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Discrete Singular Convolution and Differential Quadrature Method for Buckling Analysis of Laminated Composite Plates

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Abstract

Two different numerical methods for buckling analysis of laminated composite plates are presented. Main formulations are based on the first-order shear deformation theory (FSDT) have been given. The method of discrete singular convolution (DSC) and differential quadrature (DQ) are employed for numerical solution. The results obtained by DSC and DQ methods were compared.

Keywords: Differential quadrature, discrete singular convolution; Buckling; laminated composite.

1. Introduction

As parallel to the composite materials technology in 1960s composite and laminated composite structural components have been widely used in different engineering applications such as automobile, mechanical, civil, aero-space and chemical engineering. Therefore, mechanical modeling of these systems is increasing studied such as free vibration, bending and buckling analyses by many researchers. More detailed information can be found in literature [1-8].
In this paper, numerical solution of buckling analysis of laminated composite rectangular plates are obtained via discrete singular convolution (DSC) and differential quadrature methods. First-order shear deformation theory (FSDT) is used for modeling. Based on the first-order shear deformation theory, the governing equations for symmetric laminates under transverse loads are given [1]

$$D_{11}\frac{\partial^2\varphi_x}{\partial x^2}+D_{66}\frac{\partial^2\varphi_x}{\partial y^2}+D_{16}\frac{\partial^2\varphi_y}{\partial x^2}+D_{26}\frac{\partial^2\varphi_y}{\partial y^2}+2D_{16}\frac{\partial^2\varphi_x}{\partial x\partial y}$$

$$(D_{12}+D_{66})\frac{\partial^2\varphi_y}{\partial x\partial y}-kA_{45}\left(\varphi_y+\frac{\partial w}{\partial y}\right)-kA_{55}\left(\varphi_x+\frac{\partial w}{\partial x}\right)=0, \qquad (1a)$$

$$D_{16}\frac{\partial^2\varphi_x}{\partial x^2}+D_{26}\frac{\partial^2\varphi_x}{\partial y^2}+D_{66}\frac{\partial^2\varphi_y}{\partial x^2}+D_{22}\frac{\partial^2\varphi_y}{\partial y^2}+2D_{26}\frac{\partial^2\varphi_y}{\partial x\partial y}$$

$$(D_{12}+D_{66})\frac{\partial^2\varphi_x}{\partial x\partial y}-kA_{44}\left(\varphi_y+\frac{\partial w}{\partial y}\right)-kA_{55}\left(\varphi_x+\frac{\partial w}{\partial x}\right)=0, \tag{1b}$$

$$\frac{\partial}{\partial x}\left[kA_{45}\left(\varphi_y+\frac{\partial w}{\partial y}\right)+kA_{55}\left(\varphi_x+\frac{\partial w}{\partial x}\right)\right]$$

$$+\frac{\partial}{\partial y}\left[kA_{44}\left(\varphi_y+\frac{\partial w}{\partial y}\right)+kA_{55}\left(\varphi_x+\frac{\partial w}{\partial x}\right)\right]+q(x,y)$$

$$+N_x\frac{\partial^2 w}{\partial x^2}+2N_{xy}\frac{\partial^2 w}{\partial x\partial y}+N_y\frac{\partial^2 w}{\partial y^2}=0. \tag{1c}$$

Where N_x, N_{xy} and N_y are the in-plane applied forces**.** Also, mass inertias are given as

$$I_0=\int_{-h/2}^{h/2}\rho dz, \qquad I_2=\int_{-h/2}^{h/2}\rho z^2 dz. \tag{2,3}$$

Where ρ and h denote the density and total thickness of the plate, respectively. The bending moments and shear forces are given as

$$M_x=D_{11}\frac{\partial\varphi_x}{\partial x}+D_{12}\frac{\partial\varphi_y}{\partial y}+D_{16}\frac{\partial\varphi_y}{\partial x}+D_{16}\frac{\partial\varphi_x}{\partial y}, \tag{4a}$$

$$M_y=D_{12}\frac{\partial\varphi_x}{\partial x}+D_{22}\frac{\partial\varphi_y}{\partial y}+D_{26}\frac{\partial\varphi_y}{\partial x}+D_{16}\frac{\partial\varphi_x}{\partial y}, \tag{4b}$$

$$M_y=D_{16}\frac{\partial\varphi_x}{\partial x}+D_{26}\frac{\partial\varphi_y}{\partial y}+D_{66}\frac{\partial\varphi_y}{\partial x}+D_{16}\frac{\partial\varphi_x}{\partial y}, \tag{4c}$$

$$Q_x=kA_{55}\left(\varphi_x+\frac{\partial w}{\partial x}\right)+kA_{45}\left(\varphi_y+\frac{\partial w}{\partial y}\right), \tag{5a}$$

$$Q_y=kA_{45}\left(\varphi_x+\frac{\partial w}{\partial x}\right)+kA_{44}\left(\varphi_y+\frac{\partial w}{\partial y}\right). \tag{5b}$$

Where A_{ij} and D_{ij} are the stretching and bending stiffness, k is the shear correction factor taken as 5/6. Also, the *x-y* coordinate plane is located on the mid-plane of the laminate.

2. Discrete Singular Convolution (DSC)

A singular convolution can be defined, in the context of distribution theory, by [9]

$$F(t) = (T * \eta)(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)dx \tag{6}$$

where $T(t-x)$ is a singular kernel. The DSC algorithm can be realized by using many approximation kernels. However, it was shown [10-41] that for many problems, the use of the regularized Shannon kernel (RSK) is very efficient. The RSK is given by [11]

$$\delta_{\Delta,\sigma}(x-x_k) = \frac{\sin[(\pi/\Delta)(x-x_k)]}{(\pi/\Delta)(x-x_k)} \exp\left[-\frac{(x-x_k)^2}{2\sigma^2}\right]; \ \sigma>0 \tag{7}$$

where $\Delta=\pi/(N-1)$ is the grid spacing and N is the number of grid points. For numerical computations, this can be expressed as

$$\left.\frac{d^n f(x)}{dx^n}\right|_{x=x_i} = f^{(n)}(x) \approx \sum_{k=-M}^{M} \delta^{(n)}_{\Delta,\sigma}(x_i - x_k) f(x_k); \quad (n=0,1,2,...,) \tag{8}$$

where superscript n denotes the nth-order derivative with respect to x.

3. Differential Quadrature Method (DQM)

In the differential quadrature method, a partial derivative of a function with respect to a space variable at a discrete point is approximated as a weighted linear sum of the function values at all discrete points in the region of that variable [42-61]. The first derivatives at point i, at $x = x_i$ is given by [42]

$$\Psi_x(x_i) = \left.\frac{\partial \Psi}{\partial x}\right|_{x=x_i} = \sum_{j=1}^{N} A_{ij}\Psi((x_j); \qquad i = 1,2,........,N \tag{9}$$

where x_j are the discrete points in the variable domain, $\psi(x_j)$ are the function values at these points and A_{ij} are the weighting coefficients for the first order derivative attached to these function values. As similar to the first order, the second order derivative can be written as

$$\Psi_{xx}(x_i) = \left.\frac{\partial^2 \Psi}{\partial x^2}\right|_{x=x_i} = \sum_{j=1}^{N} B_{ij}\Psi(x_j); \qquad i = 1,2,........,N \tag{10}$$

According to the DSC method, the governing equations (Eqs.1c) can be discretized into the following form for buckling

$$kA_{45}\left(\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta x)\psi_{kj}^{y}+\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta x)\psi_{kj}^{x}+2\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta x)W_{kj}\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta y)W_{ik}\right)$$

$$kA_{55}\left(\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta x)\psi_{kj}^{x}+\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta x)W_{kj}\right)$$

$$+kA_{44}\left(\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta y)\psi_{ik}^{y}+\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta y)W_{ik}\right)$$

$$+N_x\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta x)W_{kj}+2N_{xy}\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta x)W_{kj}\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta y)W_{ik}$$

$$N_y\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta y)W_{ik}=0 \tag{11}$$

Similarly, DQ form of the above equation can also be given. Consequently, we solve the remaining eigenvalue problems given below to obtain the non-dimensional buckling load, such as,

$$GX=\lambda BX \tag{12}$$

4. Numerical examples

In numerical solutions of laminate are assumed to be of the same thickness and density. Linearly elastic composite material behavior is taken into consideration. In all the tables, S denotes simply supported while C means clamped. Following values for material parameters are used for numerical analysis.

$$G_{12}=G_{13}=0.6E_2;\ G_{23}=0.5E_2;\ v_{12}=0.25;\ E_1/E_2=40.$$

Only example have been solved and obtained results are compared. Uniaxial buckling loads of a SSSS laminated ($0^0/90^0/90^0/0^0$) square plate is obtained by the DSC method using the 13 grid points. The results in Table 1 are compared respectively to the analytical solutions based on first-order shear deformation theory (FSDT) and higher-order shear deformation theory by Khdeir and Librescu [52], the three-dimensional linear elasticity solutions of Noor [53]. Compared with the data given by Khdeir and Librescu [52], it is shown that the present results are in close agreement using the 13 grid points. Table 2 also listed same results for different grid numbers and methods.

Table 1. Comparisons of uniaxially buckling loads of a SSSS laminated ($0^0/90^0/90^0/0^0$) square plate (a/h=10; $\lambda = N_x a^2 / E_2 h^3$)

E_1/E_2	Sources			Present study
	Noor [53]	HSDT Khdeir and Librescu [52]	FSDT Khdeir and Librescu [52]	
20	15.0191	15.418	15.351	15.352
30	19.3040	19.813	19.757	19.759
40	22.8807	23.489	23.453	23.456

Table 2. Comparison of bucking loads of Table for different methods

E_1/E_2	FSDT Khdeir and Librescu [52]	DSC Results		DQ Results	
		N=11	N=13	N=11	N=13
20	15.351	15.354	15.352	15.352	15.352
30	19.757	19.761	19.759	19.762	19.760
40	23.453	23.456	23.456	23.454	23.454

The method of DSC and DQ are very effective and practical methods both macro scaled mechanical problems [13-34] and the nano scale problems [62-68]. Nonlinear analysis of nano-scaled mechanical systems will also been solved via these methods and results will presented in the next.

5. Conclusions

In the present study, buckling loads of laminated composite square plates are obtained by the methods of DQ and DSC. The first-order shear deformation theory (FSDT) is used in the study with the governing differential equations transformed into a standard eigenvalue problem by these methods. It is concluded that both the DQ and DSC methods give reasonable accurate results for buckling.

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