based Euler beam theory. Akgöz and Civalek [33] considers influences of thermal and shear deformations on the vibrations of a functionally graded thick micro composite beam.

In this theoretical work, we report a new beam theory by reducing the unknown variables from a regenerated shear deformation theory. The functionally graded (FG) nanobeam is modeled as an elastic beam which is subjected to unidirectional compressive load. The influence of stress nonlocality is examined by using nonlocal elasticity theory of Eringen which leads to a size-dependent equation. Furthermore, Navier's technique is exerted to solve the stability equation by assuming simply-supported boundary condition for both edges of the beam. To approve the present formulation, various beam theories have been analyzed resulted from several well-known references.

2. Mathematical Formulation

Fig. 1 displays a realistic model for the nanobeam subjected to unidirectional compressive loads with length *L*, outer diameter *d* and thickness *h* parallel to *x* and *z*-axes, respectively. First, according to first-order shear deformation beam (FSDT) theory, the displacement field is presented as below [13, 34-36]:

$$\begin{Bmatrix} U(x,z) \\ V(x,z) \\ W(x,z) \end{Bmatrix} = \begin{Bmatrix} u(x)+z\varphi(x) \\ 0 \\ w(x) \end{Bmatrix} \tag{1a-c}$$

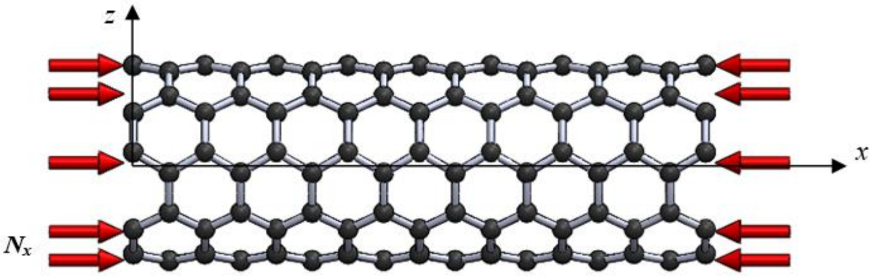

Fig. 1. The SWCNT subjected to the unidirectional compressive load

In Eq. (1), the vector quantities of the neutral axis at directions of *x* and *z* are *u* and *w*, respectively. Moreover, for defining of the rotation of beam elements around the *x* axis, φ is used. First off, let us reconsider the simple first-order shear deformation theory (S-FSDT) by which the deflections were re-formulated in the following equation [37-39]:

$$w = w(bending) + w(shear) \tag{2}$$

Also, φ parameter was developed as below:

$$\{\varphi\} = \left\{ -\frac{dw_b}{dx} \right\} \tag{3}$$

By replacement Eqs. (2-3) in Eq. (1) the displacement field of the S-FSDT was rewritten as follows [37-39]:

$$\begin{Bmatrix} U(x,z) \\ W(x,z) \end{Bmatrix} = \begin{Bmatrix} u(x) - z\dfrac{dw_b(x)}{dx} \\ w_b(x) + w_s(x) \end{Bmatrix} \tag{4a-b}$$

Use of $w = w_b + w_s$ might not be conceptual; Therefore, Eq. 4 would be refined in the following:

$$\begin{Bmatrix} U(x,z) \\ W(x,z) \end{Bmatrix} = \begin{Bmatrix} u(x) - z\dfrac{dw_b(x)}{dx} \\ w_b(x) + W' \end{Bmatrix} \tag{5a-b}$$

So, we could use bending deflection to find the value of w_s:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{xz} \end{Bmatrix} = \begin{Bmatrix} E(z)\varepsilon_{xx} \\ 2G(z)\gamma_{xz} \end{Bmatrix} \tag{6a-b}$$

After obtaining Eq. (6) from S-FSDT the stresses can be found and then by substituting Eq. (6) in the S-FSDT stress resultants, Eq. 7 will be calculated:

$$\begin{Bmatrix} M_x \\ Q_x \end{Bmatrix} = \int_A \begin{Bmatrix} \sigma_x z \\ \sigma_{xz} \end{Bmatrix} dA \tag{7a-b}$$

Let us use fourth equation of FSDT's governing equations in order to calculate w_s based on w_b:

$$\frac{dM_x}{dx} - Q_x = 0 \tag{8}$$

By imposing Eq. (8) into the stress resultants of Eq. (7):

$$E(z)I_c \frac{d^3 w_b}{dx^3} - AG(z)\frac{dw_s}{dx} = 0 \tag{9}$$

By integrating from Eq. (9) based on x, simplifying and then ignoring the integral constant terms, the shear deflection will now be obtained as follows:

$$w_s = W' = B\frac{d^2 w_b}{dx^2} \tag{10}$$

Term B could be in both positive and negative signs that is explained:

$$B = \frac{E(z)I_c}{AG},\ G(z) = \frac{E(z)}{2(1+\upsilon)} \tag{11}$$

where G represents the shear modulus, E is the Young's modulus, $I_c\left(\pi d^4/64\right)$ denotes the moment of area of the cross-section, A is the cross-sectional area and v is the Poisson's ratio for isotropic nanobeams. Afterwards, the new beam theory will now be achieved as:

$$\text{Now: } w_b = w \quad ; \quad \begin{Bmatrix} U(x,z) \\ W(x,z) \end{Bmatrix} = \begin{Bmatrix} u(x) - z\dfrac{dw(x)}{dx} \\ w(x) + B\dfrac{d^2 w(x)}{dx^2} \end{Bmatrix} \tag{12a-b}$$

Regarding Hamilton's principle, the potential energy in the whole domain of the beam (V) is made available and is written in the variational form as below [40]:

$$\delta V = \delta S + \delta\Omega = 0 \tag{13}$$

In which δS is the variation of strain energy and δV is the variation of works, which are done by external forces. The strain energy by variational formulation will be calculated:

$$\delta S = \iiint_v \sigma_{ij}\,\delta\varepsilon_{ij}\,dV = 0 \tag{14}$$

The strain tensor in Eq. (14) is expanded as follows:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \dfrac{du}{dx} - z\dfrac{d^2 w}{dx^2} + \dfrac{1}{2}\left(B\dfrac{d^3 w}{dx^3} + \dfrac{dw}{dx} \right)^2 \\ B\dfrac{d^3 w}{dx^3} \end{Bmatrix} \tag{15a-b}$$

With applying the variational formulation (δV=0) the nonlinear governing equation of motion is derived:

$$\delta w = 0;\ \frac{d^2 M_x}{dx^2} - B\frac{d^3 Q_x}{dx^3} - N_x\left(B^2\frac{d^6 w}{dx^6} + 2B\frac{d^4 w}{dx^4} + \frac{d^2 w}{dx^2} \right) = q_0 \tag{16}$$

In which M_x, Q_x, and N_x are nonlocal stress resultants, respectively and q_0 is the transverse static load which is ignored in this paper. Here, the quantity N_x is the resultant with respect to the axial applied compressive force. With regard to nonlocal theory of Eringen, the following equation is employed [13, 40]:

$$\left(1-\mu\nabla^2\right)\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl}\ ;\quad \mu(nm^2) = \left(e_0 a\right)^2,\ \nabla^2 = \frac{d^2}{dx^2} \tag{17}$$

where μ is the nonlocality factor and a is an interior determined length.
The material property gradation considering power law in the FG nanobeams is expressed as [41-43]:

$$E(z) = E_m + \left(E_c - E_m\right)\left(\frac{1}{2} + \frac{z}{h}\right)^k \tag{18}$$

Here E_c and E_m are the Young's modulus corresponding to ceramic and metal, respectively, and k is volume fraction exponent or material grading/power law index. Due to insignificant variation of the Poisson's ratio, this variant is assumed to be constant along the thickness (v (z) = v). From Eq. (18), when $k\rightarrow\infty$, the FG nanobeam reduces to a pure metal one and for case k=0, the plate becomes pure ceramic.

The stress resultants in local form are specified by relations below:

$$\begin{Bmatrix} M_x \\ Q_x \end{Bmatrix} = \int_A \begin{Bmatrix} \sigma_x z \\ \sigma_{xz} \end{Bmatrix} dA \tag{19a-b}$$

Now, by substituting Eq. (15) into the Eq. (19) the stress resultants will be given as follows:

$$\begin{Bmatrix} M_x \\ Q_x \end{Bmatrix} = \begin{Bmatrix} -EI_c \dfrac{d^2w}{dx^2} \\ AGB \dfrac{d^3w}{dx^3} \end{Bmatrix} \tag{20a-b}$$

The compressive force is assumed as follows [40, 44]:

$$N_x = -P_{Cr} \tag{21}$$

Now, incorporating Eq. (17, 20-21) and inserting them into Eq. (16) and also some manipulating, lead to the stability equation of one variable first-order shear deformation theory (OVFSDT) as:

$$\begin{aligned} \delta w = 0: EI_c \frac{d^4w}{dx^4} + B^2 AG \frac{d^6w}{dx^6} - P_{Cr}\left(B^2 \frac{d^6w}{dx^6} + 2B \frac{d^4w}{dx^4} + \frac{d^2w}{dx^2}\right) + \\ \mu P_{Cr}\left(B^2 \frac{d^8w}{dx^8} + 2B \frac{d^6w}{dx^6} + \frac{d^4w}{dx^4}\right) = 0 \end{aligned} \tag{22}$$

Also by using Eq. (4) the S-FSDT equations could be obtained as follows:

$$\begin{aligned} \delta w_b = 0: EI_c \frac{d^4w_b}{dx^4} - P_{Cr}\left(\frac{d^2w_b}{dx^2} + \frac{d^2w_s}{dx^2}\right) + \mu P_{Cr}\left(\frac{d^4w_b}{dx^4} + \frac{d^4w_s}{dx^4}\right) = 0 \\ \delta w_s = 0: AG \frac{d^2w_s}{dx^2} - P_{Cr}\left(\frac{d^2w_b}{dx^2} + \frac{d^2w_s}{dx^2}\right) + \mu P_{Cr}\left(\frac{d^4w_b}{dx^4} + \frac{d^4w_s}{dx^4}\right) = 0 \end{aligned} \tag{23a-b}$$

On the other hand, by using Eq. (1) the FSDT equations could be obtained as follows:

$$\delta w = 0: k_s AG\left(\frac{d^2w}{dx^2} - \frac{d\varphi}{dx}\right) - P_{Cr}\frac{d^2w}{dx^2} + \mu P_{Cr}\frac{d^4w}{dx^4} = 0 \tag{24a}$$

$$\delta\varphi = 0: EI_c \frac{d^2\varphi}{dx^2} + k_s AG\left(\frac{dw}{dx} - \varphi\right) = 0 \tag{24b}$$

Furthermore, for CPT the stability equation is obtained in the following form:

$$\delta w = 0: EI_c \frac{d^4w}{dx^4} - P_{Cr}\frac{d^2w}{dx^2} + \mu P_{Cr}\frac{d^4w}{dx^4} = 0 \tag{25}$$

3. Navier's technique

The Navier solution method has been applied to present simply-supported boundary condition according to Eq. (26) [44].

$$w(x,t)=\sum_{m=1}^{\infty}W_m \sin\left(\frac{m\pi}{L}x\right)e^{i\omega t} \tag{26a}$$

$$\varphi(x,t)=\sum_{m=1}^{\infty}\Phi_m \cos\left(\frac{m\pi}{L}x\right)e^{i\omega t} \tag{26a}$$

where *m* is the half-wave number as a integer one, W_m and Φ_m are the unknown terms which should be determined and also ω is the natural frequency in vibrational analysis. Substituting Eq. (26) into Eqs. (22-25), the algebraic equation is obtained from which the critical buckling load equation is calculated as follows:

- OVFSDT:

$$P_{Cr}=\frac{EI_c\left(\frac{m\pi}{L}\right)^4-\frac{(EI_c)^2}{AG}\left(\frac{m\pi}{L}\right)^6}{\left(\frac{EI_c}{AG}\right)^2\left(\frac{m\pi}{L}\right)^6-\frac{2EI_c}{AG}\left(\frac{m\pi}{L}\right)^4+\left(\frac{m\pi}{L}\right)^2+\mu\left(\left(\frac{EI_c}{AG}\right)^2\left(\frac{m\pi}{L}\right)^8-\frac{2EI_c}{AG}\left(\frac{m\pi}{L}\right)^6+\left(\frac{m\pi}{L}\right)^4\right)} \tag{27}$$

- CPT:

$$P_{Cr}=\frac{EI_c\left(\frac{m\pi}{L}\right)^4}{\left(\frac{m\pi}{L}\right)^2+\mu\left(\frac{m\pi}{L}\right)^4} \tag{28}$$

- S-FSDT:

$$\begin{bmatrix} EI_c\left(\frac{m\pi}{L}\right)^4+P_{Cr}\left(\frac{m\pi}{L}\right)^2+\mu P_{Cr}\left(\frac{m\pi}{L}\right)^4 & P_{Cr}\left(\frac{m\pi}{L}\right)^2+\mu P_{Cr}\left(\frac{m\pi}{L}\right)^4 \\ P_{Cr}\left(\frac{m\pi}{L}\right)^2+\mu P_{Cr}\left(\frac{m\pi}{L}\right)^4 & -AG\left(\frac{m\pi}{L}\right)^2+P_{Cr}\left(\frac{m\pi}{L}\right)^2+\mu P_{Cr}\left(\frac{m\pi}{L}\right)^4 \end{bmatrix}\begin{Bmatrix} w_b \\ w_s \end{Bmatrix}=0 \tag{29}$$

- FSDT:

$$\begin{bmatrix} -k_sAG\left(\frac{m\pi}{L}\right)^2+P_{Cr}\left(\frac{m\pi}{L}\right)^2+\mu P_{Cr}\left(\frac{m\pi}{L}\right)^4 & k_sAG\left(\frac{m\pi}{L}\right) \\ k_sAG\left(\frac{m\pi}{L}\right) & -EI_c\left(\frac{m\pi}{L}\right)^2-k_sAG \end{bmatrix}\begin{Bmatrix} w \\ \varphi \end{Bmatrix}=0 \tag{30}$$

If determinant of coefficients of Eqs. (29) and (30) is set to zero, the critical buckling load of S-FSDT and FSDT can be calculated.

4. Numerical results

In the first glance it is required to consider the precision of the numerical results obtained from the proposed beam theory with other theories. Hence, as can be seen in Tables 1 and 2, references [45-47] are employed. In [45] a nano rod was based on the both Euler and Timoshenko (Table 1) beam theories and the equations were solved by using an explicit analytical method and differential transform method. On the other hand, in ref. [47] Euler and Timoshenko nano rods were modeled and Navier solution method was utilized in order to obtain numerical results. In fact, both thin and moderately thick beams are compared and carried out with both ends simple boundaries. It is worth noting that with increasing length to diameter ratio of the nano rod the results in the Tables are becoming closer to one another. This means that for thin beams the proposed theory makes same predictions with Euler beam theory which is an acceptable conclusion. Because thin beam theories like Euler can predict appropriate results only for thin beams due to lack of considering transverse strain influences in such a theory. This strain is fundamentally required for response of moderately thick beams which is embedded in proposed theory. It can be seen that for lower values of length to diameter ratio which the rod goes into moderately thick and thick cases the results of Euler beam theory are in a major difference with present formulation. Furthermore, increasing small-scale parameter decreases the gap between the results of current beam theory and others. It is interesting to note that the results of S-FSDT and OVFSDT are corresponded to each other completely. Note that the shear correction factor used in Timoshenko theory can be a serious defect in light of the approximate quantity of it (k_s=5/6). Although this value has been applied for moderately thick models, it cannot be an exact value to analyze several cases, in particular nanostructures. But in the proposed beam theory this extra factor is vanished from the governing equation leads to further accurate results.

To have further comparison, Table 3 is presented in which the proposed theory is compared with ref. [48] within which a functionally graded nanobeam was analyzed with both Euler and Timoshenko beam theories and the equations were solved by Navier solution method. This Table approved the results of previous Tables for thin beams in light of the proximity of all of the beam theories to one another. Moreover, it can be seen that by an increase in the material grading index the difference of the present theory with others will be increased; however, this difference for moderately thick beams is further than thin ones. Generally, Tables 1 to 3 show the close numerical results between the present theory and others from which the theory can be confirmed. Although the new theory of beam which is used could not be a complete theory, by carrying out the errors and refining them the more appropriate numerical results will be obtained.

Table 1. Results of critical buckling load (*nN*) developed from several theories for a rod *(E=1TPa, υ= 0.19, d=1nm)*

	P_{Cr} *(nN)*									
	e_0a=0 nm		*e_0a=0.5 nm*		*e_0a=1 nm*		*e_0a=1.5 nm*		*e_0a=2 nm*	
L (nm)	*EB* [45-46], TB** [45]*	*OVFSDT FSDT*** S-FSDT*	*EB* [45-46], TB** [45]*	*OVFSDT FSDT*** S-FSDT*	*EB* [45-46], TB** [45]*	*OVFSDT FSDT*** S-FSDT*	*EB* [45-46], TB** [45]*	*OVFSDT FSDT*** S-FSDT*	*EB* [45-46], TB** [45]*	*OVFSDT FSDT*** S-FSDT*
10	4.8447 4.7670	4.7609 4.7609 4.7609	4.7281 4.654	4.7985 4.6462 4.7985	4.4095 4.3450	4.4752 4.3332 4.4752	3.9644 3.9121	4.0234 3.8957 4.0234	3.4735 3.4333	3.5252 3.4133 3.5252
12	3.3644 3.3267	3.3991 3.3237 3.3991	3.3077 3.2713	3.3418 3.2677 3.3418	3.1486 3.1156	3.181 3.1105 3.181	2.9149 2.8865	2.9449 2.8797 2.9449	2.6405 2.6172	2.6677 2.6086 2.6677
14	2.4718 2.4514	2.4905 2.4498 2.4905	2.4411 2.4212	2.4595 2.4193 2.4595	2.3533 2.3348	2.3711 2.3323 2.3711	2.2202 2.2038	2.237 2.2005 2.237	2.0574 2.0432	2.0729 2.0391 2.0729
16	1.8925 1.8805	1.9034 1.8795 1.9034	1.8744 1.8626	1.8852 1.8616 1.8852	1.8222 1.8111	1.8327 1.8098 1.8327	1.7414 1.7313	1.7515 1.7295 1.7515	1.6396 1.6306	1.6491 1.6284 1.6491
18	1.4953 1.4878	1.5021 1.4872 1.5021	1.484 1.4766	1.4907 1.476 1.4907	1.4511 1.4440	1.4577 1.4432 1.4577	1.3994 1.3928	1.4057 1.3918 1.4057	1.3329 1.3269	1.3389 1.3257 1.3389
20	1.2112 1.2063	1.2156 1.2059 1.2156	1.2038 1.1989	1.2082 1.1985 1.2082	1.182 1.1773	1.1864 1.1768 1.1864	1.1475 1.1431	1.1517 1.1424 1.1517	1.1024 1.0983	1.1064 1.0975 1.1064

* Euler beam (EB).
** Timoshenko beam (TB), k_s=5/6.
*** Timoshenko beam (FSDT), Navier, k_s=5/6.

Note that in [45] an explicit solution and in [46] differential transform method (DTM) were applied, respectively. Also for EB in ref. [45-46] for e_0a=0, 1 and 2 nm only the validation was existed, but others are appeared by solving CPT in this paper.

Table 2. Results of dimensionless critical buckling load developed from several theories for a rod (*E=1TPa, υ= 0.3, d=1nm,* $\overline{P_{Cr}} = \frac{P_{Cr}L^2}{EI_c}$)

	$\overline{P_{Cr}}$									
	μ=0 nm²		*μ=1 nm²*		*μ=2 nm²*		*μ=3 nm²*		*μ=4 nm²*	
L/d	EB[47], TB[47]	OVFSDT FSDT S-FSDT CPT	EB[47], TB[47]	OVFSDT FSDT S-FSDT CPT	EB[47], TB[47]	OVFSDT FSDT S-FSDT CPT	EB[47], TB[47]	OVFSDT FSDT S-FSDT CPT	EB[47], TB[47]	OVFSDT FSDT S-FSDT CPT
10	9.8696 9.6227	10.0305 9.6832 10.0305 9.8696	8.9830 8.7583	9.1294 8.8134 9.1294 8.9830	8.2426 8.0364	8.3769 8.0869 8.3769 8.2426	7.6149 7.4244	7.739 7.4711 7.739 7.6149	7.0761 6.8990	7.1914 6.9424 7.1914 7.0761
20	9.8696 9.8067	9.9093 9.8223 9.9093 9.8696	9.6319 9.5705	9.6707 9.5858 9.6707 9.6319	9.4055 9.3455	9.4433 9.3604 9.4433 9.4055	9.1894 9.1308	9.2263 9.1453 9.2263 9.1894	8.9830 8.9258	9.0191 8.94 9.0191 8.9830
50	9.8696 9.8595	9.8759 9.8620 9.8759 9.8696	9.8308 9.8207	9.8371 9.8232 9.8371 9.8308	9.7923 9.7822	9.7985 9.7847 9.7985 9.7923	9.7541 9.7440	9.7603 9.7466 9.7603 9.7541	9.7161 9.7062	9.7224 9.7087 9.7224 9.7161

Table 3. Results of dimensionless critical buckling load developed from several theories for a FG nanobeam (*E_1=1TPa, E_2=0.25TPa, υ= 0.3,* $\overline{P_{Cr}} = \frac{P_{Cr}L^2}{EI_c}$)

	$\overline{P_{Cr}}$									
	e_0a=0 nm		*e_0a=0.5 nm*		*e_0a=1 nm*		*e_0a=1.5 nm*		*e_0a=2 nm*	
k	EB[48], TB[48]	OVFSDT FSDT S-FSDT CPT	EB[48], TB[48]	OVFSDT FSDT S-FSDT CPT	EB[48], TB[48]	OVFSDT FSDT S-FSDT CPT	EB[48], TB[48]	OVFSDT FSDT S-FSDT CPT	EB[48], TB[48]	OVFSDT FSDT S-FSDT CPT
	L/h=10									
0	2.4674 2.4056	2.5213 2.4056 2.5213 2.4674	2.4079 2.3477	2.4606 2.3477 2.4606 2.4079	2.2457 2.1895	2.2948 2.1895 2.2948 2.2457	2.0190 1.9685	2.0631 1.9685 2.0631 2.0190	1.7690 1.7247	1.8076 1.7247 1.8076 1.7690
0.3	4.0925 3.9921	4.1820 3.9901 4.1820 4.0925	3.9940 3.8959	4.0813 3.8941 4.0813 3.9940	3.7249 3.6335	3.8063 3.6317 3.8063 3.7249	3.3488 3.2667	3.4219 3.2650 3.4219 3.3488	2.9341 2.8621	2.9982 2.8607 2.9982 2.9341
1	5.4282 5.3084	5.5468 5.2924 5.5468 5.4282	5.2975 5.1805	5.4133 5.1650 5.4133 5.2975	4.9406 4.8315	5.0485 4.8170 5.0485 4.9406	4.4418 4.3437	4.5389 4.3307 4.5389 4.4418	3.8918 3.805	3.9769 3.7944 3.9769 3.8918
3	6.8176 6.6720	6.9666 6.6470 6.9666 6.8176	6.6534 6.5113	6.7988 6.4870 6.7988 6.6534	6.2051 6.0727	6.3407 6.0498 6.3407 6.2051	5.5787 5.4596	5.7006 5.4391 5.7006 5.5787	4.8879 4.7835	4.9947 4.7656 4.9947 4.8879
10	8.3176 8.1289	8.4993 8.1095 8.4993 8.3176	8.1173 7.9332	8.2947 7.9142 8.2947 8.1173	7.5704 7.3987	7.7358 7.3810 7.7358 7.5704	6.8062 6.6518	6.9549 6.6359 6.9549 6.8062	5.9633 5.8281	6.0936 5.8141 6.0936 5.9633

					L/h=30					
		2.4732		2.4665		2.4464		2.4137		2.3693
0	2.4674	2.4603	2.4606	2.4536	2.4406	2.4336	2.4079	2.4011	2.3637	2.3569
	2.4603	2.4732	2.4536	2.4665	2.4336	2.4464	2.4011	2.4137	2.3569	2.3693
		2.4674		2.4606		2.4406		2.4079		2.3637
		4.1022		4.0910		4.0577		4.0035		3.9298
0.3	4.0925	4.0808	4.0813	4.0697	4.0481	4.0366	3.9940	3.9826	3.9205	3.9094
	4.0811	4.1022	4.0699	4.0910	4.0368	4.0577	3.9828	4.0035	3.9096	3.9298
		4.0925		4.0813		4.0481		3.9940		3.9205
		5.4412		5.4263		5.3821		5.3101		5.2124
1	5.4282	5.4128	5.4134	5.3980	5.3694	5.3541	5.2975	5.2824	5.2001	5.1853
	5.4146	5.4412	5.3998	5.4263	5.3559	5.3821	5.2843	5.3101	5.1871	5.2124
		5.4282		5.4134		5.3694		5.2975		5.2001
		6.8338		6.8151		6.7596		6.6693		6.5466
3	6.8176	6.7982	6.7989	6.7796	6.7436	6.7244	6.6534	6.6345	6.5311	6.5125
	6.8011	6.8338	6.7825	6.8151	6.7273	6.7596	6.6373	6.6693	6.5153	6.5466
		6.8176		6.7989		6.7436		6.6534		6.5311
		8.3374		8.3147		8.2470		8.1366		7.9871
10	8.3176	8.2939	8.2949	8.2713	8.2274	8.2040	8.1173	8.0942	7.9681	7.9454
	8.2962	8.3374	8.2735	8.3147	8.2062	8.2470	8.0964	8.1366	7.9476	7.9871
		8.3176		8.2949		8.2274		8.1173		7.9681
					L/h=100					
		2.4679		2.4673		2.4654		2.4624		2.4582
0	2.4674	2.4667	2.4667	2.4661	2.4649	2.4643	2.4619	2.4613	2.4576	2.4570
	2.4667	2.4679	2.4661	2.4673	2.4643	2.4654	2.4613	2.4624	2.4570	2.4582
		2.4674		2.4667		2.4649		2.4619		2.4576
		4.0934		4.0924		4.0894		4.0842		4.0773
0.3	4.0925	4.0914	4.0915	4.0905	4.0885	4.0874	4.0834	4.0823	4.0764	4.0753
	4.0915	4.0934	4.0905	4.0924	4.0874	4.0894	4.0824	4.0842	4.0754	4.0773
		4.0925		4.0915		4.0885		4.0834		4.0764
		5.4294		5.4281		5.4241		5.4173		5.4080
1	5.4282	5.4268	5.4269	5.4255	5.4229	5.4215	5.4162	5.4148	5.4069	5.4055
	5.4270	5.4294	5.4257	5.4281	5.4217	5.4241	5.4150	5.4173	5.4057	5.4080
		5.4282		5.4269		5.4229		5.4162		5.4069
		6.8191		6.8173		6.8123		6.8039		6.7922
3	6.8176	6.8159	6.8159	6.8141	6.8108	6.8090	6.8025	6.8007	6.7908	6.7890
	6.8161	6.8191	6.8144	6.8173	6.8094	6.8123	6.8010	6.8039	6.7893	6.7922
		6.8176		6.8159		6.8108		6.8025		6.7908
		8.3194		8.3173		8.3111		8.3010		8.2867
10	8.3176	8.3155	8.3155	8.3134	8.3094	8.3072	8.2992	8.2971	8.2849	8.2828
	8.3157	8.3194	8.3136	8.3173	8.3075	8.3111	8.2972	8.3010	8.2830	8.2867
		8.3176		8.3155		8.3094		8.2992		8.2849

6. Conclusions

This article investigated stability of functionally graded nanobeams exposed to the axial compressive loads. To obtain this, a novel beam approach was re-formulated to present governing equations. Nanoscale influences were evaluated by use of a non-classical elasticity theory. Moreover, to calculate the numerical results the Navier's approach was used. The greatness outcomes proved that the Euler beam theory has not satisfactory results for moderately thick and thick beams. On the other hand, although the impacts of transverse shear strains has been taken into account by Timoshenko beam, the used shear correction factor deviates outcomes of this beam approach slightly. The appropriate amount of this factor for nanostructures has not been already calculated and the used value cannot be appropriate at all.

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