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Frequencies Values of Orthotropic Composite Circular and Annular Plates

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Abstract

Free vibration analysis of orthotropic composite annular plate is investigated. First-order shear deformation theory (FSDT) is used for equation of motion. Two different kernels such as Regularized Shannon delta (RSD) kernel and Lagrange delta sequence (LDS) kernel are used. The method of discrete singular convolution (DSC) is used for numerical simulation of governing equations to obtain the frequency values. It is shown that the convergence and accuracy of the DSC method is very good for vibration problem of orthotropic annular plate.

Keywords: *Frequency, annular plate, discrete singular convolution, composite laminated.*

1. Introduction

Free vibration analyses of shells and plates have widely studied by this time. Frequencies values of shell structures have major importance for their design in different fields. In literature, it is possible to find a few books on analysis and design of these structures [1-11]. Some important studies have been listed in references [9-42].

This paper is summarized in a few sections. In section 2, just main formulations for truncated conical shells and annular plates are given via Tong's [43] paper. The method of discrete singular convolution (DSC) is given in section 3. DSC solution for free vibration of orthotropic annular plates with is briefly defined in section 4. Results are listed in Section 5. Finally, a conclusion is located at the end of the paper.

2. Fundamental Equations

Geometry and parameters of conical shells and annular plates are depicted in Fig. 1.

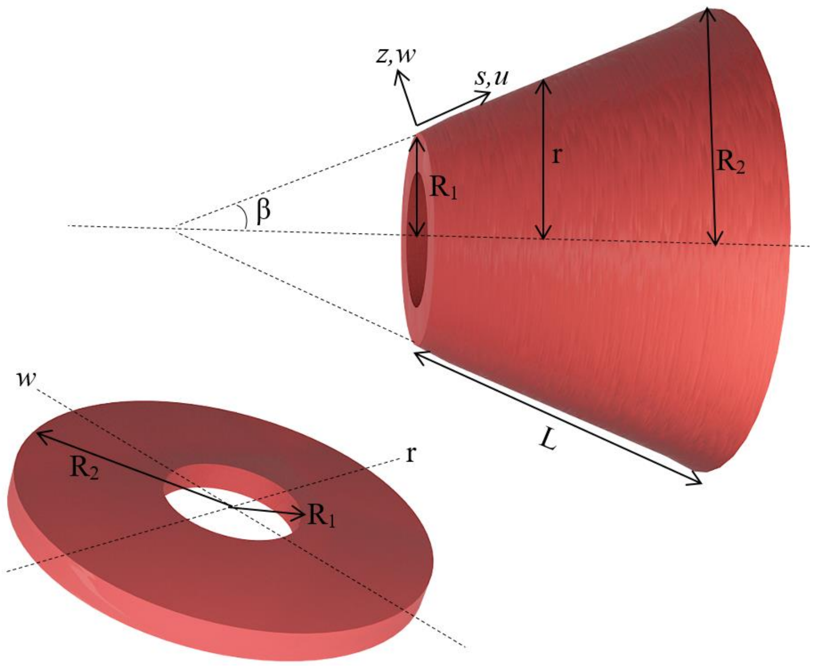

Fig. 1. Demonstration and notation of conical shell and annular plate

The equations of motion are [43]

$$\frac{\partial N_x}{\partial x}+\frac{1}{R(x)}\frac{\partial N_{xs}}{\partial s}+\frac{\sin\alpha}{R(x)}(N_x-N_s)=\rho h\frac{\partial^2 u}{\partial t^2} \tag{1}$$

$$\frac{\partial N_{xs}}{\partial x}+\frac{1}{R(x)}\frac{\partial N_s}{\partial s}+\frac{\cos\alpha}{R(x)}V_s+2\frac{\sin\alpha}{R(x)}N_{xs}=\rho h\frac{\partial^2 v}{\partial t^2} \tag{2}$$

$$\frac{\partial V_x}{\partial x}+\frac{\sin\alpha}{R(x)}V_x+\frac{1}{R(x)}\frac{\partial V_s}{\partial s}-\frac{\cos\alpha}{R(x)}N_s=\rho h\frac{\partial^2 w}{\partial t^2} \tag{3}$$

$$\frac{\partial V_x}{\partial x}+\frac{1}{R(x)}\frac{\partial V_s}{\partial s}-\frac{\cos\alpha}{R(x)}N_s+\frac{\sin\alpha}{R(x)}V_x=\frac{\rho h^3}{12}\frac{\partial^2 \varphi_x}{\partial t^2} \tag{4}$$

$$\frac{\partial M_{xs}}{\partial x}+2M_{xs}\frac{\sin\alpha}{R(x)}+\frac{1}{R(x)}\frac{\partial M_s}{\partial s}-V_s=\frac{\rho h^3}{12}\frac{\partial^2 \varphi_s}{\partial t^2} \tag{5}$$

Moment and forces components can be defined as:

$$\tilde{N}=\begin{Bmatrix} N_x \\ N_s \\ N_{xs}\end{Bmatrix}=\int_{-h/2}^{h/2}\begin{Bmatrix}\sigma_x \\ \sigma_s \\ \tau_{xs}\end{Bmatrix}dz \tag{6}$$

$$\tilde{M}=\begin{Bmatrix} M_x \\ M_s \\ M_{xs}\end{Bmatrix}=\int_{-h/2}^{h/2}\begin{Bmatrix}\sigma_x \\ \sigma_s \\ \tau_{xs}\end{Bmatrix}zdz \tag{7}$$

$$\tilde{V} = \begin{Bmatrix} V_s \\ V_x \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \tau_{sz} \\ \tau_{xz} \end{Bmatrix} dz \tag{8}$$

For annular plates (α=90; φ=360) based on the FSDT the differential equations of motion can be defined in each direction:

$$\begin{aligned}
&A_{11}\frac{\partial^2 u}{\partial x^2} + \frac{A_{11}}{R(x)}\sin\alpha\frac{\partial u}{\partial x} - \frac{A_{22}}{R^2(x)}\sin^2\alpha\cdot u + \frac{A_{33}}{R^2(x)}\frac{\partial^2 u}{\partial s^2} + \frac{(A_{12}+A_{33})}{R(x)}\frac{\partial^2 v}{\partial x\partial s} \\
&-\frac{(A_{22}+A_{33})}{R^2(x)}\sin\alpha\frac{\partial v}{\partial s} + \frac{A_{12}}{R(x)}\cos\alpha\frac{\partial w}{\partial x} - \frac{A_{22}}{R^2(x)}\sin\alpha\cdot\cos\alpha\cdot w - B_{11}\frac{\partial^2\varphi_x}{\partial x^2} \\
&+\frac{B_{11}}{R(x)}\sin\alpha\frac{\partial\varphi_x}{\partial x} - \frac{B_{22}}{R^2(x)}\sin^2\alpha\cdot\varphi_x + \frac{B_{33}}{R^2(x)}\frac{\partial^2\varphi_x}{\partial s^2} \\
&+\frac{(B_{12}+B_{33})}{R(x)}\frac{\partial^2\varphi_s}{\partial x\partial s} - \frac{(B_{22}+B_{33})}{R^2(x)}\frac{\partial\varphi_s}{\partial s}\sin\alpha = \rho h\frac{\partial^2 u}{\partial t^2}
\end{aligned} \tag{9}$$

$$\begin{aligned}
&\frac{(A_{12}+A_{33})}{R(x)}\frac{\partial^2 u}{\partial x\partial s} + \frac{(A_{22}+A_{33})}{R^2(x)}\sin\alpha\frac{\partial u}{\partial s} + A_{33}\frac{\partial^2 v}{\partial x^2} + A_{33}\frac{\sin\alpha}{R(x)}\frac{\partial v}{\partial x} \\
&-\frac{A_{33}}{R^2(x)}\sin^2\alpha\cdot v + \frac{A_{22}}{R^2(x)}\frac{\partial^2 v}{\partial s^2} - \frac{A_{44}}{R^2(x)}\cos^2\alpha\cdot v + \frac{(A_{22}+A_{44})}{R^2(x)}\cos\alpha\frac{\partial w}{\partial s} \\
&+\frac{(B_{12}+B_{33})}{R(x)}\frac{\partial^2\varphi_x}{\partial x\partial s} + \frac{(B_{22}+B_{33})}{R^2(x)}\sin\alpha\frac{\partial\varphi_x}{\partial s} + B_{33}\frac{\partial^2\varphi_s}{\partial x^2} + B_{33}\frac{\sin\alpha}{R(x)}\frac{\partial\varphi_s}{\partial x} \\
&-\frac{B_{33}}{R^2(x)}\sin^2\alpha\cdot\varphi_s + \frac{B_{22}}{R^2(x)}\frac{\partial^2\varphi_s}{\partial s^2} + A_{44}\frac{\cos\alpha}{R(x)}\cdot\varphi_s = \rho h\frac{\partial^2 v}{\partial t^2}
\end{aligned} \tag{10}$$

$$\begin{aligned}
&-\frac{A_{12}}{R(x)}\cos\alpha\frac{\partial u}{\partial x} - \frac{A_{22}}{R^2(x)}\cdot u\cdot\sin\alpha\cdot\cos\alpha - \frac{(A_{22}+A_{44})}{R^2(x)}\cdot\cos\alpha\frac{\partial v}{\partial s} + A_{55}\frac{\partial^2 w}{\partial x^2} \\
&+\frac{A_{55}}{R(x)}\sin\alpha\cdot\frac{\partial w}{\partial s} + \frac{A_{44}}{R^2(x)}\frac{\partial^2 w}{\partial s^2} - \frac{A_{22}}{R^2(x)}\cdot w\cdot\cos^2\alpha + A_{55}\frac{\partial\varphi_x}{\partial x} - \frac{B_{12}}{R(x)}\cos\alpha\cdot\frac{\partial\varphi_x}{\partial x} \\
&+\frac{A_{55}}{R(x)}\sin\alpha\cdot\varphi_x - \frac{B_{22}}{R^2(x)}\sin\alpha\cdot\cos\alpha\cdot\varphi_x + \frac{A_{44}}{R(x)}\cdot\frac{\partial\varphi_s}{\partial s} \\
&-\frac{B_{22}}{R^2(x)}\cdot\cos\alpha\frac{\partial\varphi_s}{\partial s} = \rho h\frac{\partial^2 w}{\partial t^2}
\end{aligned} \tag{11}$$

$$\begin{aligned}
&B_{11}\frac{\partial^2 u}{\partial x^2} + \frac{B_{11}}{R(x)}\sin\alpha\frac{\partial u}{\partial x} - \frac{B_{22}}{R^2(x)}\cdot u\cdot\sin^2\alpha + \frac{B_{33}}{R^2(x)}\frac{\partial^2 u}{\partial s^2} + \frac{(B_{12}+B_{33})}{R(x)}\frac{\partial^2 v}{\partial x\partial s} \\
&-\frac{(B_{22}+B_{33})}{R^2(x)}\sin\alpha\frac{\partial v}{\partial s} - A_{55}\frac{\partial w}{\partial x} + B_{12}\frac{\cos\alpha}{R(x)}\frac{\partial w}{\partial x} - \frac{B_{22}}{R^2(x)}\cdot w\cdot\sin\alpha\cos\alpha \\
&+D_{11}\frac{\partial^2\varphi_x}{\partial x^2} + D_{11}\frac{\sin\alpha}{R(x)}\frac{\partial\varphi_x}{\partial x} - \frac{D_{22}}{R^2(x)}\varphi_x\sin^2\alpha + \frac{D_{33}}{R^2(x)}\frac{\partial^2\varphi_x}{\partial s^2} - A_{55}\varphi_x \\
&+\frac{(D_{12}+D_{33})}{R(x)}\frac{\partial^2\varphi_s}{\partial x\partial s} - \frac{(D_{22}+D_{33})}{R^2(x)}\frac{\partial\varphi_s}{\partial s}\sin\alpha = \rho h\frac{\partial^2\varphi_x}{\partial t^2}
\end{aligned} \tag{12}$$

$$\frac{(B_{12}+B_{33})}{R(x)}\frac{\partial^2 u}{\partial x \partial s}+\frac{(B_{22}+B_{33})}{R^2(x)}\frac{\partial u}{\partial s}\sin\alpha+B_{33}\frac{\partial^2 v}{\partial x^2}+B_{33}\frac{\sin\alpha}{R(x)}\frac{\partial v}{\partial x}$$
$$-B_{33}\frac{\sin^2\alpha}{R^2(x)}\cdot v+\frac{B_{22}}{R^2(x)}\frac{\partial^2 v}{\partial s^2}+\frac{A_{44}}{R(x)}\cdot v\cdot\cos\alpha-\frac{A_{44}}{R(x)}\frac{\partial w}{\partial s}+\frac{B_{22}}{R^2(x)}\cos\alpha\frac{\partial w}{\partial s}$$
$$+\frac{(D_{12}+D_{33})}{R(x)}\frac{\partial^2\varphi_x}{\partial x\partial s}+\frac{(D_{22}+D_{33})}{R^2(x)}\sin\alpha\frac{\partial\varphi_x}{\partial s}-D_{33}\frac{\partial^2\varphi_s}{\partial x^2}$$
$$+D_{33}\frac{\sin\alpha}{R(x)}\frac{\partial\varphi_s}{\partial x}-\frac{D_{33}}{R^2(x)}\sin^2\alpha\cdot\varphi_s+\frac{D_{22}}{R^2(x)}\frac{\partial^2\varphi_s}{\partial s^2}-A_{44}\cdot\varphi_s=\rho h\frac{\partial^2\varphi_s}{\partial t^2} \tag{13}$$

3. Discrete Singular Convolution (DSC)

The method is originally introduced by Wei [44-47]. After the Wei's paper, the method of DSC have been used in many problems related to static, dynamic, free vibration and buckling analysis of structures [48-74]. A singular convolution F can be formulated as [44]

$$F(t)=(T*\eta)(t)=\int_{-\infty}^{\infty}T(t-x)\eta(x)dx \tag{14}$$

In the study, regularized Shannon kernel (RSK) and Lagrange kernels are used.
Regularized Shannon kernel (RSK)
RSK kernel can be listed below [45-47]

$$\delta_{\Delta,\sigma}(x-x_k)=\frac{\sin[(\pi/\Delta)(x-x_k)]}{(\pi/\Delta)(x-x_k)}\exp\left[-\frac{(x-x_k)^2}{2\sigma^2}\right];\ \sigma>0 \tag{15}$$

Gaussian envelope is showed by symbol σ. In discrete form, any derivation can be written as

$$\left.\frac{d^n f(x)}{dx^n}\right|_{x=x_i}=f^{(n)}(x)\approx\sum_{k=-M}^{M}\delta^{(n)}_{\Delta,\sigma}(x_i-x_k)f(x_k)\quad;\quad(n=0,1,2,...,) \tag{16}$$

Lagrange delta sequence (LDS) kernel
LDS kernel is defined for $i = 0,1,\dots, N-1$ and $j = -M,\dots,M$ is given by [44-50]

$$\Re_{i,j}(x) = \begin{cases} \prod_{k=i-M, k \neq i+j}^{i+M} \dfrac{x - x_k}{x_{i+j} - x_k}, & x_{i-M} \leq x \leq x_{i+M}, \\ 0 & \text{otherwise.} \end{cases} \tag{17}$$

In this case, the first and second order derivatives are given as

$$\delta^{(1)}_{\Delta,\sigma}(x) = \sum_{i=-M; i \neq k}^{M} \left(\frac{1}{x_k - x_i} \right) \prod_{i=-M, k \neq i}^{i+M} \frac{x - x_i}{x_k - x_i} \tag{18}$$

$$\delta^{(2)}_{\Delta,\sigma}(x) = \sum_{\substack{i,m=-M; i \neq k \\ m \neq k, i \neq m}}^{M} \left(\frac{1}{(x - x_i)(x - x_m)} \right) \prod_{i=-M, k \neq i}^{i+M} \frac{x - x_i}{x_k - x_i} \tag{19}$$

4. Results

In this section, two examples are solved via two different kernels such as Regularized Shannon delta (RSD) kernel and Lagrange delta sequence (LDS). Frequency values for annular and circular plates have been obtained and results are listed in Tables 1-2 for orthotropic case. Results are obtained for clamped cases for annular and circular plates. Both kernels are useful for numerical discretization via DSC. It is shown that the 9*7 grids are efficient for best convergence.

Table 1. Frequency values ($\Omega = \omega R_1 \sqrt{\rho(1-\upsilon_r \upsilon_\theta)/E_r}$) for orthotropic annular plate with C-C edges (R_1/R_2=2; R_1/h=1000; E_θ=70 GPa, υ_c=0.3, ρ_c=5700 kg/m^3, E_r=1400 GPa, υ_r=0.3, ρ =7850 kg/m^3)

Modes	Present DSC Results- RSD kernel			
(σ=2.8)	7×*7(M=14)*	9×*7(M=14)*	9×*9(M=14)*	11×*9(M=14)*
1	4.52420	4.52413	4.52413	4.52413
2	4.74045	4.74038	4.74038	4.74038
3	5.31453	5.31449	5.31442	5.31442
4	6.10096	6.10090	6.10085	6.10085
5	7.04989	7.04978	7.04976	7.04976
	Present DSC Results- LDS kernel			
(σ=2.8)	7×*7(M=14)*	9×*7(M=14)*	9×*9(M=14)*	11×*9(M=14)*
1	4.52443	4.52438	4.52438	4.52438
2	4.74059	4.74053	4.74051	4.74051
3	5.31504	5.31493	5.31493	5.31493
4	6.10103	6.10098	6.10094	6.10094
5	7.05068	7.05016	7.05003	7.05003

Table 2. Frequency values ($\Omega = \omega R_1 \sqrt{\rho(1-\upsilon_r \upsilon_\theta)/E_r}$) for orthotropic circular plate with clamped edges (R_1/h=1000; E_θ=70 GPa, υ_c=0.3, ρ_c=5700 kg/m^3, E_r=2800 GPa, υ_r=0.3, ρ =7850 kg/m^3)

Modes	Present DSC Results- RSD kernel			
(σ=2.8)	9×*9(M=14)*	9×*7(M=14)*	11×*9(M=14)*	11×*11(M=14)*
1	2.72081	2.72081	2.72081	2.72081
2	3.37236	3.37236	3.37236	3.37236
3	4.50756	4.50753	4.50753	4.50753
4	4.98188	4.98182	4.98182	4.98182
5	5.60235	5.60227	5.60227	5.60227
	Present DSC Results- LDS kernel			
(σ=2.8)	9×*9(M=14)*	9×*7(M=14)*	11×*9(M=14)*	11×*11(M=14)*
1	2.72086	2.72086	2.72086	2.72086
2	3.37244	3.37240	3.37240	3.37240
3	4.50767	4.50760	4.50760	4.50760
4	4.98195	4.98190	4.98188	4.98188
5	5.60242	5.60236	5.60234	5.60234

5. Discussions

In this paper discrete singular convolution method via FSDT shell theory is used for free vibration of annular and circular plates with orthotropic case. Two kernels namely Regularized Shannon delta (RSD) kernel and Lagrange delta sequence (LDS) kernel are used. The effects of grid numbers and kernel types on results have been investigated.

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