



## Free Vibration Analysis of a Cross-Ply Laminated Plate in Thermal Environment

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### Abstract

This paper presents free vibration analysis of a cross-ply laminated plate under temperature rising with considering temperature-dependent physically properties. Material properties of laminas are orthotropic and temperature-dependent. In the kinematic model of the plate, first-order shear deformation plate theory is used. In solution method, the Navier procedure is used for a simply supported plate. The vibration frequencies of the laminated plate are obtained and discussed for different values of temperature, sequence of laminas and orientation angle of layers. Also, the difference between temperature dependent and independent physical properties is investigated.

**Keywords:** Composite Materials; Laminated Plates; Free Vibration; Temperature Rising.

### 1. Introduction

Laminated composite structures have been used a lot of engineering applications, for example; aircrafts, space vehicles, automotive engineering, defence industries and civil engineering applications because these structures have higher strength-weight ratios, more lightweight and ductile properties than classical materials. In generally, laminated composite structures are used in higher thermal systems. Hence, the temperature effect is very important issue of laminated composite structures and their design. In the literature, studies about temperature problems in composite plates are; Pal [1] analyzed nonlinear vibrations of plates under thermal loading. Chen and Chen [2] examined thermal buckling of laminated plates by finite element method. Chen and Chen [3] studied thermal post-buckling of laminated plates under thermal loading. Liu and Huang [4] analyzed vibration of laminated plates under thermal loading with first shear deformation plate theory (FSDPT). F. Lee et al. [5] studied free vibration of symmetrically laminated plates with FSDPT. Reddy and Chin [6] investigated dynamic thermo-elastic analysis of functionally graded cylinders and plates. Lee and Saravanos [7] studied thermo-piezoelectric composite materials with thermal effects with temperature dependent material properties. Reddy [8] performed static analysis of functionally graded plates by using FSDPT. Jane and Hong [9] investigated thermal problems of thin laminated rectangular orthotropic plates by



using generalized differential quadrature method. Shen [10] examined thermal post-buckling of laminated plate resting on elastic foundation. Singha et al. [11] studied thermal postbuckling of graphite/epoxy laminated plates of various by finite element method. Sayman [12] analyzed elastic-plastic behavior of aluminum metal-matrix laminated plate under temperature effect. Patel et al. [13] examined flexural analysis of laminated plates of bimodulus materials under temperature effect. Shukla et al. [14] investigated postbuckling of laminated plates under temperature effect. Liew et al. [15] examined thermal buckling/post-buckling of thick laminated plates uniform temperature rising. Emery et al. [16] investigated thermoelastic stress analysis of laminated orthotropic plates. Shen [17] examined nonlinear analysis of functionally graded nanocomposite plates reinforced by single-walled carbon nanotubes under temperature effect. Zenkour and Alghamdi [18] examined bending of functional graded layered plates under thermal and mechanical loads. Vosoughi et al. [19] examined thermal postbuckling thermal postbuckling behavior of laminated composite skew with temperature dependent material properties. Kishore et al. [20] investigated nonlinear analysis of magnetostrictive layered plate by using third order shear deformation theory. Sahoo and Singh [21] presented static analysis of layered plates by using the hyperbolic zigzag theory. Carrera et al. [22] analyzed static stress problems in multi-layer plates. Sahoo and Singh [23] examined static analysis of layered plates by using a new inverse trigonometric ZigZag theory. Chen et al. [24] investigated thermal buckling and vibration of composite plates with temperature-dependent material properties and initially stressed. Torabizadeh and Fereidoon [25] solved general laminated composite plates under mechanical and thermal loading. Houmat [26] investigated the geometrically nonlinear free vibration of laminated composite rectangular plates with curvilinear fibers. Khorshid and Farhadi [27] investigated hydrostatic vibration analysis of a laminated composite rectangular plate partially contacting with a bounded fluid. Akbaş [28,29,30,31,32,33,34,35,36,37,38] investigated dynamics and stability of functionally graded composite beams by using finite element method. Sayyad et al. [39,40] solved thermoelastic analysis of laminated plates under thermal loading. Li and Qiao [41,42] examined thermal postbuckling analysis of laminated composite beams under thermal loading. Akbaş [43] examined a nano-plate by using generalized differential quadrature method. Ramos et al. [44] investigated thermoelastic static analysis of composite plates by using a new combined trigonometric equation. Akbaş [45,46] investigated functionally graded porous plates. Choudhury et al. [47] solved stress analysis of composite plate under thermo mechanical loads. Akbaş [48,49,50] investigated thee laminated beams with nonlinear behavior. Akbaş [51] examined bi-material composite beams by using finite element method. Yüksel et al. [52] examined temperature dependent vibration of a simply supported plate by using the Navier method. Yüksel and Akbaş [53] investigated the stress analysis of a laminated composite plate under temperature rising. Also, many researchers investigated vibration, buckling, post-buckling analysis of nano composites, functionally graded composite structures in thermal and mechanical loads [54-73].

In this paper, free vibration of cross-play laminated plate examined under thermal effects. In constitute model of lamina, orthotropic and temperature-dependent properties are used. FSDPT is used in plate model. The Navier procedure is used for a simply supported plate. Effects of temperature, sequence of lamina and orientation angle of layers on the vibration characterises of laminated plate are investigated in temperature-dependent physically property.

## 2. Theory and Formulations

In figure 1, a simply supported rectangular cross-ply laminated composite plate with thickness  $h$ , the length of  $L_{X_1}$  and  $L_{X_2}$  is displayed. Laminated composite plate is subjected to a non-uniform temperature rising with temperature rising values at the bottom surface  $\Delta T_B$  and top surface  $\Delta T_T$ . Height of face sheet layers is equal to each other. In this study, numbers of the laminae are selected as two and three.

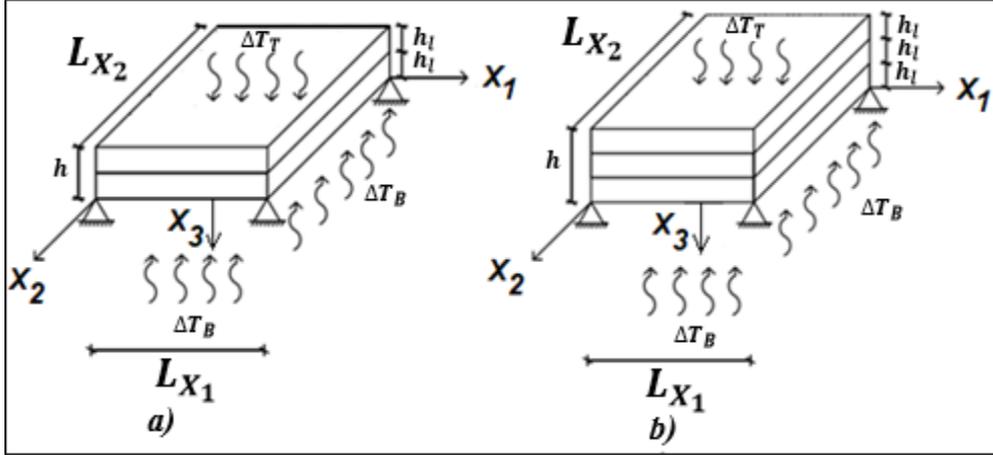


Fig. 1. A simply supported laminated rectangular composite plate under non-uniform temperature rising for a) two layer and b) three layer.

Based on FSDPT, the strain-displacement relations are expressed as;

$$\varepsilon_{X_1 X_1} = \frac{\partial u_{01}}{\partial X_1} + X_3 \frac{\partial \phi_{X_1}}{\partial X_1} \quad \varepsilon_{X_2 X_2} = \frac{\partial u_{02}}{\partial X_2} + X_3 \frac{\partial \phi_{X_2}}{\partial X_2} \quad (1)$$

$$\gamma_{X_1 X_2} = \frac{\partial u_{02}}{\partial X_2} + \frac{\partial u_{01}}{\partial X_1} + X_3 \left( \frac{\partial \phi_{X_1}}{\partial X_2} + \frac{\partial \phi_{X_2}}{\partial X_1} \right) \quad (2)$$

$$\gamma_{X_1 X_3} = \frac{\partial u_{03}}{\partial X_1} + \phi_{X_1}, \quad \gamma_{X_2 X_3} = \frac{\partial u_{03}}{\partial X_2} + \phi_{X_2}, \quad \varepsilon_{X_3 X_3} = 0 \quad (3)$$

where  $u_{01}$ ,  $u_{02}$ ,  $u_{03}$  indicate displacements in  $X_1$ ,  $X_2$  and  $X_3$  directions, respectively. Constitutive expressions of orthotropic laminated plate for  $n$ th layer with temperature effect are given as follows:

$$\begin{Bmatrix} \sigma_{X_1 X_1} \\ \sigma_{X_2 X_2} \\ \sigma_{X_1 X_2} \end{Bmatrix}^{(n)} = \begin{bmatrix} \bar{Q}_{11}(T) & \bar{Q}_{12}(T) & \bar{Q}_{16}(T) \\ \bar{Q}_{12}(T) & \bar{Q}_{22}(T) & \bar{Q}_{26}(T) \\ \bar{Q}_{16}(T) & \bar{Q}_{26}(T) & \bar{Q}_{66}(T) \end{bmatrix}^{(n)} \left\{ \begin{array}{l} \frac{\partial u_{01}}{\partial X_1} - X_3 \frac{\partial^2 u_{03}}{\partial X_1^2} - \bar{\alpha}_{11}(T)\Delta T \\ \frac{\partial u_{02}}{\partial X_2} - X_3 \frac{\partial^2 u_{03}}{\partial X_2^2} - \bar{\alpha}_{22}(T)\Delta T \\ \frac{\partial u_{01}}{\partial X_2} + \frac{\partial u_{02}}{\partial X_1} - X_3 \frac{\partial^2 u_{03}}{\partial X_2^2} - X_3 \frac{\partial^2 u_{03}}{\partial X_1^2} - 2\bar{\alpha}_{12}(T)\Delta T \end{array} \right\}^{(n)} \quad (4a)$$

$$\begin{Bmatrix} \sigma_{X_2 X_3} \\ \sigma_{X_1 X_3} \end{Bmatrix}^{(n)} = \begin{bmatrix} \bar{Q}_{44}(T) & \bar{Q}_{45}(T) \\ \bar{Q}_{45}(T) & \bar{Q}_{55}(T) \end{bmatrix}^{(n)} \left\{ \begin{array}{l} \frac{\partial u_{02}}{\partial X_2} - \frac{\partial u_{03}}{\partial X_2} \\ \frac{\partial u_{03}}{\partial X_1} - \frac{\partial u_{03}}{\partial X_1} \end{array} \right\}^{(n)} \quad (4b)$$

where  $\bar{Q}_{ij}(T)$  is the transformed reduced material properties which depends the temperature ( $T$ ) are given as follows:

$$\begin{aligned} \bar{Q}_{11}(T) &= Q_{11}(T)\cos^4\theta + 2(Q_{12}(T) + 2Q_{66}(T))\sin^2\theta\cos^2\theta + Q_{22}(T)\sin^4\theta \\ \bar{Q}_{12}(T) &= (Q_{11}(T) + Q_{22}(T) - 4Q_{66}(T))\sin^2\theta\cos^2\theta + Q_{12}(T)(\sin^4\theta + \cos^4\theta) \\ \bar{Q}_{22}(T) &= Q_{11}(T)\sin^4\theta + 2(Q_{12}(T) + 2Q_{66}(T))\sin^2\theta\cos^2\theta + Q_{22}(T)\cos^4\theta \\ \bar{Q}_{16}(T) &= (Q_{11}(T) - Q_{12}(T) - 2Q_{66}(T))\sin\theta\cos^3\theta + (Q_{12}(T) - Q_{22}(T) + 2Q_{66}(T))\sin^3\theta\cos\theta \\ \bar{Q}_{26}(T) &= (Q_{11}(T) - Q_{12}(T) - 2Q_{66}(T))\sin^3\theta\cos\theta + (Q_{12}(T) - Q_{22}(T) + 2Q_{66}(T))\sin\theta\cos^3\theta \end{aligned}$$

$$\begin{aligned}
 \bar{Q}_{66}(T) &= (Q_{11}(T) + Q_{22}(T) - 2Q_{12}(T) - 2Q_{66}(T))\sin^2\theta\cos^2 + Q_{66}(T)(\sin^4\theta + \cos^4\theta) \\
 \bar{Q}_{44}(T) &= Q_{44}(T)\cos^2\theta + Q_{55}(T)\sin^2\theta \\
 \bar{Q}_{45}(T) &= (Q_{55}(T) - Q_{44}(T))\cos\theta\sin\theta \\
 \bar{Q}_{55}(T) &= Q_{44}(T)\sin^2\theta + Q_{55}(T)\cos^2\theta
 \end{aligned} \tag{5}$$

where,  $\theta$  is the fiber orientation angle. Components of the  $Q_{ij}$  are given as follows;

$$\begin{aligned}
 Q_{11}(T) &= \frac{E_1(T)}{1-\nu_{12}\nu_{21}}, & Q_{22}(T) &= \frac{E_2(T)}{1-\nu_{12}\nu_{21}} \\
 Q_{12}(T) &= \frac{\nu_{12}E_2(T)}{1-\nu_{12}\nu_{21}} = \frac{\nu_{21}E_1(T)}{1-\nu_{12}\nu_{21}} & Q_{44}^{(n)}(T) &= G_{23}^{(n)}(T) & Q_{55}^{(n)}(T) &= G_{13}^{(n)}(T) \\
 Q_{21}(T) &= \frac{\nu_{12}E_2(T)}{1-\nu_{12}\nu_{21}} = \frac{\nu_{21}E_1(T)}{1-\nu_{12}\nu_{21}} & Q_{66}(T) &= G_{12}(T)
 \end{aligned} \tag{6}$$

The material properties of orthotropic laminated plate is a function of temperature ( $T$ ) as follows (Shen[67]; Li and Qiao[68]).

$$\begin{aligned}
 E_1(T) &= E_1(1 - 0,5 * 10^{-3}\Delta T)GPa \\
 E_2(T) &= E_2(1 - 0,2 * 10^{-3}\Delta T)GPa \\
 G_{12}(T) &= G_{13}(T) = G_{12}(1 - 0,2 * 10^{-3}\Delta T)GPa \\
 G_{23}(T) &= G_{23}(1 - 0,2 * 10^{-3}\Delta T)GPa \\
 \alpha_1(T) &= \alpha_1(1 + 0,5 * 10^{-3}\Delta T)/^\circ C \\
 \alpha_2(T) &= \alpha_2(1 + 0,5 * 10^{-3}\Delta T)/^\circ C
 \end{aligned} \tag{7}$$

The transformed the thermal expansion coefficients  $\alpha_{X_1X_1}$ ,  $\alpha_{X_2X_2}$ ,  $\alpha_{X_1X_2}$  are given as follows;

$$\begin{aligned}
 \alpha_{X_1X_1} &= \alpha_1\cos^2\theta + \alpha_2\sin^2\theta \\
 \alpha_{X_2X_2} &= \alpha_2\cos^2\theta + \alpha_1\sin^2\theta \\
 2\alpha_{X_1X_2} &= 2(\alpha_1 - \alpha_2)\sin\theta\cos\theta
 \end{aligned} \tag{8}$$

where  $\alpha_1$  and  $\alpha_2$  are thermal expansion coefficients in  $X_1$  and  $X_2$  directions, respectively. Stress resultants are given as follows;

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} [A(T)] & [B(T)] \\ [B(T)] & [D(T)] \end{bmatrix} \begin{Bmatrix} \{\varepsilon^0\} \\ \{\varepsilon^1\} \end{Bmatrix} - \begin{Bmatrix} \{N^T\} \\ \{M^T\} \end{Bmatrix} \tag{9}$$

where  $N$  is normal force and  $M$  is moment.  $\{N^T\}$  and  $\{M^T\}$  are thermal force resultants:

$$\{N^T\} = \sum_{n=1}^N \int_{z_n}^{z_{n+1}} \bar{Q}_{ij}(T)^n \{\bar{\alpha}(T)\}^n \Delta T dX_3 \tag{10a}$$

$$\{M^T\} = \sum_{n=1}^N \int_{z_n}^{z_{n+1}} \bar{Q}_{ij}(T)^n \{\bar{\alpha}(T)\}^n \Delta T X_3 dX_3 \tag{10b}$$

$\{\varepsilon^0\}$  and  $\{\varepsilon^1\}$  are given as follows;

$$\{\varepsilon^0\} = \begin{Bmatrix} \frac{\partial u_{01}}{\partial X_1} \\ \frac{\partial u_{02}}{\partial X_2} \\ \frac{\partial u_{01}}{\partial X_2} + \frac{\partial u_{02}}{\partial X_1} \end{Bmatrix}, \quad \{\varepsilon^1\} = \begin{Bmatrix} \frac{\partial \phi_{X_1}}{\partial X_1} \\ \frac{\partial \phi_{X_2}}{\partial X_2} \\ \frac{\partial \phi_{X_1}}{\partial X_2} + \frac{\partial \phi_{X_2}}{\partial X_1} \end{Bmatrix} \tag{11}$$

where  $A_{ij}$  is extensional stiffness,  $D_{ij}$  is bending stiffness, and  $B_{ij}$  is bending – extensional coupling stiffness.  $A_{ij}$ ,  $B_{ij}$  and  $D_{ij}$  are expressed as follows:

$$A_{ij} = \sum_{k=1}^n \bar{Q}_{ij}^{(n)} (z_{n+1} - z_n) \quad (12a)$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^n \bar{Q}_{ij}^{(n)} (z_{n+1}^2 - z_n^2) \quad (12b)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^n \bar{Q}_{ij}^{(n)} (z_{n+1}^3 - z_n^3) \quad (12c)$$

The elastic strain energy ( $U_i$ ) and the kinetic energy ( $T$ ) of laminated plate are expressed as follows:

$$U_i = \frac{1}{2} \int_V \sigma_{ij} \varepsilon_{ij} dV \quad (13a)$$

$$T = \frac{1}{2} \int_V \rho \left[ \left( \frac{\partial u_{01}}{\partial t} \right)^2 + \left( \frac{\partial u_{02}}{\partial t} \right)^2 + \left( \frac{\partial u_{03}}{\partial t} \right)^2 \right] dV \quad (13b)$$

The Hamilton's principle of the problem is as follows;

$$\delta \int_0^t [T - U_i] dt \quad (14)$$

After using Hamilton's principle, governing equations of the laminated plate can be obtained;

$$\frac{\partial N_{X_1 X_1}}{\partial X_1} + \frac{\partial N_{X_1 X_2}}{\partial X_2} = I_0 \frac{\partial^2 u_{01}}{\partial t^2} + I_1 \frac{\partial^2 \phi_{X_1}}{\partial t^2} \quad (15a)$$

$$\frac{\partial N_{X_1 X_2}}{\partial X_1} + \frac{\partial N_{X_2 X_2}}{\partial X_2} = I_0 \frac{\partial^2 u_{02}}{\partial t^2} + I_1 \frac{\partial^2 \phi_{X_2}}{\partial t^2} \quad (15b)$$

$$\frac{\partial Q_{X_1}}{\partial X_1} + \frac{\partial Q_{X_2}}{\partial X_2} = I_0 \frac{\partial^2 u_{03}}{\partial t^2} \quad (15c)$$

$$\frac{\partial M_{X_1 X_1}}{\partial X_1} + \frac{\partial M_{X_1 X_2}}{\partial X_2} - Q_{X_1} = I_2 \frac{\partial^2 \phi_{X_1}}{\partial t^2} + I_1 \frac{\partial^2 u_{01}}{\partial t^2} \quad (15d)$$

$$\frac{\partial M_{X_1 X_2}}{\partial X_1} + \frac{\partial M_{X_2 X_2}}{\partial X_2} - Q_{X_2} = I_2 \frac{\partial^2 \phi_{X_2}}{\partial t^2} + I_1 \frac{\partial^2 u_{02}}{\partial t^2} \quad (15e)$$

where

$$\begin{Bmatrix} N_{X_1 X_1} \\ N_{X_2 X_2} \\ N_{X_1 X_2} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{X_1 X_1} \\ \sigma_{X_2 X_2} \\ \sigma_{X_1 X_2} \end{Bmatrix} dX_3 \quad (16a)$$

$$\begin{Bmatrix} M_{X_1 X_1} \\ M_{X_2 X_2} \\ M_{X_1 X_2} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{X_1 X_1} \\ \sigma_{X_2 X_2} \\ \sigma_{X_1 X_2} \end{Bmatrix} X_3 dX_3 \quad (16b)$$

$$\begin{Bmatrix} Q_{X_2} \\ Q_{X_1} \end{Bmatrix} = K \begin{bmatrix} A_{44}(T) & A_{45}(T) \\ A_{45}(T) & A_{55}(T) \end{bmatrix} \begin{bmatrix} \frac{\partial u_{03}}{\partial X_2} + \phi_{X_2} \\ \frac{\partial u_{03}}{\partial X_1} + \phi_{X_1} \end{bmatrix} \quad (16c)$$

$$\begin{Bmatrix} I_0 \\ I_1 \\ I_2 \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} 1 \\ X_3 \\ X_3^2 \end{Bmatrix} \rho_0 dX_3 \quad (16d)$$

In solution of problem, Navier method is implemented in the solution of the problem. In Navier solution, boundary conditions and displacement fields the plate are given the following equations:

$$u_{01}(X_1, 0, t) = 0, \quad u_{01}(X_1, b, t) = 0, \quad u_{02}(0, X_2, t) = 0, \quad u_{02}(a, X_2, t) = 0, \quad (17a)$$

$$u_{03}(X_1, 0, t) = 0, \quad u_{03}(X_1, b, t) = 0, \quad u_{03}(0, X_2, t) = 0, \quad u_{03}(a, X_2, t) = 0, \quad (17b)$$

$$\phi_{X_1}(X_1, 0, t) = 0, \quad \phi_{X_1}(X_1, b, t) = 0, \quad \phi_{X_2}(0, X_2, t) = 0, \quad \phi_{X_2}(a, X_2, t) = 0, \quad (17c)$$

$$N_{X_1X_1}^T(0, X_2, t) = 0, \quad N_{X_1X_1}^T(a, X_2, t) = 0, \quad N_{X_2X_2}^T(X_1, 0, t) = 0, \quad N_{X_2X_2}^T(X_1, b, t) = 0 \quad (17d)$$

$$M_{X_1X_1}^T(0, X_2, t) = 0, \quad M_{X_1X_1}^T(a, X_2, t) = 0, \quad M_{X_2X_2}^T(X_1, 0, t) = 0, \quad M_{X_2X_2}^T(X_1, b, t) = 0 \quad (17e)$$

$$u_{01}(X_1, X_2, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} U_{1mn}(t) \cos kX_1 \sin lX_2 e^{-i\beta t} \quad (18a)$$

$$u_{02}(X_1, X_2, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} U_{2mn}(t) \sin kX_1 \cos lX_2 e^{-i\beta t} \quad (18b)$$

$$u_{03}(X_1, X_2, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} U_{3mn}(t) \sin kX_1 \sin lX_2 e^{-i\beta t} \quad (18c)$$

$$\phi_{X_1}(X_1, X_2, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} X_{X_1mn}(t) \cos kX_1 \sin lX_2 e^{-i\beta t} \quad (18d)$$

$$\phi_{X_2}(X_1, X_2, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Y_{X_2mn}(t) \sin kX_1 \cos lX_2 e^{-i\beta t} \quad (18e)$$

where  $U_{1mn}$ ,  $U_{2mn}$ ,  $U_{3mn}$ ,  $X_{X_1mn}$ ,  $Y_{X_2mn}$  are displacement coefficients,  $k = m\pi/L_{X_1}$ ,  $l = n\pi/L_{X_2}$ ,  $\beta$  is the natural frequency and  $i = \sqrt{-1}$ . The temperature rising is defined as follows in the Navier solution;

$$\Delta T(X_1, X_2, X_3, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} T_{mn}(X_3, t) \sin kX_1 \sin lX_2 \quad (19a)$$

$$T_{mn}(X_3, t) = \frac{4}{L_X L_Y} \int_0^a \int_0^b \Delta S(X_1, X_2, X_3, t) \sin kX_1 \sin lX_2 dX_1 dX_2 \quad (19b)$$

Substituting Eqs. (17-19) into Eqs. (15), and then using matrix procedure, the algebraic equations of free vibration problem can be expressed as follows;

$$\begin{pmatrix} p_{11} & p_{12} & 0 & p_{14} & p_{15} \\ p_{12} & p_{22} & 0 & p_{24} & p_{25} \\ 0 & 0 & p_{33} & p_{34} & p_{35} \\ p_{14} & p_{24} & p_{34} & p_{44} & p_{45} \\ p_{15} & p_{25} & p_{35} & p_{45} & p_{55} \end{pmatrix} - \omega^2 \begin{pmatrix} m_{11} & 0 & 0 & 0 & 0 \\ 0 & m_{22} & 0 & 0 & 0 \\ 0 & 0 & m_{33} & 0 & 0 \\ 0 & 0 & 0 & m_{44} & 0 \\ 0 & 0 & 0 & 0 & m_{55} \end{pmatrix} \begin{Bmatrix} U_{1mn} \\ U_{2mn} \\ U_{3mn} \\ X_{X_1mn} \\ Y_{X_2mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (20)$$

where

$$\begin{aligned} p_{11} &= (A_{11}(T)k^2 + A_{66}(T)l^2), & p_{12} &= (A_{12}(T) + A_{66}(T))kl \\ p_{14} &= (B_{11}(T)k^2 - B_{66}(T)l^2), & p_{15} &= (B_{12}(T) + B_{66}(T))kl, \\ p_{22} &= (A_{66}(T)k^2 + A_{22}(T)l^2), & p_{24} &= p_{15}, \\ p_{25} &= (B_{66}(T)k^2 + B_{22}(T)l^2), & p_{33} &= K(A_{55}(T)k^2 + A_{44}(T)l^2), \\ p_{34} &= KA_{55}(T)k, & p_{35} &= KA_{44}(T)l, \\ p_{44} &= (D_{11}(T)k^2 + D_{22}(T)l^2 + KA_{55}(T)) \\ p_{45} &= (D_{12}(T) + D_{66}(T))kl, & p_{55} &= (D_{66}(T)k^2 + D_{22}(T)l^2 + KA_{44}(T))k \\ m_{11} &= I_0, & m_{22} &= I_0, & m_{33} &= I_0, & m_{44} &= I_2, & m_{55} &= I_2 \end{aligned} \quad (21)$$

where  $K$  is shear correction factor. Dimensionless fundamental frequency  $\bar{\omega}$  is defined as follows;

$$\bar{\omega}_{mn} = \omega_{mn} (L_{X_2}^2 / \pi^2) \sqrt{\rho h / D_{22}} \quad (22)$$

### 3. Numerical Results

In numerical study, dimensionless frequencies of cross-ply laminated simply-supported plate are calculated obtained in figures for different temperature values, orientation angles and sequence of laminas in temperature-dependent physically property. The mechanical properties of manufactured using graphite epoxy and its material parameters are;  $E_1=150$  GPa,  $E_2=9$  GPa,  $E_3=9$  GPa,  $G_{12}=7,1$  GPa,  $G_{23}=2,5$  GPa,  $G_{13}=7,1$  GPa,  $\rho=1600$  kg/m<sup>3</sup>,  $\nu_{12}=\nu_{21}=0.3$ ,  $\alpha_1 = 1,1 \cdot 10^{-6}$ ,  $\alpha_2 = 25,2 \cdot 10^{-6}$  at  $30^\circ\text{C}$  (Li and Qiao [68], Oh vd. [69]). The dimensions of plate are considered as follows:  $L_{X_1} = 4\text{m}$ ,  $L_{X_2} = 4\text{m}$ ,  $h=0.2$  m. In the obtaining the numerical results and figures, MATLAB program is used. It is noted that temperature rising of bottom surface  $\Delta T_B$  is changed and the temperature of the top surface  $\Delta T_T$  is constant  $\Delta T_T = 20^\circ\text{C}$  in the numerical calculations.

In the numerical results, the relation between temperature rising and dimensionless natural frequencies is presented for different orientation angles and sequence of laminas. Also the difference between temperature dependent and independent physical properties on the dimensionless natural frequencies of laminated composite plate is discussed. For this purpose, figures 2,3,4 and 5 show the effect of the temperature rising on the first three lower dimensionless natural frequencies of the laminated plate for 0/0, 0/90, 90/0 and 90/90, respectively in two layer sequence in both temperature dependent and independent physical properties. Also, figures 6,7,8,9 and 10 show effect of temperature rising on first three lower dimensionless natural frequencies of the laminated plate for 0/0/0, 0/90/0, 90/0/90, 0/90/90 and 90/90/90, respectively in three layer sequence in both temperature dependent and independent physical properties.

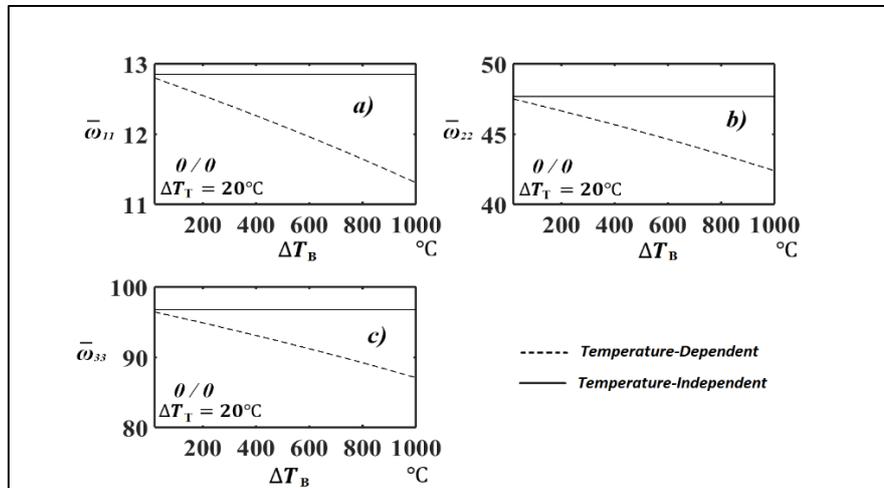


Fig. 2. The natural frequencies versus temperature rising for the two layers for stacking sequence 0/0 for a)  $\bar{\omega}_{11}$  b)  $\bar{\omega}_{22}$  and c)  $\bar{\omega}_{33}$ .

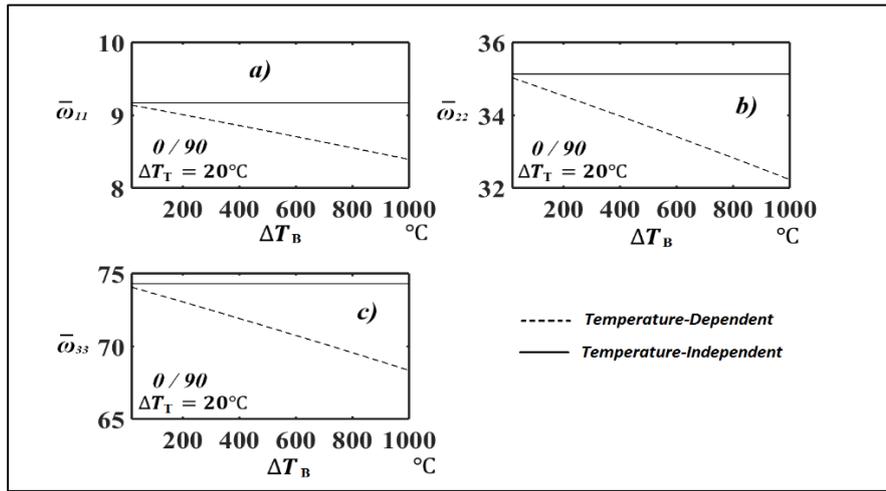


Fig. 3. The natural frequencies versus temperature rising for the two layers for stacking sequence 0/90 for a)  $\bar{\omega}_{11}$  b)  $\bar{\omega}_{22}$  and c)  $\bar{\omega}_{33}$ .

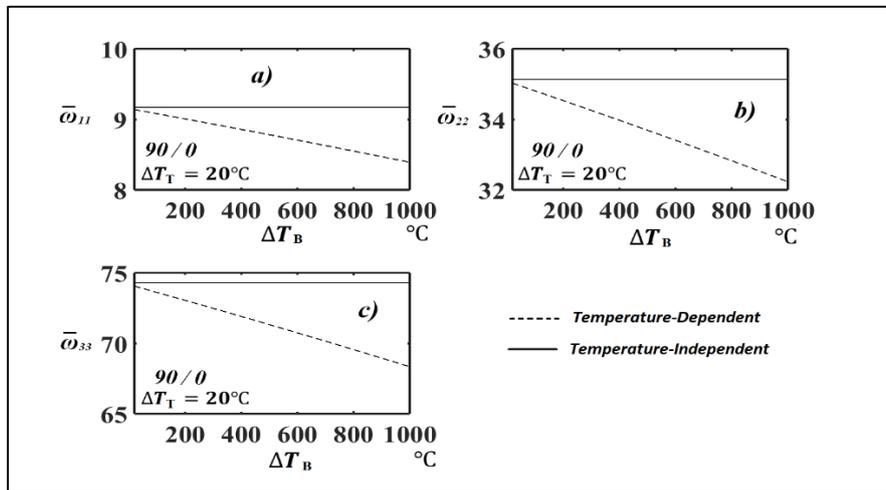


Fig. 4. The natural frequencies versus temperature rising for the two layers for stacking sequence 90/0 for a)  $\bar{\omega}_{11}$  b)  $\bar{\omega}_{22}$  and c)  $\bar{\omega}_{33}$ .

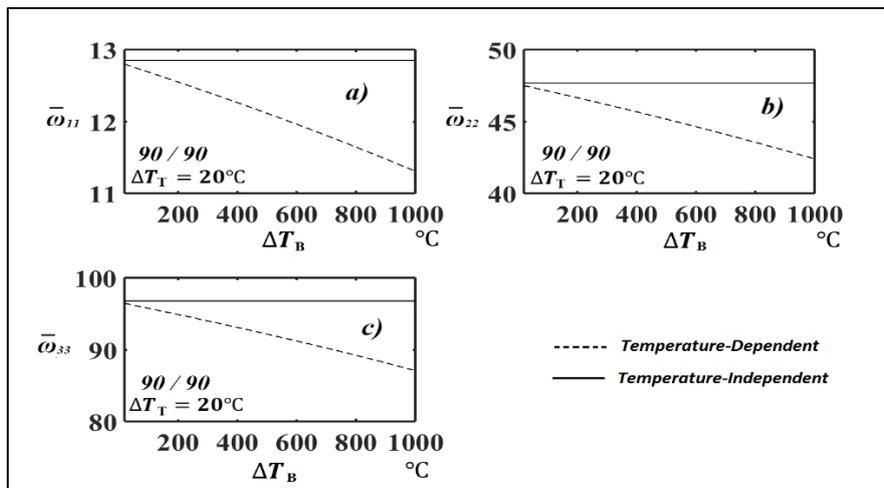


Fig. 5. The natural frequencies versus temperature rising for the two layers for stacking sequence 90/90 for a)  $\bar{\omega}_{11}$  b)  $\bar{\omega}_{22}$  and c)  $\bar{\omega}_{33}$ .

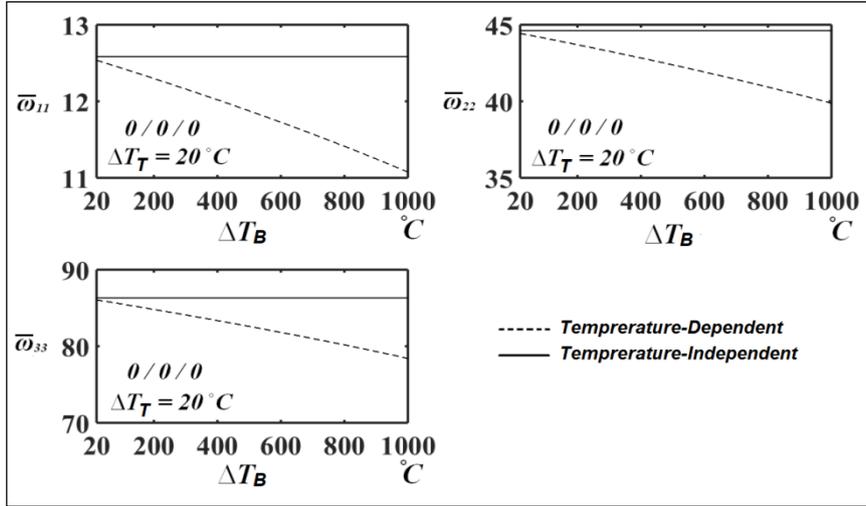


Fig. 6. The natural frequencies versus temperature rising for the three layers for stacking sequence 0/0/0 for a)  $\bar{\omega}_{11}$  b)  $\bar{\omega}_{22}$  and c)  $\bar{\omega}_{33}$ .

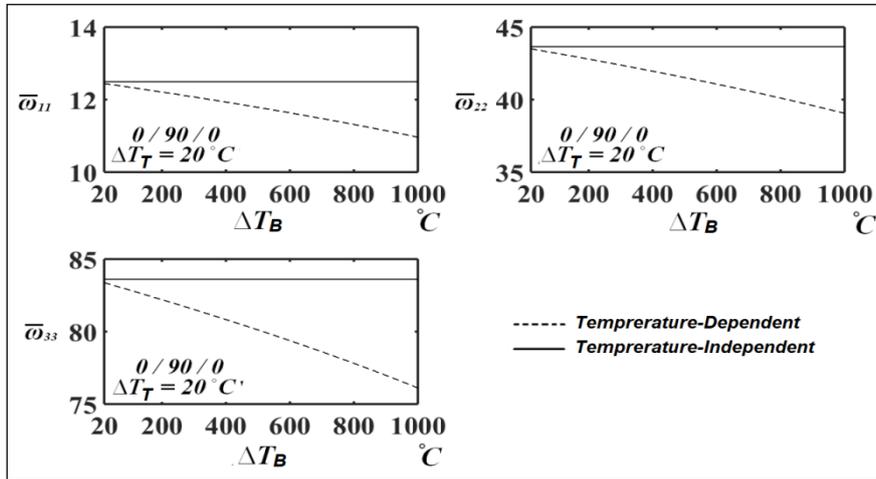


Fig. 7. The natural frequencies versus temperature rising for the three layers for stacking sequence 0/90/0 for a)  $\bar{\omega}_{11}$  b)  $\bar{\omega}_{22}$  and c)  $\bar{\omega}_{33}$ .

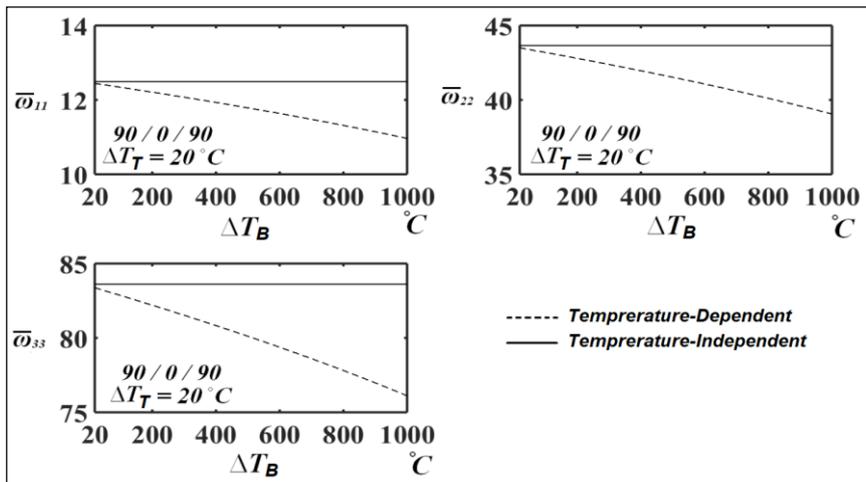


Fig. 8. The natural frequencies versus temperature rising for the three layers for stacking sequence 90/0/90 for a)  $\bar{\omega}_{11}$  b)  $\bar{\omega}_{22}$  and c)  $\bar{\omega}_{33}$ .

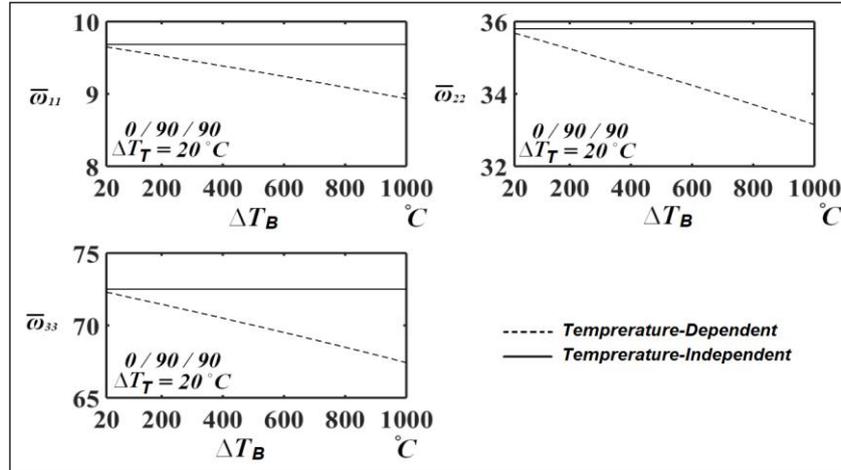


Fig. 9. The natural frequencies versus temperature rising for the three layers for stacking sequence 0/90/90 for a)  $\bar{\omega}_{11}$  b)  $\bar{\omega}_{22}$  and c)  $\bar{\omega}_{33}$ .

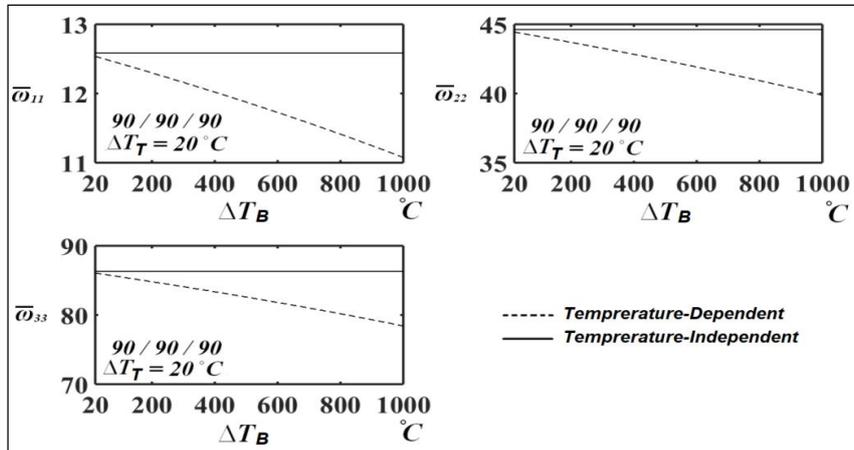


Fig. 10. The natural frequencies versus temperature rising for the three layers for stacking sequence 90/90/90 for a)  $\bar{\omega}_{11}$  b)  $\bar{\omega}_{22}$  and c)  $\bar{\omega}_{33}$ .

Figures 2-10 display that increasing in temperature, dimensionless frequency of laminated plate decreases significantly. With increasing temperature, the results of difference between temperature dependent and independent properties increase considerably.

Frequencies of temperature dependent are smaller than the frequencies of temperature independent. This is because; with the temperature increase, the strength of laminated plate decreases in the temperature dependent physical properties, so the frequencies decrease naturally. However, the strength of the laminated plate does not change with temperature increase in the temperature independent physical properties.

With changing the orientation angles, the dimensionless frequency change significantly. With increasing the orientation angles from 0 degree, the dimensionless frequency decrease considerably. Also, the stacking sequence play important role on vibration characterises of the laminated composite plate. In is observed from these figures that stacking sequence is very effective on thermal vibration responses.

#### 4. Conclusions

In the presented paper, free vibration of a laminated plate is studied under thermal loading by using FSDPT in temperature-dependent physical properties. Cross-ply laminated sequence and simply-supported boundary conditions are considered. The Navier solution is implemented in the solution method. Effects of temperature, sequence of laminas and orientation angle of layers on the vibration characteristics of laminated plate are investigated in temperature dependent physical properties. Also, difference between temperature dependent and independent are examined on the vibration results. As seen from the graphs that increasing temperature yields to increasing difference between the temperature dependent and independent results. Increasing fiber orientation angles and temperature yields to decreasing the frequency values. Frequencies of temperature dependent physical properties are smaller than those of temperature independent's. Stacking sequence and orientation angle of layers play important role on vibration behavior of composite laminated plates.

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