2024

INTERNATIONAL ANTALYA MATHEMATICS OLYMPIAD

6TH GRADE QUESTION BOOKLET

NAME SURNAME : ....

SCHOOL : .... GRADE : ....

SIGNATURE : ....

EXAMINATION RULES

1. It is forbidden to take the exam with a phone. Please hand in your phone to the attendant. This exam consists of 25 multiple-choice questions and the exam duration is 120 minutes.

2. Each question has only one correct answer. Mark your correct answer by completely crossing out the relevant box on your answer sheet. No marking in the question booklet will be evaluated.

3.All questions are of equal value and four wrong answers will cancel one correct answer. Questions left blank will not have a positive or negative effect on the evaluation.

4. The questions are NOT in order of difficulty. Therefore, it is recommended that you review all questions before you start answering.

5. It is forbidden to use aids such as compasses, rulers, calculators and scratch paper. Do all your work on the question booklet.

6. During the exam, you will not talk to the staff and you will not ask them any questions. It is unlikely that there will be a mistake in the questions. If this happens, the exam academic board will take appropriate action. In this case, you should mark the option that you think is the most correct.

7. Students are not allowed to ask each other for pencils, erasers, etc.

8. It is forbidden to leave the exam for the first 60 minutes. A candidate who goes out will not be allowed to take the exam again.

9. Do not forget to hand in your answer sheet and question booklet to the staff before leaving the exam hall.

1. $n!$ denotes the product $1 \cdot 2 \cdot 3 \cdots n$. What is the sum of the numerator and denominator in the simplest form of the fraction below?

$$\frac{5! + 6! + 7!}{6! + 7!}$$

A) $\mathbf{90}$ B) $\mathbf{84}$ C) $\mathbf{85}$ D) $\mathbf{97}$ E) $\mathbf{86}$

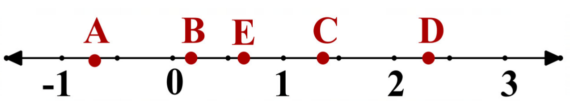
2. The absolute value of a real number a, denoted $|a|$ is the non-negative value of a without regard to its sign. Namely, $|a| = a$ if a is a positive number, and $|a| = -a$ if a is negative. According to the equation

$$\left|\mathbf{n} - \left|-3^2 - \left|-5^2 - \left|-2^3\right|\right|\right|\right| = 12,$$

what is the sum of the possible values of n?

A) $\mathbf{80}$ B) $\mathbf{84}$ C) $\mathbf{16}$ D) $\mathbf{24}$ E) $\mathbf{76}$

3. Below, some numbers are given on the number line with the letters $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ and $\mathbf{E}$. According to this, which of the following fractions is greater than the others?



A) $\frac{\mathbf{D}}{\mathbf{A}}$ B) $\frac{\mathbf{B}}{\mathbf{E}}$ C) $\frac{\mathbf{C}}{\mathbf{B}}$ D) $\frac{\mathbf{C}}{\mathbf{E}}$ E) $\mathbf{E} \cdot \mathbf{C}$

4. Some books will be chosen at random from a library containing $\mathbf{25}$ Turkish, $\mathbf{20}$ Mathematics, $\mathbf{10}$ Science and $\mathbf{9}$ English books. What is the minimum number of books to be selected so that there are at least $\mathbf{13}$ books of the same course?

A) $\mathbf{43}$ B) $\mathbf{44}$ C) $\mathbf{53}$ D) $\mathbf{50}$ E) $\mathbf{39}$

5.

$$\frac{2^{16} + 2 \cdot 2^3}{2^{17}} - \frac{2^{14} + 8}{2^{16}}$$

What is the simplest form of the above rational expression?

A) $\frac{3}{8}$ B) $\mathbf{2}$ C) $\frac{1}{8}$ D) $\frac{1}{4}$ E) $\frac{1}{2}$

6. There are $\mathbf{6}$ shelves in a bookcase and there are $\mathbf{23, 25, 32, 29, 26, 33}$ books on each shelf respectively. At least how many books should be moved so that each shelf has exactly the same number of books?

A) $\mathbf{12}$ B) $\mathbf{13}$ C) $\mathbf{10}$ D) $\mathbf{8}$ E) $\mathbf{9}$

7. The area of a trapezoid is equal to half of the sum of the upper and lower base multiplied by the height. A ruler measures lengths 2% more than their actual value. Pınar calculates the area of the trapezoid using the lengths of the bottom base, top base and height that she found with the ruler. If the actual area of the trapezoid is 10000, how much larger is the area found by Pınar than the real area?

A) 404 B) 402 C) 401 D) 400 E) 398

8. The code for a positive number n is constructed as follows. First, the number n is written in powers of all primes from smallest to largest. Then, the code of the number is formed by writing these powers side by side with a comma between them, including zero.

$$n = 2^a \cdot 3^b \cdot 5^c \cdot 7^d \cdots p^k \xrightarrow{\text{CODE}} (a, b, c, d, ..., k)$$

For example,

$$20 = 2^2 \cdot 3^0 \cdot 5^1 \xrightarrow{\text{CODE}} (2, 0, 1)$$
$$30 = 2^1 \cdot 3^1 \cdot 5^1 \xrightarrow{\text{CODE}} (1, 1, 1)$$
$$315 = 2^0 \cdot 3^2 \cdot 5^1 \cdot 7 \xrightarrow{\text{CODE}} (0, 2, 1, 1)$$

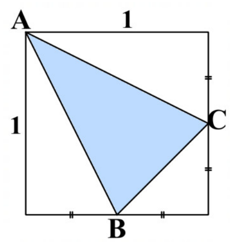
According to this, multiplying the number whose code is $(1, 2, 3, 4)$ by which number gives the number whose code is $(4, 2, 4, 5)$?

9. How many of the digits of the following number are zeros?

$$(10^7 + 777) \cdot 10^7 + 77 \cdot 10^8 - 1$$

A) 10 B) 7 C) 9 D) 3 E) 4

10. The area of a triangle is equal to half of the base times height. The area of a right triangle is equal to half of the product of the lengths of the perpendicular sides. Use this information to solve the following question.



In the figure above, a triangle is drawn inside a square with a side of 1 unit and two of its vertices are on the midpoints of the sides of the square. Find the area of this shaded triangle.

A) $\frac{1}{5}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) $\frac{3}{8}$ E) $\frac{5}{8}$

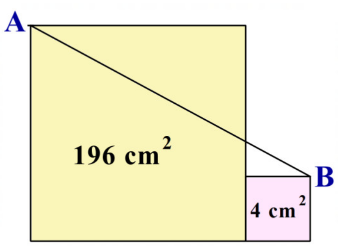
11. The teacher wrote $\mathbf{101}$ on the board. The students began to write the next number one by one, by adding $\mathbf{5}$, then by adding $\mathbf{10}$, then by adding $\mathbf{15}$, so in this way they created a sequence given below:

$$\mathbf{101, 106, 116, 131, 151, \ldots}$$

If there are $\mathbf{22}$ students in the class, what number will the last student write on the board?

A) $\mathbf{1361}$ B) $\mathbf{1366}$ C) $\mathbf{1356}$ D) $\mathbf{1351}$ E) $\mathbf{1371}$

12. If the area of the large square is $\mathbf{196}$ cm$^{\mathbf{2}}$ and the area of the small square is $\mathbf{4}$ cm$^{\mathbf{2}}$, then find the length |AB|.



A) $\mathbf{16}$ B) $\mathbf{20}$ C) $\mathbf{19}$ D) $\mathbf{15}$ E) $\mathbf{80}$

13. For any number $\boldsymbol{A}$, $\boldsymbol{k(A)}$, $\boldsymbol{b(A)}$ and $\boldsymbol{t(A)}$ are defined as follows.

- $\boldsymbol{k(A)}$: The smallest digit of the number $\boldsymbol{A}$
- $\boldsymbol{b(A)}$: The largest digit of the number $\boldsymbol{A}$
- $\boldsymbol{t(A)}$: Sum of the digits of the number $\boldsymbol{A}$

For example, for the number $\boldsymbol{A = 45601}$, $\boldsymbol{k(A) = 0}$, $\boldsymbol{b(A) = 6}$ and $\boldsymbol{t(A) = 4 + 5 + 6 + 0 + 1 = 16.}$

How many five-digit even numbers with different digits are there such that $\boldsymbol{b\,(A) = 7,\ t\,(A) = 25}$ and $\boldsymbol{k\,(A)}$ is a prime number?

A) $\mathbf{48}$ B) $\mathbf{120}$ C) $\mathbf{240}$ D) $\mathbf{24}$ E) $\mathbf{64}$

14. For any real number $\boldsymbol{x}$, the representations $\lfloor \boldsymbol{x} \rfloor$ and $\lceil \boldsymbol{x} \rceil$ are called the floor integer of real number x and the ceiling integer of real number $\boldsymbol{x}$, respectively, and are defined as follows.

If $\boldsymbol{x}$ is an integer,

$$\lfloor \boldsymbol{x} \rfloor = \lceil \boldsymbol{x} \rceil = \boldsymbol{x};$$

If $\boldsymbol{x}$ is not an integer,

$\lfloor \boldsymbol{x} \rfloor$ = The largest integer less than $\boldsymbol{x}$

$\lceil \boldsymbol{x} \rceil$ = The smallest integer greater than $\boldsymbol{x}$.

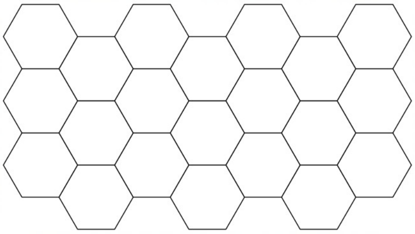
For example, $\lfloor \mathbf{3,4} \rfloor = \mathbf{3}$, $\lceil \mathbf{3,4} \rceil = \mathbf{4}$, $\lfloor \mathbf{3} \rfloor = \lceil \mathbf{3} \rceil = \mathbf{3}.$

If $\mathbf{10} < \boldsymbol{x} < \mathbf{24}$ and $\mathbf{10} < \boldsymbol{y} < \mathbf{24}$, what is the maximum value of the following expression?

$$\left\lfloor \frac{\boldsymbol{x}}{\mathbf{4}} \right\rfloor - \left\lceil \frac{\boldsymbol{y}}{\mathbf{3}} \right\rceil$$

A) $\mathbf{3}$ B) $\mathbf{0}$ C) $\mathbf{2}$ D) $\mathbf{1}$ E) $\mathbf{4}$

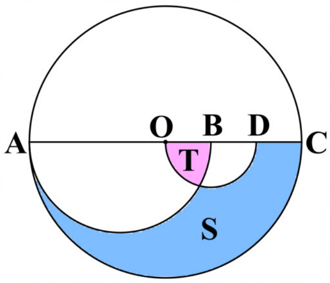
15.



The figure above is created with regular hexagons. We want to color each hexagon with red, blue and black. How many different ways can this figure be colored, if no two adjacent hexagons are the same color?

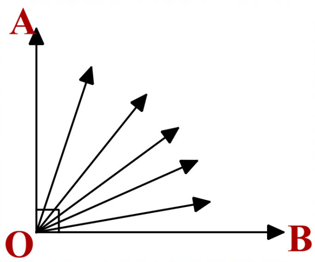
A) **8** B) **6** C) **9** D) **10** E) **120**

16. In the figure below, two semicircles of different radii are drawn inside the great circle centered at **O** with a radius of **6** cm. If $|\mathbf{OB}| = |\mathbf{BD}| = |\mathbf{DC}|$, how much larger is the shaded area indicated by $\boldsymbol{S}$ than the shaded area indicated by $\boldsymbol{T}$?



A) $\mathbf{8\pi}$ B) $\mathbf{7\pi}$ C) $\mathbf{9\pi}$ D) $\mathbf{10\pi}$ E) $\mathbf{6\pi}$

17. In the figure, **OA** and **OB** are perpendicular to each other. How many acute angles are there whose vertex is point **O**?



A) **22** B) **18** C) **15** D) **6** E) **20**

18. The following operations are allowed on the given numbers:

■ Multiplying the number by **2**.

■ Add **2** to the number.

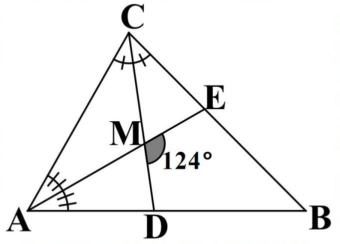
According to this, at least how many operations must be done to get the number **400** from the number **1**?

A) **8** B) **10** C) **9** D) **12** E) **16**

19. Two types of tickets are sold for a concert: standing or seated. Three quarters of the participants in this concert are sitting in four fifths of the seats reserved for the concert. $\mathbf{24}$ of the seat tickets are not sold and these seats remain empty. According to this, how many people attended the concert standing?

A) $\mathbf{24}$ B) $\mathbf{32}$ C) $\mathbf{36}$ D) $\mathbf{42}$ E) $\mathbf{30}$

20. In the figure, $\boldsymbol{AE}$ and $\boldsymbol{CD}$ are angle bisectors of the triangle $\boldsymbol{ABC}$. If $\boldsymbol{AE}$ and $\boldsymbol{CD}$ intersect at point $\boldsymbol{M}$ and $\boldsymbol{m(\angle DME) = 124^\circ}$, what is the measure of the angle $\angle \boldsymbol{ABC}$?



A) $\mathbf{60^\circ}$ B) $\mathbf{72^\circ}$ C) $\mathbf{68^\circ}$ D) $\mathbf{82^\circ}$ E) $\mathbf{76^\circ}$

21. How many three-digit numbers less than $\mathbf{500}$ are exactly divisible by the number in the hundreds place?

A) $\mathbf{200}$ B) $\mathbf{209}$ C) $\mathbf{198}$ D) $\mathbf{211}$ E) $\mathbf{199}$

22. If $\mathbf{-5 \leq x \leq 6}$ and $\mathbf{-6 \leq y \leq 10}$, what is the sum of all integer values that the product of $\boldsymbol{x \cdot y}$ can take?

A) $\mathbf{480}$ B) $\mathbf{1200}$ C) $\mathbf{980}$ D) $\mathbf{555}$ E) $\mathbf{500}$

23. Alp and Berk, without showing each other, find four different integers whose product is 360 and they add the four numbers they find. Then, they both write their sums on the board. What is the maximum difference between the two numbers written on the board?

A) 134 B) 130 C) 122 D) 356 E) 80

24. A villager selling eggs, buys 10 liters of milk from another villager by exchanging 1 liter of milk for 8 eggs and starts selling milk along with eggs. When he sold all the products he had as a result of this exchange, he made 200TL more profit. If the villager sold 1 egg for 4 TL, how much did he sell 10 liters of milk in total?

A) 500 B) 520 C) 540 D) 560 E) 550

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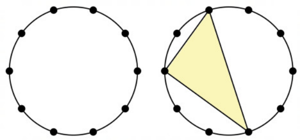
25. $n!$ denotes the product $1 \cdot 2 \cdot 3 \cdots n$. For $1 \leq k < n$, we can choose k out of n different objects in

$$\frac{n!}{k!\,(n-k)!}$$

different ways. This expression is simply denoted by $\binom{n}{k}$. For example, 3 out of 4 objects can be chosen in

$$\binom{4}{3} = \frac{4!}{3!\,(4-3)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 1} = 4$$

different ways.



There are 10 points lie on a circle. By joining points how many triangle can be drawn from these points? One of these triangles is shown in the figure.

A) 100 B) 54 C) 108 D) 120 E) 144