International Journal of Engineering & Applied Sciences (IJEAS)
Vol.8, Issue 3(2016) 58-66

The Control of The Lorenz Chaotic System Blended With The Noise

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Received date: October 2016
Accepted date: October 2016

Abstract

In this study, the Lorenz chaotic system behavior is investigated according to the change in the noise of the system and measurement. The noise was emerged as an important impact on the Lorenz chaotic system. The discrete-time LQR (Linear Quadratic Regulator) control method and discrete-time PI controller method have been used to the noisy system controls. Noise was accepted as a destructor that changes the output behavior of the chaotic system. Noisy system outputs were enforced to move to the equilibrium points by the support of controllers. Optimization of the controller parameters for noiseless system models was performed using a genetic algorithm method. Then, both control system algorithms determined as successful in transporting to the equilibrium point for state variables which are blended with the noise. Changes, associated with the noise conditions are given in the graphics and tables. Besides, the error performance criteria and the control performance criteria were obtained to evaluate the results. This study was performed by MATLAB-Simulink simulation program.

Keywords: Lorenz chaotic system, Gaussian white noise, Discrete-time PI, LQR, controller, Genetic algorithm, Matlab-Simulink

1. Introduction

The initial condition of the state variables changes dramatically the behavior of chaotic systems seen as a disorder of the order [1]. The chaotic systems having nonlinear properties provide the convenience for analyzing the behavior of many systems in real life. In 1960, the first studies related to the chaos have been performed by Lorenz to understand and estimate the weather events. Nowadays, the chaotic systems are widely used for the more reliable, efficient, less energy expenditure of many systems like encrypted communications, mixers, dishwashers and washing machines [2, 3].

The noise factor should be added to the defined expression of the chaotic systems for expressing the chaotic systems like improved truth appropriate models in the other systems. The noise is very important factor in the real life system. For example, if the sound and vibration produced by the aircraft engine express with the chaotic system, the air pressure and impacts depended on the variation of temperature created by the aircraft is called as the noise. All of the factors should be added in the expression of this model to express this dynamic structure as closed to the real systems. The consideration of other many factors in the noise can be better solution method for expressing with the better model of the real system.

The examination of the chaotic systems with the noise is the very important as the scientifically. The noise can be used in the chaotic systems to determine what extent distruction of the dynamic nature of chaotic systems, controllability of system. Furthermore, this study can be provided the benefit for new studies to decrease effects of the noise on the chaotic systems and make the noise factor unimportant.

There has been a significant amount of studies in the literature to analyze the effects of noise on chaotic systems. Some of them are as follows: An iterative method has been proposed by Çoban et al. to determine the noise level of the chaotic time series [4]. It is shown that small value having 2% standard deviation of the noise is the destructive for the signal [5]. Lei et al. have been examined as in detail the chaos noise of the duffing type oscillator [6]. Hassan has suggested a secure chaotic communication in public communication channel with the noisy [7]. Majhi et al. have investigated the effects of noise-induced functional delay on chaotic synchronization [8]. Longtin has studied the effects of noise on harmonic resonance [9]. Behera et al. have used to assessed artificial neural network for active noise control [10].

In this study, the parameters of the discrete-time PI controller and LQR (Linear Quadratic Regulator) of the equation Ricatti Located in the optimization calculations Q and R matrix parameters were optimized by a genetic algorithm. Then, performance on a Lorenz chaotic system with the noise were analyzed in the MATLAB / Simulink simulation program.

2. Description of system

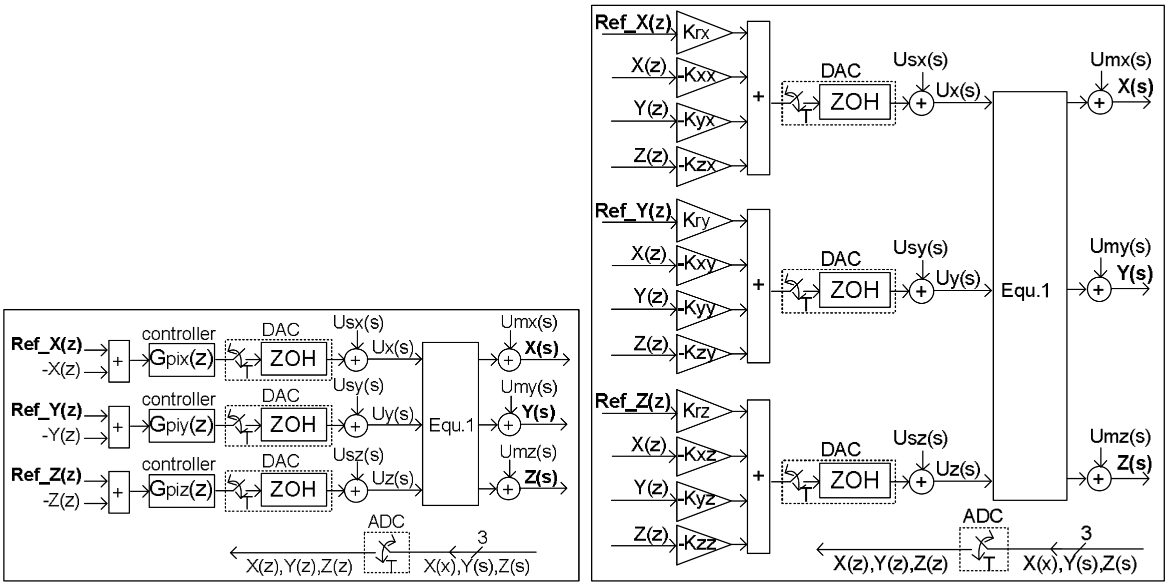
The mathematical model of the Lorenz chaotic system has given in Eq. (1) where x, y and z are state variables; a, b and c are positive constant parameters.

$$\left.\begin{aligned} \dot{x} &= \mathrm{a}(y-x) \\ \dot{y} &= (\mathrm{c}-z)x - y \\ \dot{z} &= xy - \mathrm{b}z \end{aligned}\right\} \tag{1}$$

The system have been added to the controller state variables ($U_x(s)$, $U_y(s)$, $U_z(s)$) for controlling of the Lorenz chaotic system given in Eq. 1. If the system and measurement noise is added to the Lorenz chaotic system, and then the Laplace transform is applied to the system, as a result, Eq. 2 is obtained.

$$\left.\begin{aligned} X(s) &= \frac{1}{s}\Big[aY(s) - aX(s) + U_x(s) + U_d(s)\Big] + U_{\ddot{o}}(s) \\ Y(s) &= \frac{1}{s}\Big[X(s)(c - Z(s)) - Y(s) + U_y(s) + U_d(s)\Big] + U_{\ddot{o}}(s) \\ Z(s) &= \frac{1}{s}\Big[X(s)Y(s) - bZ(s) + U_z(s) + U_d(s)\Big] + U_{\ddot{o}}(s) \end{aligned}\right\} \tag{2}$$

The non-linear Lorenz chaotic system structure block diagram with the inclusion of the system and measurement noise is shown in Figure 1. Figure 1(a) has given with the discrete-time PID controller model. Also, Figure 1(b) has given with the discrete-time LQR controller model.



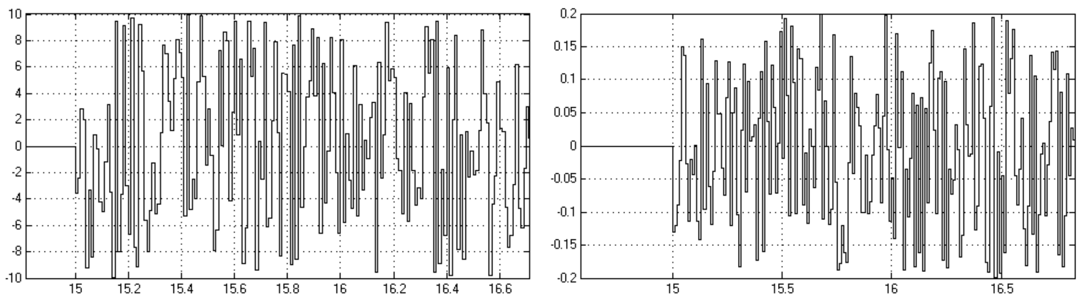
(a) **(b)**

Figure 1. The Lorenz chaos control model with the noise, **(a)** PI, **(b)** LQR control model

The model given in Eq. 2 has been transformed into the Z-domain using the S-domain integral operator method. Using the given operator (forward difference method) with the Eq. 3, this conversion was performed. Here, T_s show the sampling period [11].

$$s \approx \frac{1}{T_s}\left(\frac{1-z^{-1}}{z^{-1}}\right) \tag{3}$$

The located the closed-loop control system structure in Figure 1 which are Gaussian white measurement noise $U_{system}(s)$ and white Gaussian noise $U_{measure}(s)$ is shown in Figure 2. While, the amplitude of the white Gaussian system noise varies in the range of ∓ 10, the amplitude of white Gaussian measurement noise change in the range of ∓ 0.2. Both the noise was applied to the system after 15 seconds.



(a)

(b)

Figure 2. Applied white Gaussian noise,(a) system noise, (b) measurement noise

2.1 Genetic Algorithm

The studies related to genetic algorithm are described in detail in conducted a study earlier. There might be studied, genetic algorithm coefficients, flow diagram and the obtained results in detail [12].

Using the LQR method, the Q and R parameters located within a matrix equation has to be optimum selected in order to optimize the control coefficients of the Riccati equation. These values of the conventional method, there are experimentally. In this study, the matrix parameters are optimized using a genetic algorithm and is given in Table 1. The Kr variable in the Table 1 is the correction variable. The Kr values were calculated using state variables belonging to the steady-state error.

Table 1. The LQR optimization controller criteria

Control State Variable	Kx	Ky	Kz	Kr
X	3.6	6.49	5.456	6
Y	3	10.89	3.02	400
Z	2.94	-5.878	4.948	7.625

The parameters of the PI controller was found to as optimal with using the genetic algorithm. Results are given in Table 2.

Table 2. The PI optimization controller criteria

	Gpi_x	Gpi_y	Gpi_z
Kp	211.2	183.1	249
Ki	17623.9	10840.3	22184

3. Simulation Results

The simulation results are shown in Figures 4, 5, 6, 7, 8, 9, and 10. Uncontrolled and noiseless version of the z state variable in the Figure 4, the version of the z state variable uncontrolled and with system noise in the Figure 5, the version of the z state variable uncontrolled and with the measurement noise in the Figure 6 (a) and with both the measurement noise and system noise in the Figure 6 (b) are shown.

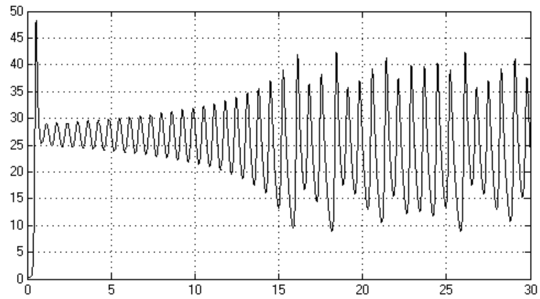


Figure 4. Uncontrolled and noiseless version of the z state variable

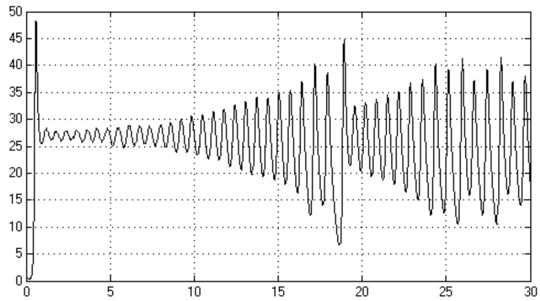
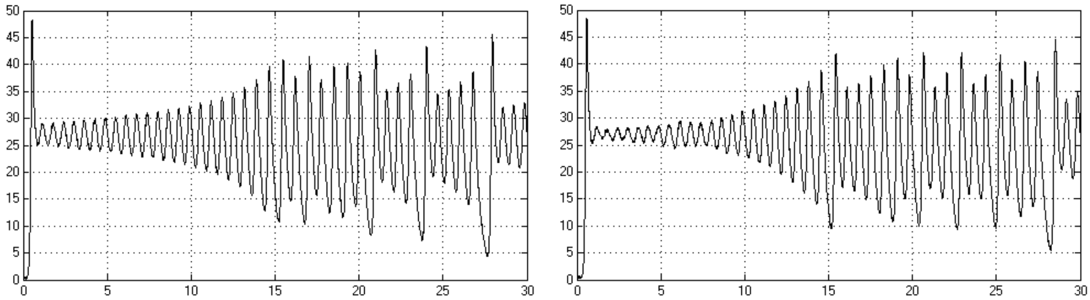


Figure 5. The version of the z state variable uncontrolled and system noise



(a) **(b)**

Figure 6. (a) The version of the z state variable uncontrolled and the measurement noise, **(b)** The version of the z state variable uncontrolled and the measurement noise

The variations of the state variables obtained against noise by applying closed-loop control system the discrete-time LQR control method are shown in the Figure 7. When the method is also applied to discrete-time PI controller, change of the state variables are given in Figure 8. While, the noise has been activated after 15 seconds and the control has been activated after 25 seconds passed.

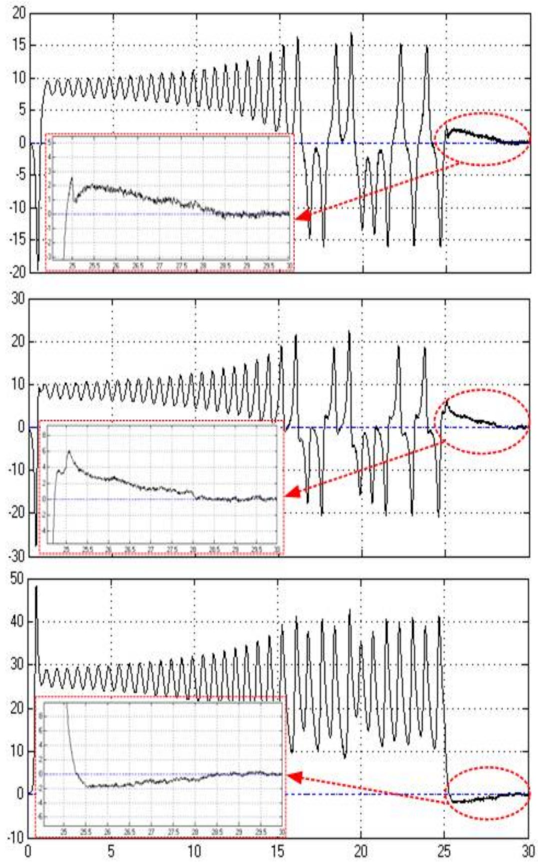


Figure 7. Under discrete-time LQR control methods, the x, y and z state variables change, respectively

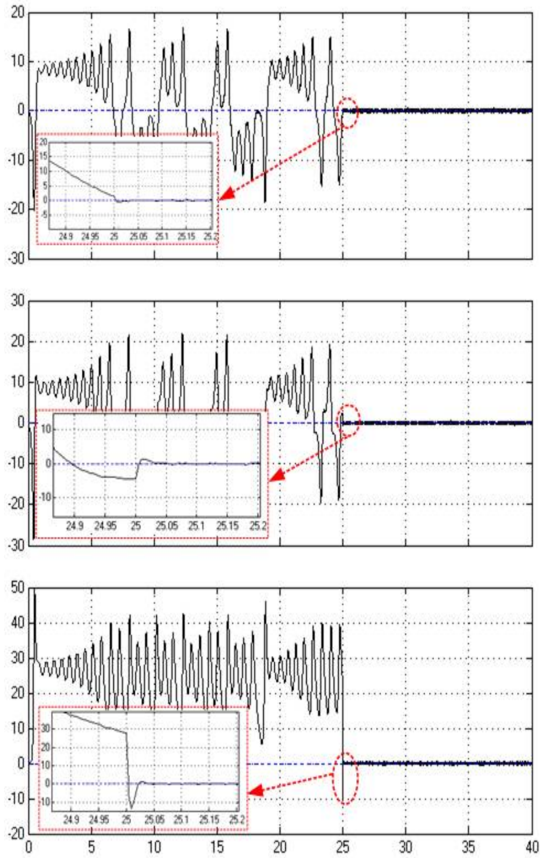


Figure 8. Under PI control methods, the x, y and z state variables change, respectively

According to the results obtained in the Figure 7 and 8, controller performance criteria for both controllers are given in Table 3.

Table 3. The controllers performance criteria

State Variable	Controller	Settling Time	Steady-State Error
x	LQR	3200 (ms)	±0.10
	PI	10 (ms)	±1e-4
y	LQR	3400 (ms)	±0.12
	PI	25 (ms)	±1e-4
z	LQR	2900 (ms)	±0.14
	PI	30 (ms)	±1e-4

Additionally, the results obtained using both control method, xy, xz and yz the phase portraits are shown in Figures 9 and 10.

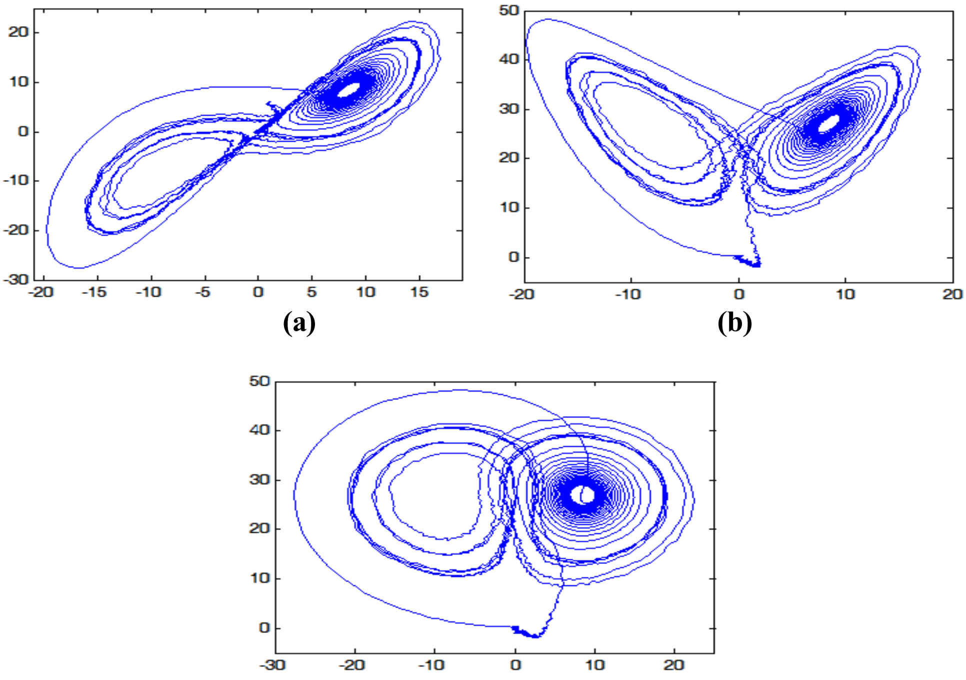


Figure 9. Under discrete-time LQR control methods, (a) xy , (b) xz and (c) yz phase portraits

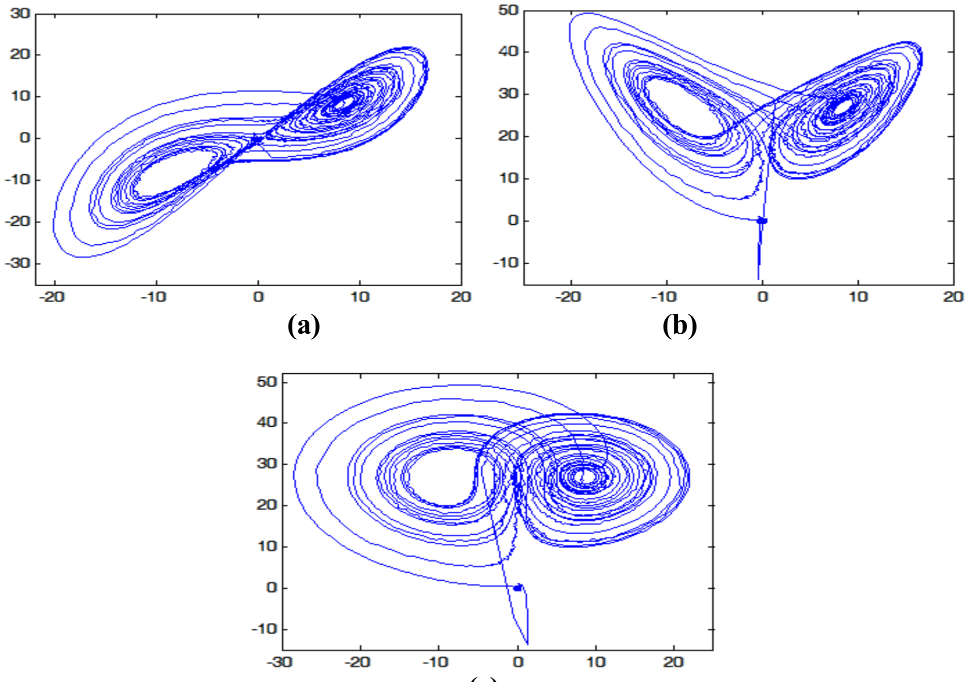


Figure 10. Under discrete-time PI control methods, (a) xy , (b) xz and (c) yz phase portraits

The error performance indicators frequently used in the literature are an important indicator to determine the performance of the controller. The exchange of error is given in Table 4. The integral of absolute error (IAE), integral of squared error (ISE), integral of time multiplied by absolute error (ITAE) has been calculated according to the Eq. (4).

$$\left.\begin{aligned} \text{IAE} &= \int |e(t)| dt \\ \text{ISE} &= \int e^2(t) dt \\ \text{ITEA} &= \int t|e(t)| dt \end{aligned}\right\} \tag{4}$$

Table 4. The error performance criteria of controller

	Controller	IAE	ISE	ITEA
x	LQR	4.085	5.545	8.77
	PI	1.138	0.1618	8.528
y	LQR	6.898	18.9	11.244
	PI	1.14	0.211	8.359
z	LQR	5.611	23.81	9.814
	PI	1.367	2.527	8.91

4. Conclusions

Lorenz or other chaotic systems are extremely sensitive to parameter variations. The waveform of a chaotic system output signals were affected directly even too low parameter or proceeding percentile deviations were occurred. Indeed, the white Gaussian noise that has been injected into Lorenz chaotic system made proceeding deviations so that altered wave form of the chaotic signals based on the initial conditions were generated completely altered. The discrete time PI controller has been observed to be more successful as compared to the discrete time LQR controller for controlling the non-linear Lorenz's chaotic system states with noise injection. Also, the success has been confirmed with ISE, IAE and ITAE objective functions.

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