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Defination of length-scale parameter in Eringen's Nonlocal Elasticity via Nolocal Lattice and Finite Element Formulation

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Abstract

Nonlocal elasticity theory is one of the popular approaches for nano mechanic problems. In this study, nonlocal parameter is defined via different approach. Nonlocal finite element formulations for axial vibration of nanorods have been given and some parameters are compared with the lattice dynamics. Weak form and final finite element formulation for axial vibration case have been derived.

Keywords: Nonlocal elasticity, finite element, axial vibration, rod, lattice dynamics.

1. Introduction

It is known that the general forms of the nonlocality are as follows [1]:

$$t_{kl,k} + \rho(f_1 - \ddot{u}_1) = 0 \tag{1}$$

$$t_{kl}(x) = \int_V \alpha(|x' - x|, \tau)\sigma_{kl}(x')dv(x'), \tag{2}$$

$$\sigma_{kl}(x') = \lambda e_{rr}(x')\delta_{kl} + 2\mu e_{kl}(x'), \tag{3}$$

$$e_{kl}(x') = \frac{1}{2}\left(\frac{\partial u_k(x')}{\partial x_l'} + \frac{\partial u_l(x')}{\partial x_k'}\right), \tag{4}$$

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If the Eq. (2) is write in Eq. (1), we obtain

$$\frac{\partial \alpha}{\partial x_k}\sigma_{kl}(x') = -\frac{\partial \alpha}{\partial x'_k}\sigma_{kl}(x')$$

$$= -\frac{\partial}{\partial x'_k}\left[\alpha\sigma_{kl}(x')\right] + \alpha\frac{\partial \sigma_{kl}}{\partial x'_k} - \int_{\partial V}\alpha(|x'-x|)\sigma_{kl}(x')n'_k da(x') + \int_V \alpha(|x'-x|) \times \sigma_{kl,k}dv(x') + \rho(f_1 - \ddot{u}_1) = 0 \tag{5}$$

Here, the first integral through the surface represents the surface stresses. As a result, non-local elasticity theory includes surface physics, an important entity not included in classical theories. Again Eq. (3) and (4) introduce in Eq.(5), ones obtain

$$-\int_{\partial V}\alpha(|x'-x|)\left[\lambda u'_{r,r}\delta_{kl} + \mu(u'_{k,l} + u'k)\right]n'_k da' + \int_V \alpha(|x'-x|) \left[(\lambda+\mu)u'_{k,lk} + \mu u'_{l,kk}\right]dv' + \rho(f_1 - \ddot{u}_1) = 0 \tag{6}$$

In above equations the *(')* means depending on x'. Namely, $u' = u(x')$. If we solve the Eq.(6) under the suitable boundary and initial conditions, $u(x,t)$ displacement vector can be obtained. Initial conditions are depend t_{kl} not σ_{kl}. So we can easily write $\tau_{\mathrm{kl}}n_k = t_{(n)l}$

2. Definition of nonlocal parameter in nonlocal elasticity

It is shown that the unit of nonlocal parameter located in Eq. (2) ($\alpha|x-x'|$) is (length)$^{-3}$. Thus, the non-local parameter will be dependent on a characteristic length ratio (*a/l*) in which an internal characteristic length a, (eg. lattice parameter, granular distance) and an external characteristic length *l* (eg, crack length, wavelength) is present. Thus it defined as

$$\alpha = \left(\alpha|x'-x|, \tau\right),\ \tau = \frac{e_0 a}{l} \tag{7}$$

Here e_0 will be different constant for each material. Some properties of nonlocal parameter are as follows:

- It reach the maximum value at $x' = x$ and decrease the value via $|x'-x|$ calculation.
- When $\tau \to 0$, the statement of α is become Dirac delta function. Hence, the boundary (limit) of the classical elasticity introduce the nearly zero value boundary of the internal length scale (a). Namely:

$$\lim_{\tau\to 0} = \left(\alpha|x'-x|, \tau\right) = \delta\left(\alpha|x'-x|\right) \tag{8}$$

So, it is easily said that “a” is a delta array.

- For small internal length scale value (such as $\tau \to 1$) nonlocal elasticity theory behave as atomic lattice dynamics.

By matching the wave distribution curves with the distribution curves of the atomic lattice dynamics (or experiments), we can determine "a"value for a certain material. Various forms have been obtained as a result of research [2-16]. Some of them are as follows:

One-dimensional parameter

$$\alpha(|x|,\tau)=\frac{1}{l\tau}\left(1-\frac{|x|}{l\tau}\right),|x|\le l\tau$$
$$\alpha(|x|,\tau)=0,\qquad |x|\ge l\tau \tag{9}$$

$$\alpha(|x|,\tau)=\frac{1}{2l\tau}e^{\sim\frac{|x|}{l\tau}} \tag{10}$$

$$\alpha(|x|,\tau)=\frac{1}{l\sqrt{\pi\tau}}e^{\frac{-x^2}{l^2\tau}} \tag{11}$$

Two-dimensional parameter

$$\alpha(|x|,\tau)=\frac{1}{2\pi l^2\tau^2}K_0\left(\frac{\sqrt{x\cdot x}}{l\tau}\right) \tag{12}$$

Here K_0 is a modified Bessel function.

Three-dimensional parameter

$$\alpha(|x|,t)=\frac{1}{8(\pi t)^{3/2}}e^{-\frac{x\cdot x}{4t}},t=\frac{l^2\tau}{4} \tag{13}$$

$$\alpha(|x|,\tau)=\frac{1}{4\pi l^2\tau^2\sqrt{x\cdot x}}e^{-\frac{\sqrt{x\cdot x}}{\pi l}} \tag{14}$$

When the equation 10 is examined, it is seen that the one-dimensional plane waves based on the Born-Kármán model, which is based on the theory of non-local elasticity and atomic lattice dynamics, fits perfectly with the distribution curve. When the two-dimensional parameter is analyzed, it is seen that the maximum error is 1.2% [1]. It is seen that all non-local parameters are normalized when the integrals are taken (over the length, area or volume). In addition, for

$\tau \to 0$ the Dirac delta function is obtained. With this feature, it is seen that when the term Dirac delta function is used in Equation 2, classical theory of elasticity is reverted and Hooke's law becomes valid. This observation was developed by Eringen [2] as follows:

If α is the linear differential operator Green function, we write

$$L\alpha\left(\left|x'-x\right|,\tau\right)=\delta\left(\left|x'-x\right|\right) \tag{15}$$

After used this Equation in Eq.(2), ones obtain

$$Lt_{kl}=\sigma_{kl} \tag{16}$$

Let be consider the L is a differential operator having constant coefficients

$$\left(Lt_{kl}\right)_{,k}=Lt_{kl,k} \tag{17}$$

So, we obtain the below equation

$$\sigma_{kl,k}+L\rho(f_1-\ddot{u}_1)=0 \tag{18}$$

Hence we obtain the differential equation instead of partial integral. For static case $L\rho(f_1-\ddot{u}_1)=0$

Finally, we can write below form

$$\sigma_{kl,k}=0 \tag{19}$$

If we sued the Eq. (19) in Eq.(3) we obtain the well-known Navier equation. So, differential operator as define via Eq.(3)

$$L=1-\tau^2 l^2\nabla^2 \tag{20}$$

After using this equation in Eq. (17) we obtained the following form

$$\left(1-\tau^2 l^2\nabla^2\right)t=\sigma \tag{21}$$

The accuracy of this result can be demonstrated by the atomic distribution relationship. For this purpose, the frequency expression obtained from the Born-Kármán model must be equal to the expression of non-local elasticity for plane waves.

3. Modeling by lattice dynamics

Lattice dynamics is known as harmonic approach provided that the displacements are small. In the chain, atoms can be connected with elastic springs (Figure 1).

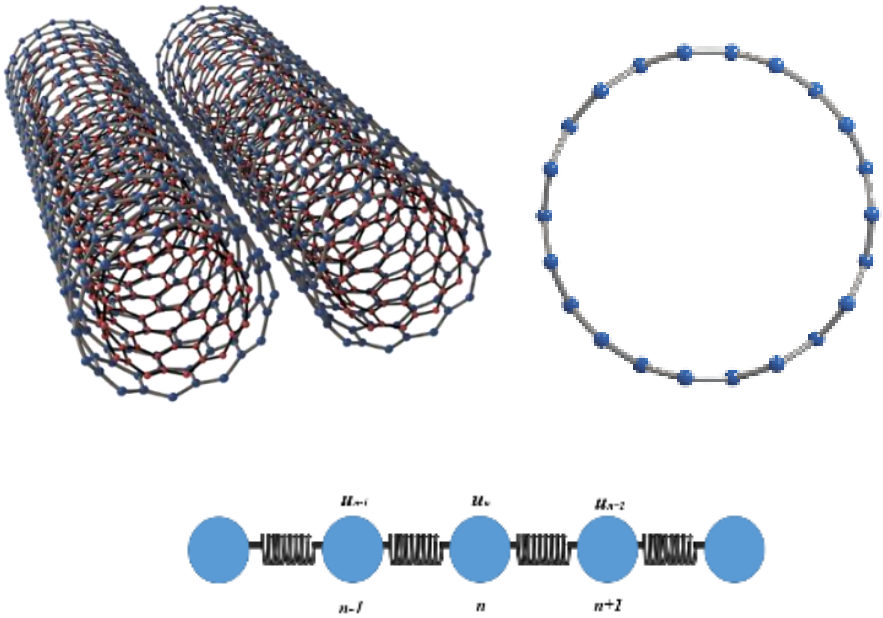

Fig. 1. One-dimensional lattice model

Therefore, force applied to the n^{th} atom can be written as

$$F_n = K(u_{n+1} - u_n) + K(u_{n-1} - u_n) \tag{22}$$

Here K is the inter-atomic force (elastic) constant. Newton's second law applied to the n^{th} atom

$$M\frac{d^2u_n}{dt^2} = F_n = K(u_{n+1} - u_n) + K(u_{n-1} - u_n) = -K(2u_n - u_{n+1} - u_{n-1}) \tag{23}$$

In the above expression M denotes the mass of the atom. Similarly, the equation for each atom in the cage should be written. N; In order to express the total number of atoms, the result is the N equation which must be solved simultaneously. In addition, the boundary conditions applied to the end of the cage must also be taken into account. The following conversion will be used for the solution

$$u_n = Ae^{i(kx_n - \omega t)} \tag{24}$$

Here, x_n n. refers to the position of the atom and $x_n = na$. This equation represents a moving wave with q wavelength where all atoms oscillate at the same frequency (q) with the same A amplitude. Equation (24) is written in Equation (23)

$$M(-\omega^2)e^{ikna} = -C(2e^{ikna} - e^{ik(n+1)a} - e^{ik(n-1)a}) \tag{25}$$

After some manipulation

$$M\omega^2 = C(2 - e^{ika} - e^{-ika}) = 2C(1 - \cos ka) \tag{26}$$

$$\frac{M}{C}\omega^2 = 2(1 - \cos ka) \tag{27}$$

Finally

$$\frac{M}{C}\omega^2 = 2(1 - \cos ka) = 4\sin^2\frac{ka}{2} \tag{28}$$

So, frequency and distribution relation is

$$\sqrt{\omega_j{}^2} = \sqrt{\frac{M}{C}\omega^2} = 2\left|\sin\frac{ka}{2}\right| \tag{29}$$

We expect the results obtained with the cage dynamics between the two atoms to be the same as the non-local elasticity results. In this case, the ratio of cage dynamics frequency distribution relation to bar frequency distribution relation

$$\alpha(k) = \frac{\omega_j{}^2(k)}{\omega_{oj}{}^2} = \left(\frac{2\kappa}{\pi k}\right)^2 \sin^2\left(\frac{\pi k}{2\kappa}\right) \tag{30}$$

Here angular frequency can be write as

$$\omega_{oj}{}^2 = c_0{}^2 k^2 \tag{31}$$

Also $c_0 = \sqrt{E/\rho}$. κ is the Brillouin region is the value of the upper limit of k. This value is for one dimensional mesh dynamics. In the light of this information Equation (31) is reorganized

$$\alpha(k) = \frac{\omega_j{}^2(k)}{\omega_{oj}{}^2} = \left(\frac{2}{ka}\right)^2 \sin^2\left(\frac{ka}{2}\right) \tag{32}$$

If we write above equation as terms of cosine

$$\alpha(k) = \frac{\omega_j{}^2(k)}{\omega_{oj}{}^2} = \frac{2}{k^2a^2}(1 - \cos ka) \tag{33}$$

If the equation opens into the Maclaurin series

$$\cos ka = 1 - \frac{(ka)^2}{2!} + \frac{(ka)^4}{4!} - \frac{(ka)^6}{6!} + ... \tag{34}$$

$$1 - \cos ka = \frac{(ka)^2}{2!} - \frac{(ka)^4}{4!} + \frac{(ka)^6}{6!} - .... \tag{35}$$

After using these equations in Eq.(32), we obtain

$$\alpha(k) = \frac{2}{(ka)^2}(1 - \cos ka) = 1 - \frac{(ka)^2}{12} + \frac{(ka)^4}{360} - .... \tag{36}$$

Using the first-order approach and by using the first two-term of the Eq.36

$$\frac{1}{\alpha} = 1 + (e_0 a)^2 k^2 \tag{37}$$

After some arrangement

$$\alpha = \left(1 + e_0^2 a^2 k^2\right)^{-1},\ \left(1 + e_0^2 a^2 k^2\right)\bar{t}_{kl} = \bar{\sigma}_{kl},\ \left(1 - e_0^2 a^2 \nabla^2 - ...\right)t_{kl} = \sigma_{kl} \tag{38}$$

Hence, non-dimensional frequency via nonlocal elasticity is

$$\frac{\omega a}{c_0} = ka\left(1 + e_0^2 a^2 k^2\right)^{-1/2} \tag{39}$$

The relation of frequency distribution via lattice dynamics is as follows with the help of equation (29)

$$\frac{\omega a}{c_0} = 2\sin(ka/2) \tag{40}$$

If the Eq. (39) and Eq. (40) are equalized for $ka = \pi$

$$ka\left(1 + e_0^2 a^2 k^2\right)^{-1/2} = 2\sin(ka/2)$$

$$\pi\left(1 + e_0^2 \pi^2\right)^{-1/2} = 2\sin(\pi/2) \tag{41}$$

$$e_0 = 0.39$$

If second-order approach Eq. (36) extract the polynomial form in three-terms as

$$\frac{1}{\alpha} = 1 + (e_0 a)^2 k^2 + (\gamma_0 a)^4 k^4,\ \alpha = \left(1 + e_0^2 a^2 k^2 + {\gamma_0}^4 a^4 k^4\right)^{-1} \tag{42}$$

And nonlocal stress equation

$$\left(1 + e_0^2 a^2 k^2 + {\gamma_0}^4 a^4 k^4\right)\bar{t}_{kl} = \bar{\sigma}_{kl} \tag{43}$$

In this last equation $\bar{t}_{kl}$ and $\bar{\sigma}_{kl}$, means the Fourier transforms of t_{kl} and σ_{kl}', respectively. If we take the inverse Fourier transform of Eq. (43)

$$\left(1 - e_0^2 a^2 \nabla^2 + {\gamma_0}^4 a^4 \nabla^4 - ...\right)t_{kl} = \sigma_{kl} \tag{44}$$

Also, the governing equation for stress in nonlocal case is

$$\sigma_{kl,k} + \left(1 - e_0^2 a^2 \nabla^2 + {\gamma_0}^4 a^4 \nabla^4\right)\left(\rho f_l - \rho \ddot{u}_l\right) = 0 \tag{45}$$

These equations replaced Navier's classical elasticity equations. New dimensionless frequency according to non-local elasticity theory is

$$\frac{\omega a}{c_0} = ka\left(1 + e_0^2 a^2 k^2 + {\gamma_0}^4 a^4 k^4\right)^{-1/2} \tag{46}$$

Lazar et al. [16]. Stated that $\varepsilon^4 = 4\gamma^4$. Frequency distribution relationship via lattice dynamics is as follows

$$\frac{\omega a}{c_0} = 2\sin(ka/2) \tag{47}$$

Eq. (46) and Eq. (47) are equalized for $ka = \pi$

$$\begin{gathered} ka\left(1 + e_0^2 a^2 k^2 + {\gamma_0}^4 a^4 k^4\right)^{-1/2} = 2\sin(ka/2) \\ \pi\left(1 + 2\gamma_0^2 \pi^2 + {\gamma_0}^4 \pi^4\right)^{-1/2} = 2\sin(\pi/2) \\ \gamma_0 = 0.24,\ \varepsilon_0 = 0.339 \end{gathered} \tag{48}$$

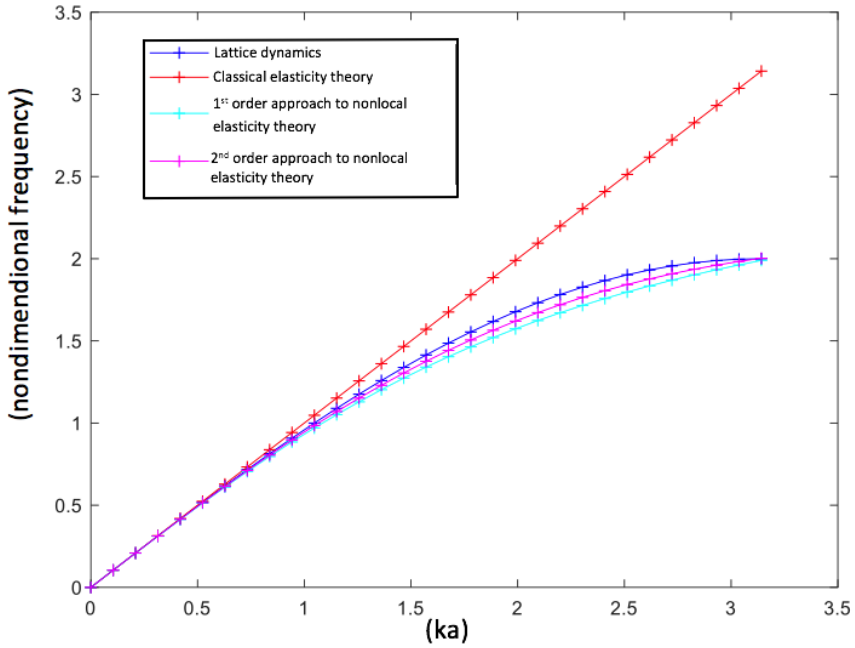

Fig. 2. Convergence of frequency value

The first and secon-order approaches of the theory of non-local elasticity, classical elasticity theory, and the frequency comparison of the lattice dynamics are presented in Figure 2. As can be seen from the figure, the frequency results for the second order approximation of non-local elasticity theory are closer to the lattice dynamic than the first-order approach. In more general case (under the axial foundation effect and thermal effect) free vibration form of axial vibration of elastic nanorod have been given as two different forms:

$$EA\frac{\partial^2 u}{\partial x^2} = \left(EA\alpha\Delta T\right)' - f(x) + k_w u(x) + \rho A\frac{\partial^2 u}{\partial t^2} + (e_0 a)^2 \frac{\partial^2}{\partial x^2}\left(f - k - \rho A\frac{\partial^2 u}{\partial t^2}\right) \tag{49}$$

$$\int_0^L \left[EA\frac{\partial u}{\partial x}\frac{\partial w}{\partial x} + EA\alpha_T\Delta_T\frac{\partial w}{\partial x} + wf(x) - wku(x) - \rho Aw\frac{\partial^2 u}{\partial t^2} - \mu f(x)\frac{\partial^2 w}{\partial x^2} - \mu k\frac{\partial w}{\partial x}\frac{\partial u}{\partial x} - \mu\rho A\frac{\partial^3 u}{\partial x\partial t^2}\frac{\partial w}{\partial x}\right]dx \tag{50}$$

Eq. (50) is the weak form for FEM approach of axial vibration. Some applications have also been listed in references related to macro, micro and nanomechanics [17-39].

4.Conclusion

Some comparison has been made for axial vibration. First and second order approach for nonlocal elasticity and lattice dynamics results have also been compared. Finally, weak form is given for axial vibration problem of nanorods.

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