2024

INTERNATIONAL ANTALYA MATHEMATICS OLYMPIAD

8TH GRADE QUESTION BOOKLET

NAME SURNAME : .....

SCHOOL : ..... GRADE : .....

SIGNATURE : .....

EXAMINATION RULES

1. It is forbidden to take the exam with a phone. Please hand in your phone to the attendant. This exam consists of 25 multiple-choice questions and the exam duration is 120 minutes.

2. Each question has only one correct answer. Mark your correct answer by completely crossing out the relevant box on your answer sheet. No marking in the question booklet will be evaluated.

3.All questions are of equal value and four wrong answers will cancel one correct answer. Questions left blank will not have a positive or negative effect on the evaluation.

4. The questions are NOT in order of difficulty. Therefore, it is recommended that you review all questions before you start answering.

5. It is forbidden to use aids such as compasses, rulers, calculators and scratch paper. Do all your work on the question booklet.

6. During the exam, you will not talk to the staff and you will not ask them any questions. It is unlikely that there will be a mistake in the questions. If this happens, the exam academic board will take appropriate action. In this case, you should mark the option that you think is the most correct.

7. Students are not allowed to ask each other for pencils, erasers, etc.

8. It is forbidden to leave the exam for the first 60 minutes. A candidate who goes out will not be allowed to take the exam again.

9. Do not forget to hand in your answer sheet and question booklet to the staff before leaving the exam hall.

1. Some books will be chosen at random from a library containing **25** Turkish, **20** Mathematics, **10** Science and **9** English books. What is the minimum number of books to be selected so that there are at least **13** books of the same course?

A) **43** B) **44** C) **53** D) **50** E) **39**

2. The area of a triangle with given side lengths can be calculated by the following formula, known as Heron's formula.

Heron Formula: The area of a triangle with side lengths a, b, c is equal to

$$\frac{1}{4}\sqrt{(a+b+c)\,(-a+b+c)\,(a-b+c)\,(a+b-c)}.$$

Using this information, calculate the area of a triangle with side lengths **13, 14, 15**.

A) **80** B) **72** C) $\mathbf{9\sqrt{6}}$ D) $\mathbf{10\sqrt{6}}$ E) **84**

3. There are **6** shelves in a bookcase and there are **23, 25, 32, 29, 26, 33** books on each shelf respectively. At least how many books should be moved so that each shelf has exactly the same number of books?

A) **10** B) **13** C) **12** D) **8** E) **9**

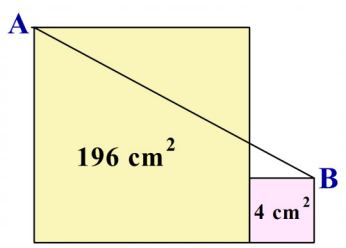
4. The teacher wrote **101** on the board. The students began to write the next number one by one, by adding **5**, then by adding **10**, then by adding **15**, so in this way they created a sequence given below:

101, 106, 116, 131, 151, ...

If there are **22** students in the class, what number will the last student write on the board?

A) **1361** B) **1366** C) **1356** D) **1351** E) **1371**

5. If the area of the large square is **196** $\mathbf{cm^2}$ and the area of the small square is **4** $\mathbf{cm^2}$, then find the length |AB|.



A) **16** B) **20** C) **19** D) **15** E) **80**

6. For any number A, $k(A)$, $b(A)$ and $t(A)$ are defined as follows.

- $k(A)$: The smallest digit of the number A
- $b(A)$: The largest digit of the number A
- $t(A)$: Sum of the digits of the number A

For example, for the number $A = 45601$, $k(A) = 0$, $b(A) = 6$ and $t(A) = 4 + 5 + 6 + 0 + 1 = 16$.

How many five-digit even numbers with different digits are there such that $b(A) = 7$, $t(A) = 25$ and $k(A)$ is a prime number?

A) 240 B) 120 C) 48 D) 24 E) 64

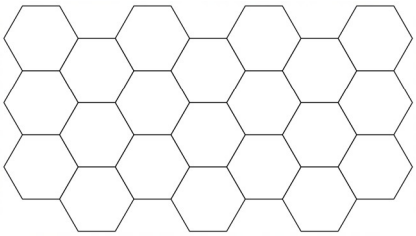
7.

$$\cfrac{1}{2-\cfrac{1}{2-\cfrac{1}{2-\cfrac{1}{2-\cdots\cfrac{1}{2-\frac{1}{2}}}}}}$$

The rational expression above has 1000 fraction lines. What is the value of this expression?

A) $\frac{1000}{1001}$ B) $\frac{1000}{999}$ C) $\frac{1}{2^{1000}}$ D) $\frac{1001}{1000}$ E) $\frac{500}{999}$

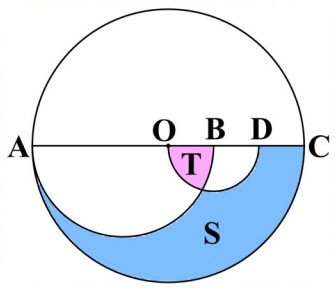
8.



The figure above is created with regular hexagons. We want to color each hexagon with red, blue and black. How many different ways can this figure be colored, if no two adjacent hexagons are the same color?

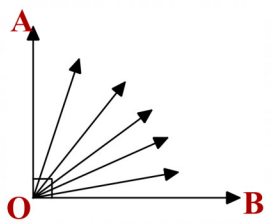
A) 8 B) 10 C) 9 D) 6 E) 120

9. In the figure below, two semicircles of different radii are drawn inside the great circle centered at O with a radius of 6 cm. If $|OB| = |BD| = |DC|$, how much larger is the shaded area indicated by S than the shaded area indicated by T?



A) 8π B) 7π C) 9π D) 10π E) 6π

10. In the figure, **OA** and **OB** are perpendicular to each other. How many acute angles are there whose vertex is point **O**?



A) **15** B) **18** C) **20** D) **6** E) **22**

11. Two types of tickets are sold for a concert: standing or seated. Three quarters of the participants in this concert are sitting in four fifths of the seats reserved for the concert. **24** of the seat tickets are not sold and these seats remain empty. According to this, how many people attended the concert standing?

A) **24** B) **42** C) **36** D) **32** E) **30**

12. If the greatest common divisor of two natural numbers, one of which is not a multiple of the other, is **9** and the least common multiple is **108**, what is the sum of these two numbers?

A) **63** B) **45** C) **117** D) **54** E) **36**

13. Let a, b, c and d be nonzero real numbers. Find c, if

$$a^b = c^d \quad \text{ve} \quad \frac{a}{2c} = \frac{b}{d} = 4.$$

A) 4 B) 2 C) $\frac{1}{2}$ D) $\frac{1}{4}$ E) $\frac{1}{16}$

14.

$$\begin{aligned} A(1) &= \frac{1}{1}, \\ A(2) &= \frac{1}{2}+\frac{2}{2}, \\ A(3) &= \frac{1}{3}+\frac{2}{3}+\frac{3}{3}, \\ A(4) &= \frac{1}{4}+\frac{2}{4}+\frac{3}{4}+\frac{4}{4} \end{aligned}$$

Continuing as follows and lastly

$$A(9) = \frac{1}{9}+\frac{2}{9}+\frac{3}{9}+\cdots+\frac{8}{9}+\frac{9}{9}$$

is written at the end. Then, what is the sum

$$A(1)+A(2)+A(3)+\cdots+A(9)?$$

A) **26** B) **25** C) **27** D) **20** E) **30**

15. Find $A - B$, if

$$A = \left(\frac{123454320}{123454321}\right)^2 + \left(\frac{123454322}{123454321}\right)^2$$

$$B = 2\left(\frac{1}{123454321}\right)^2.$$

A) $\frac{1}{2}$ B) 2 C) 1 D) $\frac{1}{3}$ E) $\frac{2}{3}$

16. How many digits 2 will be found in the writing of the whole number after the sum below is calculated?

$$\begin{array}{r} 1992 \\ 19993 \\ 199994 \\ \vdots \\ 1999999998 \\ +\quad 19999999999 \\ \hline \end{array}$$

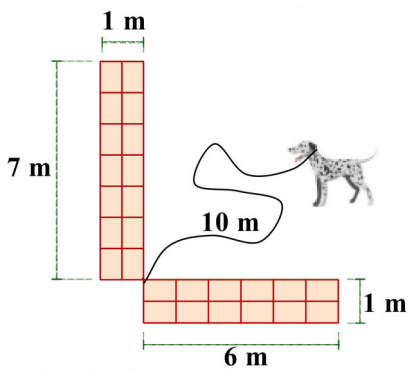
A) 5 B) 7 C) 6 D) 8 E) 1

17. If a real number x satisfies the equation $x^3 - x - 1 = 0$, what is the value of the expression

$$\frac{x^4 + x + 1}{x^6}.$$

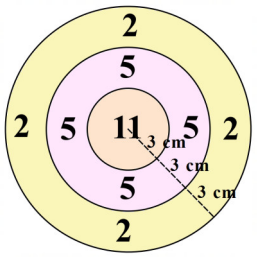
A) $\frac{1}{6}$ B) $\frac{1}{2}$ C) 1 D) 2 E) $\frac{1}{3}$

18. Öykü ties his dog with a 10-meter rope as shown in the figure between two walls 1 meter thick and 6 meters and 7 meters long, respectively. Find the total value of the areas that can be reached by the neck of the dog area where the dog's collar is attached.



A) $\frac{69}{2}\pi$ B) $\frac{61}{2}\pi$ C) $\frac{53}{2}\pi$ D) 27π E) 29π

19.



Berk continuously throws darts at a dartboard consisting of circles with the same centre and radii of $\mathbf{3}$, $\mathbf{6}$, $\mathbf{9}$ cm respectively. The dart always hits a region on the board for every throwing. What is Berk's average score if this throw continues for as long as desired?

A) $\mathbf{5,5}$ B) $\mathbf{5}$ C) $\mathbf{6}$ D) $\mathbf{4}$ E) $\mathbf{4,5}$

20. For $\mathbf{1} < x < y < z$, how many positive integer triples (x, y, z) are there satisfying the following equality?

$$x + xy + xyz = 1001$$

A) $\mathbf{1}$ B) $\mathbf{3}$ C) $\mathbf{4}$ D) $\mathbf{0}$ E) $\mathbf{7}$

21. A villager selling eggs, buys $\mathbf{10}$ liters of milk from another villager by exchanging $\mathbf{1}$ liter of milk for $\mathbf{8}$ eggs and starts selling milk along with eggs. When he sold all the products he had as a result of this exchange, he made $\mathbf{200}$TL more profit. If the villager sold $\mathbf{1}$ egg for $\mathbf{4}$ TL, how much did he sell $\mathbf{10}$ liters of milk in total?

A) $\mathbf{500}$ B) $\mathbf{520}$ C) $\mathbf{540}$ D) $\mathbf{560}$ E) $\mathbf{550}$

22. Let a, b and c be positive integers. If

$$1 \div (a + 1 \div (b + 1 \div c)) = \frac{21}{68},$$

then find the sum

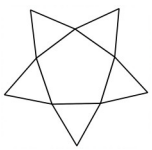
$$a + b + c.$$

A) $\mathbf{8}$ B) $\mathbf{10}$ C) $\mathbf{16}$ D) $\mathbf{15}$ E) $\mathbf{12}$

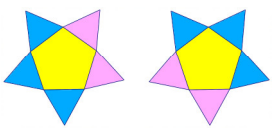
23. Berke wants to buy dollars for all the lira he has. Bank A, which offers 1 dollar for 20 liras, charges a service fee of 50 liras regardless of the amount of money exchanged. Bank B, which offers 1 dollar for 21 liras, charges a service fee of 2 dollars. Berke realizes that he can buy the same amount of dollars from both Bank A and Bank B with the money he has. How many liras does Berke have?

A) 180 B) 190 C) 210 D) 200 E) 220

24. Below, a star-shaped cardboard is created with 5 equilateral triangles and 1 regular pentagon. One side of this cardboard will be painted with yellow, blue and pink colors. How many different paintings can be done provided that at least 2 colors are used in each painting process and the pieces with common sides are painted in different colors?



(**Note :** Other colorings that can be obtained by rotating a dyed carton will be considered the same. For example, the following two colorings are identical.)



A) 18 B) 24 C) 21 D) 36 E) 27

25. What is the numerator of the rational number

$$A \cdot B - C \cdot D$$

in its simplest form, where

$$A = \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots + \frac{1}{97} + \frac{1}{99}$$

$$B = 1 + \frac{1}{5} + \frac{1}{7} + \cdots + \frac{1}{99} + \frac{1}{101}$$

$$C = 1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{97} + \frac{1}{99}$$

$$D = \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \cdots + \frac{1}{99} + \frac{1}{101}?$$

A) 98 B) 99 C) 101 D) 102 E) 100