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Static Analysis of a Fiber Reinforced Composite Beam Resting on Winkler-Pasternak Foundation

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Abstract

This paper presents static analysis of a simply supported beam made of fiber reinforced composite material resting on elastic foundation. The foundation type is considered as Winkler-Pasternak foundation type. The first-shear beam theory is used in the kinematics of the beam and the Ritz method is used and in the solution of the problem. In the Ritz method, algebraic polynomials are used with the trivial functions. In the numerical examples, the effects of fibre orientation angles, the volume fraction and foundation parameters on the static deflections of fiber reinforced composite beam are investigated. The numerical results show that fiber orientation angle, volume fraction and foundation parameter have great influence on static behavior of fiber reinforced composites.

Keywords: Fiber Reinforced Composite Material; Static Analysis; Winkler-Pasternak Foundation; Ritz Method

1. Introduction

Fiber reinforced composite (FRC) structures are used in a lot of engineering applications, for example, airplanes, machine, marine, and civil engineering projects. FRC structures mainly preferred in the engineering projects due to their higher strength-weight ratios, more lightweight and ductile properties.

In the literature, many researchers investigated the static, dynamic and stability analyses of FRC structures in last decades. Some investigations about of FRC structures are as follows; Krawczuk et al. [1] studied the vibration of cracked composite beams. Shen [2] presented post-buckling analysis of laminated plate with thermal effects resting on elastic foundation. Sayman [3] investigated elastic-plastic analysis of aluminum metal-matrix laminated plate under thermal effect. Shukla et al. [4] presented thermal postbuckling analysis of laminated plates. Emery et al. [5] analyzed thermoelastic stress analysis of laminated orthotropic plates. Shen [6] presented thermal nonlinear analysis of functionally graded nanocomposite plates reinforced by single-walled carbon nanotubes. Akgöz and Civalek [7,8,9,10] presented mechanical behavior of composite structure resting on foundation.

Kishore et al. [11] investigated nonlinear analysis of magnetostrictive layered plate by using third order shear deformation theory. Sahoo and Singh [12] analyzed static of layered composite plates by using the hyperbolic zigzag theory. Houmat [13] presented nonlinear vibration of laminated composite plates with curvilinear fibers. Khorshid and Farhadi [14] analyzed hydrostatic vibration analysis of a laminated composite rectangular plate partially contacting with a bounded fluid. DeValve and Pitchumani [15] investigated damping vibration analysis of rotating composite beams with embedded carbon nanotubes. Tornabene et al. [16] investigated static and vibration analysis of laminated doubly-curved shells and panels embedded in elastic foundation by using the generalized differential quadrature. Akbaş [17-22] presented free vibration of functionally graded composite beams. Yüksel and Akbaş [23] presented thermal effects of laminated plates by using the Navier method. Draiche et al. [24] presented static analysis of laminated reinforced composite plates based on first-order shear deformation theory by using the Navier method. Jena et al. [25] analysed dynamic behavior of cracked fiber reinforced composite beams. Zenkour et al. [26] investigated torsional dynamics of carbon nanotubes embedded in viscoelastic medium. Waddar et al. [27] investigated buckling and dynamic response of cenosphere reinforced epoxy composite core sandwich beam with sisal fabric/epoxy composite facings under compressive load by experimentally. Akbaş [28-43] investigated nonlinear behavior and forced vibration analysis of composite structures. Also, many researchers investigated mechanical analysis of composite structures resting on foundation [44-60].

The main purpose of this study is to investigate the effects of the fibre orientation angles, the volume fraction and foundation parameters on the static deflections of the FRC beam in detail. In solution of the problem, first shear deformation beam theory and the energy based Ritz method are used. In the numerical results, the effects of fibre orientation angles, the volume fraction and foundation parameters on the static deflections of the FRC beam are investigated.

2. Formulations

Figure 1 shows a simply supported FRC beam resting on Winkler-Pasternak Foundation with with spring constant k_w and k_p, the length L, the height h and width b under a point load (Q) at midpoint of the beam. When the Pasternak foundation spring constant k_p=0, the foundation model reduces to Winkler type.

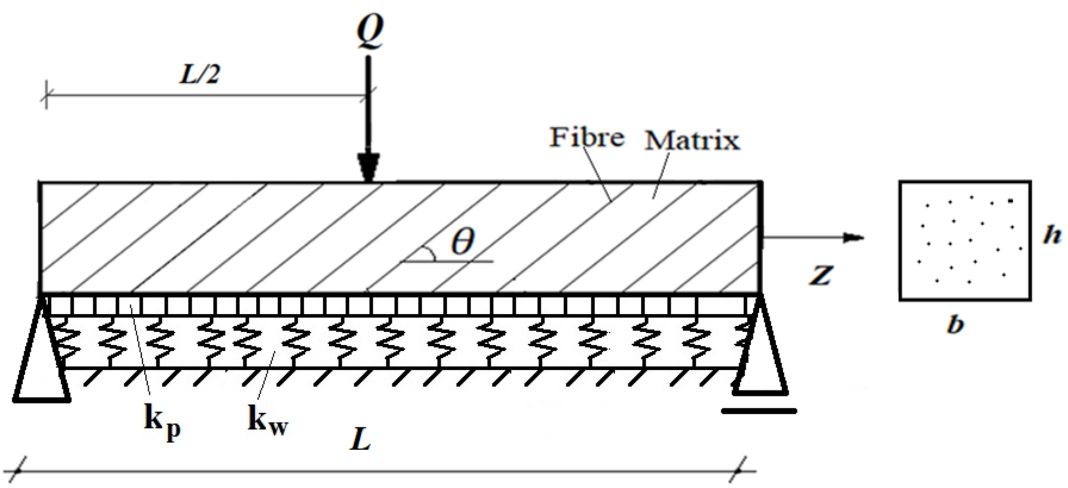

Fig.1. A simply supported FRC beam resting on Winkler-Pasternak Foundation under a point load.

The axial strain (ε_z) and shear strain (γ_{zy}) are given according to the first shear deformation

$$\varepsilon_z = \frac{\partial u}{\partial z} - Y\frac{\partial \emptyset}{\partial z} \tag{1a}$$

$$\gamma_{zy} = \frac{\partial v}{\partial z} - \emptyset \tag{1b}$$

where, u, v and $\emptyset$ are axial displacement, vertical displacement and rotation, respectively. The constitute relation is presented as follows;

$$\begin{Bmatrix} \sigma_z \\ \sigma_{zy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{16} \\ \bar{Q}_{16} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_z \\ \gamma_{zy} \end{Bmatrix} \tag{2}$$

where $\overline{Q}_{ij}$ are the transformed components of the reduced constitutive tensor. The transformed components of the reduced constitutive tensor for orthotropic material are as follows:

$$\overline{Q}_{11} = Q_{11}l^4 + 2(Q_{12} + 2Q_{66})l^2n^2 + Q_{22}n^4 \tag{3a}$$

$$\overline{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})sin^2cos^2 + Q_{12}(l^4 + n^4) \tag{3b}$$

$$\overline{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})nl^3 + (Q_{12} - Q_{22} + 2Q_{66})n^3l \tag{3c}$$

$$\overline{Q}_{22} = Q_{11}n^4 + 2(Q_{12} + 2Q_{66})n^2l^2 + Q_{22}l^4 \tag{3d}$$

$$\overline{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})n^3l + (Q_{12} - Q_{22} + 2Q_{66})nl^3 \tag{3e}$$

$$\overline{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})n^2l^2 + Q_{66}(n^4 + l^4) \tag{3f}$$

where $l=cos\ \theta$ and $n= sin\ \theta$, θ indicates the fiber orientation angle and the expressions of Q_{ij} are as follows;

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}\ , \qquad Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} \tag{4a}$$

$$Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} \tag{4b}$$

$$Q_{21} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} \tag{4c}$$

$$Q_{66} = G_{12} \tag{4d}$$

where E_1 is the Young's modulus in the X direction, E_2 is the Young's modulus in the Y direction, ν_{12} and ν_{21} are Poisson's ratios and G_{12} is the shear modulus in XY plane. The gross mechanical properties of the composite materials are calculated by using the following expression (Vinson and Sierakowski [61]):

$$E_1 = E_fV_f + E_m\left(1 - V_f\right), \tag{5a}$$

$$E_2 = E_m\left[\frac{E_f+E_m+(E_f-E_m)V_f}{E_f+E_m-(E_f-E_m)V_f}\right] \tag{5b}$$

$$\nu_{12} = \nu_f\ V_f + \nu_m\left(1 - V_f\right), \tag{5c}$$

$$G_{12} = G_m\left[\frac{G_f+G_m+(G_f-G_m)V_f}{G_f+G_m-(G_f-G_m)V_f}\right] \tag{5d}$$

$$\rho = \rho_fV_f + \rho_m\left(1 - V_f\right), \tag{5e}$$

where f indicates the fibre and m indicates the matrix. V_f is the volume fraction of fiber. E, G, ν and ρ are the Young's modulus, the shear modulus, Poisson's ratio and mass density, respectively.

The strain energy (U_i), and potential energy of the external loads (U_e) are presented as follows;

$$U_i = \frac{1}{2}\int_0^L \left[A_0\left(\frac{\partial u_0}{\partial z}\right)^2 - 2A_1\frac{\partial u_0}{\partial z}\frac{\partial \emptyset}{\partial z} + A_2\left(\frac{\partial \emptyset}{\partial z}\right)^2\right] dZ + \frac{1}{2}\int_0^L K_s B_0\left[\left(\frac{\partial v_0}{\partial z}\right)^2 - 2\frac{\partial v_0}{\partial z}\emptyset + \emptyset^2\right] dZ + \frac{1}{2}\int_0^L \left(k_w (v_0)^2 + k_p\left(\frac{\partial v_0}{\partial z}\right)^2\right) dZ \tag{6a}$$

$$U_e = -Q(t)\, v\big(z_p, t\big) \tag{6b}$$

where,

$$(A_0, A_1, A_2) = \int_A \overline{Q}_{11}(1, Y, Y^2) dA\,,\; B_0 = \int_A \overline{Q}_{66} dA, \tag{7}$$

The total potential energy of the problem is expressed as follows:

$$\Pi = (U_i - U_e) \tag{8}$$

In the solution of the problem in Ritz method, approximate solution is given as series of *i* terms of the following form:

$$u(z) = \sum_{i=1}^{\infty} \mathrm{a}_i\, \alpha_i(z) \tag{9a}$$

$$v(z) = \sum_{i=1}^{\infty} \mathrm{b}_i\, \beta_i(z) \tag{9b}$$

$$\emptyset(z) = \sum_{i=1}^{\infty} \mathrm{c}_i\, \gamma_i(z) \tag{9c}$$

where a_i, b_i and c_i are the unknown coefficients, $\alpha_i(z)$, $\beta_i(z)$, $\gamma_i(z)$ are the coordinate functions depend on the boundary conditions over the interval [0,L]. The coordinate functions for the simply supported beam are given as algebraic polynomials:

According to the minimum total potential energy principle, unknown coefficients $a_{i,}$, b_i , c_i which correspond to the minimum of the total potential energy (Π) are determined by the conditions:

$$\frac{\partial \Pi}{\partial a_i} = 0\,,\; \frac{\partial \Pi}{\partial b_i} = 0\,,\; \frac{\partial \Pi}{\partial c_i} = 0 \tag{10}$$

Differentiation of *Π* in respect to unknown coefficients produces the following equilibrium equations:

$$[\mathrm{K}]\{\mathrm{q}\} = \{\mathrm{F}\} \tag{11}$$

where $[K]$ and {F} are the stiffness matrix and load vector, respectively. The detail of these expressions are given as follows;

$$[K] = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \tag{12}$$

Where

$$K_{ij}^{11} = \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L A_0 \frac{\partial\alpha_i}{\partial z}\frac{\partial\alpha_j}{\partial z} dz \text{ , } K_{ij}^{12} = 0 \text{ ,}$$
$$K_{ij}^{13} = -\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L A_1 \frac{\partial\alpha_i}{\partial z}\frac{\partial\gamma_j}{\partial z} dz, K_{ij}^{21} = 0,$$
$$K_{ij}^{22} = \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L K_s B_0 \frac{\partial\beta_i}{\partial z}\frac{\partial\beta_j}{\partial z} + \beta_i\beta_j k_w + \frac{\partial\beta_i}{\partial z}\frac{\partial\beta_j}{\partial z} k_p dz,$$
$$K_{ij}^{23} = -\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L K_s B_0 \frac{\partial\beta_i}{\partial z}\gamma_j \, dz,$$
$$K_{ij}^{31} = -\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L A_1 \frac{\partial\gamma_i}{\partial z}\frac{\partial\alpha_j}{\partial z} dz,$$
$$K_{ij}^{32} = -\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L K_s B_0 \gamma_i \frac{\partial\beta_j}{\partial z} dz,$$
$$K_{ij}^{33} = \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L A_2 \frac{\partial\gamma_i}{\partial z}\frac{\partial\gamma_j}{\partial z} + \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L K_s B_0 \gamma_i\gamma_j \, dz, \tag{13}$$

$$\{\mathrm{F(t)}\} = Q\beta_j \tag{14}$$

The dimensionless quantities can be expressed as

$$\bar{k}_w = \frac{k_w L^4}{E_f I}, \quad \bar{k}_p = \frac{k_p L^2}{E_f I}, \quad \bar{\mathrm{v}} = \frac{\mathrm{v}}{L} \tag{15}$$

$\bar{k}_w$ and $\bar{k}_p$ are the dimensionless Winkler Pasternak parameters, $\bar{\mathrm{v}}$ is lateral dimensionless displacement.

3. Numerical Results

In the numerical study, static displacements of the FRC simply supported beam are presented and discussed. In the numerical examples, the materials of the beams are selected as made of graphite fibre-reinforced polyamide composite and its material parameters are as follows (Krawczuk et al [1]); E_m = 2.756 GPa, E_f = 275.6 GPa, Gm = 1.036 GPa, G_f = 114.8 GPa, ν_m = 0.33, νf = 0.2. The geometry properties of the beam are selected as $b = 0.1$ m, h=0.1 m and L=1.2 m. In the numerical results, number of the series term is taken as 10. The load value is selected as Q_0=1000 kN.

In figure 2, effects of the volume fraction of fiber (v_f) on the lateral static dimensionless displacements of FRC beam at midpoint ($\bar{\mathrm{v}}_m$) are presented with effects of foundation parameter for $\theta = 30$. It is seen from figure 2 that, displacements of the FRC beam decrease with increasing of the volume fraction of fiber and foundation stiffness parameters due to the bending rigidity increases according to Eq. 5. With increasing of foundation stiffness parameters, the difference among the results of v_f decreases considerably. It is seen from figures 2 that Pasternak parameter $\bar{k}_p$ is more effective than Winkler parameter $\bar{k}_w$ on the behavior of the volume fraction of fiber.

In figure 3, effects of the fiber orientation angles (θ) on the lateral static dimensionless displacements of FRC beam at midpoint ($\bar{\mathrm{v}}_m$) are presented with effects of foundation parameter for v_f=0.3. Figure 3 shows that, displacements of the FRC beam increase with increasing of the fiber orientation angles (θ) due to the bending rigidity increases according to

Eq. 3. It is observed from figure 3, Pasternak parameter $\bar{k}_p$ is more effective on the results of fiber orientation angles like the results of the volume fraction of fiber.

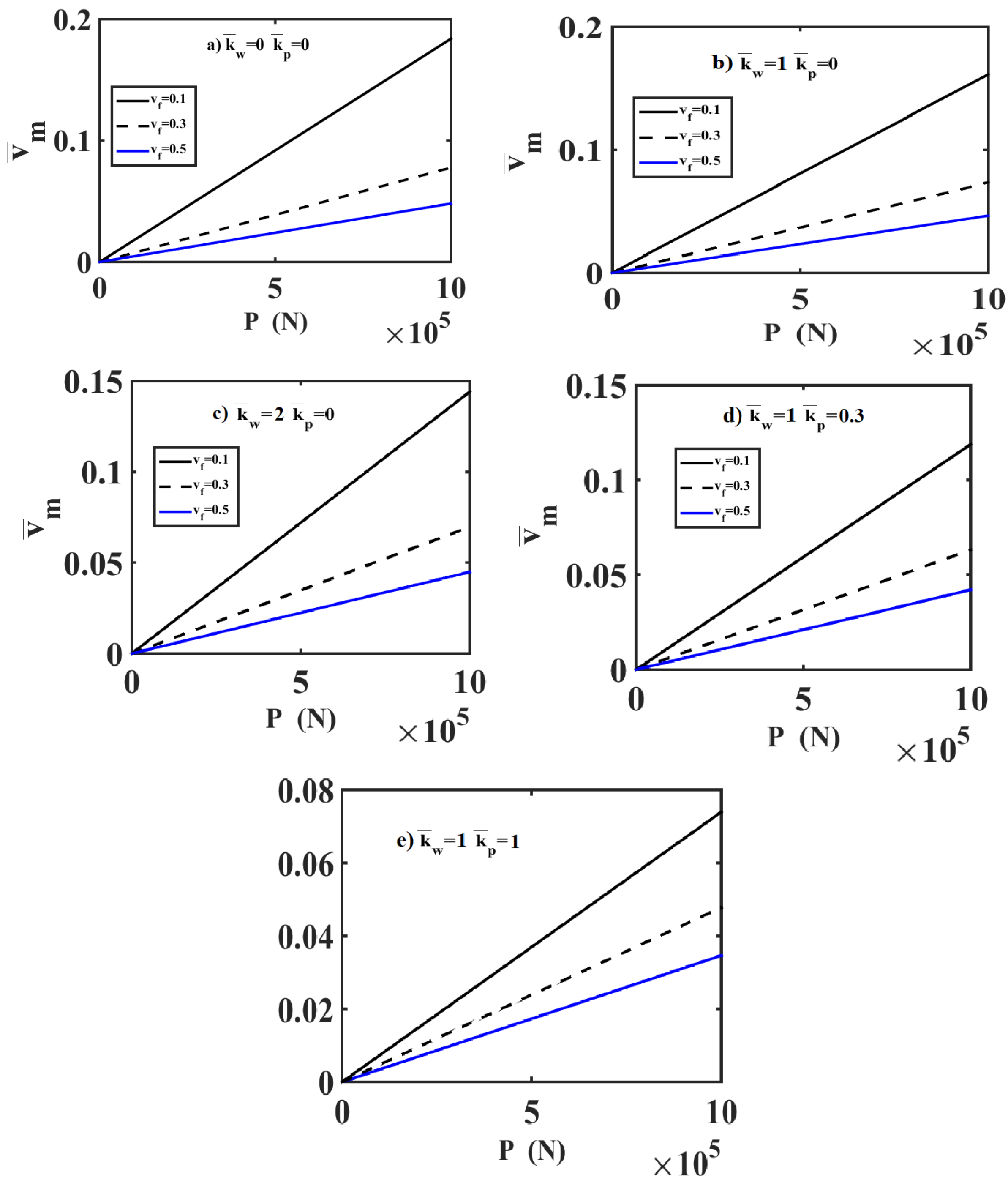

Fig.2. Load – dimensionless lateral displacement (at midpoint) relation for different values of the volume fraction of fiber (v_f) for a) $\bar{k}_w = 0$, $\bar{k}_p = 0$ b) $\bar{k}_w = 1$, $\bar{k}_p = 0$, c) $\bar{k}_w = 2$, $\bar{k}_p = 0$, d) $\bar{k}_w = 1$, $\bar{k}_p = 0.3$, e) $\bar{k}_w = 1$, $\bar{k}_p = 1$

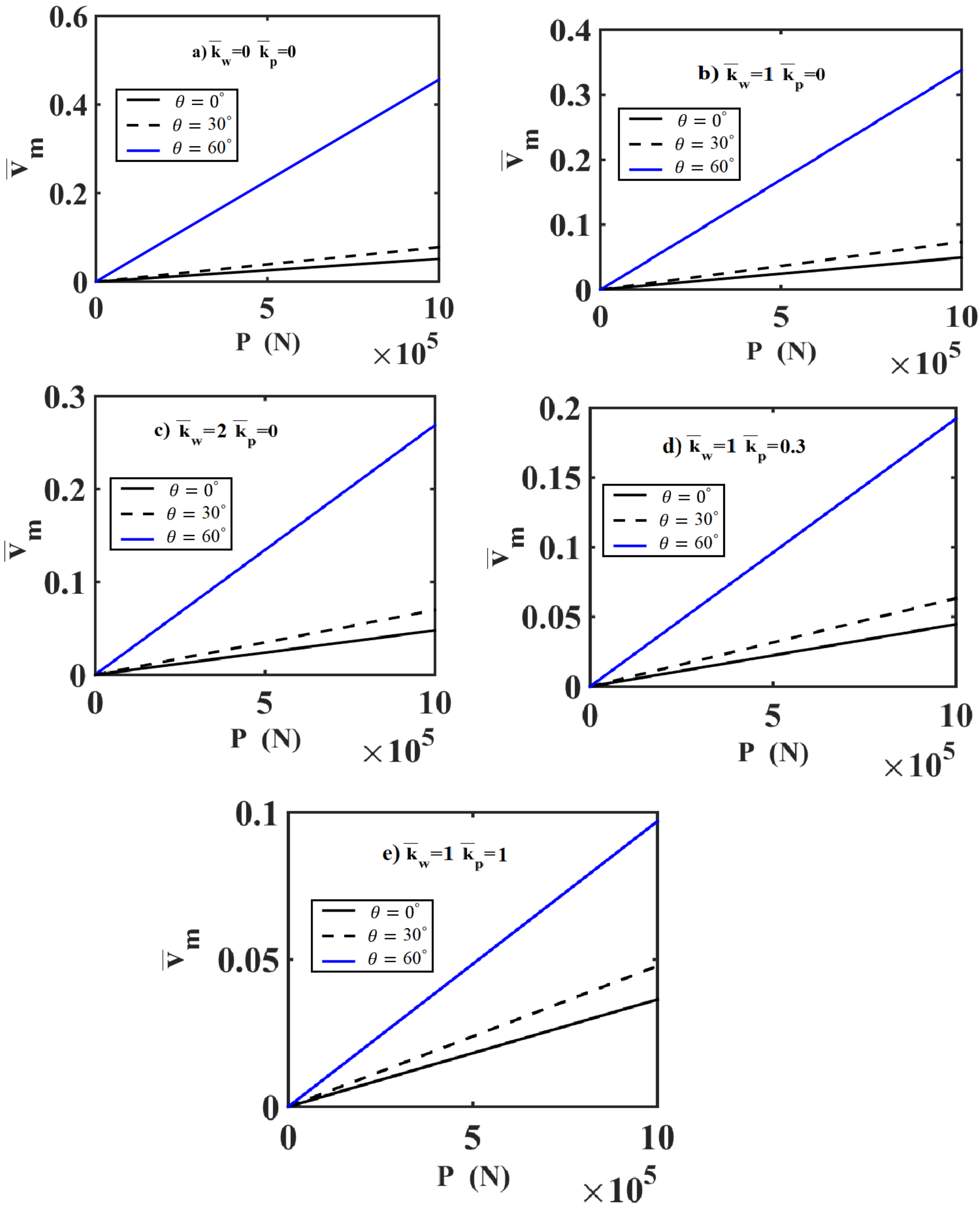

Fig.3. Load – dimensionless lateral displacement (at midpoint) relation for different values of the fiber orientation angles (θ) for a) $\bar{k}_w = 0$, $\bar{k}_p = 0$ b) $\bar{k}_w = 1$, $\bar{k}_p = 0$, c) $\bar{k}_w = 2$, $\bar{k}_p = 0$, d) $\bar{k}_w = 1$, $\bar{k}_p = 0.3$, e) $\bar{k}_w = 1$, $\bar{k}_p = 1$

4. Conclusions

Effects of Winkler-Pasternak foundation parameters and composite material parameters on the static displacements of the FRC simply supported beam are investigated in this paper by using the first shear deformation beam theory. In solution of the problem, the energy based Ritz method is implemented. The presented results show that the displacements of FRC beam change significantly with fiber orientation angle and the volume fraction. The Pasternak parameter is a great influence on behavior of material properties of FRC.

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