International Journal of Engineering & Applied Sciences (IJEAS)
Vol.9, Issue 3 (2017) 36-50
http://dx.doi.org/10.24107/ijeas.329433
Int J Eng Appl Sci 9(3) (2017) 36-50

Effects of Inhomogeneity and Thickness Parameters on the Elastic Response of a Pressurized Hyperbolic Annulus/Disc Made of Functionally Graded Material

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Received date: July 2017
Accepted date: September 2017

Abstract

A broad parametric study is carried out to investigate the effects of both the inhomogeneity parameter, and a profile index of Stodola's hyperbolic function on the static response of such structures subjected to both the inner and outer pressures. The investigation is based on the analytical formulas lately published by the author. The effects of those parameters on the variation of the radial displacement, the radial and hoop stresses are all graphically illustrated for an annulus pressurized at its both surfaces. It is observed that, especially, the variation of the hoop stress in radial coordinate is closely sensible to variation of those parameters. For the chosen problems it was observed that one of two materials whose Young's modulus is higher than the other is better to locate at the inner surface of the disc having divergent profile to get reasonable maximum hoop stresses. However much smaller radial displacements may be obtained by using positive inhomogeneity indexes for all discs whose surfaces host a material whose Young's modulus is smaller than the other. To reach a final decision, analytical formulas such as those used in the present study together with a failure criteria such as Von Mises and Tresca become indispensable means in a design process.

Keywords: Pressurized disc, hyperbolic annulus, functionally graded, variable thickness, exact solution, elasticity solution, inhomogeneity parameter, thickness parameter.

1. Introduction

A pressurized annulus or disc or collar or ring is mainly used as a pipe flange to be attached to a pipe. It fulfills some functions such as providing increased support for strength, blocking off a pipeline or implementing the attachment of more items. There are gradually increasing number of analytical and numerical studies on beams, plates, shells and annular structures made of isotropic or anisotropic functionally graded materials in the available literature since 1990s [1-30]. The reason is that the structures like discs manufactured with functionally graded materials can have much more favorable thermal and mechanical properties along the desired directions. To do this, at least two materials are combined in the way that the overall material properties along the chosen direction must obey a certain material grading rule. As one of the pioneer researches of the subject, Horgan and Chan [1, 2] showed that the stress response of an inhomogeneous cylinder (or disk) is significantly different from that of a homogeneous body. For example, the maximum hoop stress does not, in general, occur on the inner surface in contrast with the situation for the homogeneous material.

In analytical studies the well-known simple power rule was frequently used by researches like Horgan and Chan [1-2], Bayat et al. [3], Yıldırım [4] and Çallıoğlu et al. [5] since this material grading rule makes the variable coefficients of the governing differential equation to be solved turned into constant coefficients. Nejad et al. [6-7] gave a closed-form analytical solution in terms of hyper-geometric functions to elastic analysis of exponentially functionally graded stationary discs subjected to internal and external pressures. Based on the hypergeometric functions, You et al. [8] developed an analytical elastic solution for circular disks made of functionally linearly graded materials subjected to internal and/or external pressure. For an exponentially grading rule [6-7, 9-14], Whittaker / Kumer functions or Frobenius series are also involved in the solution. Saidi et al. [15] used Green function in their analytical work.

For arbitrary material grading rules, some numerical solution techniques are employed such as Fredholm integrals [16,17], the modified Runge-Kutta algorithm [12], the finite element method [18,19], the finite difference method [20, 21], finite volume method [22], complementary functions method [23,24,25] and whatnot numerical techniques.

Uniform discs [1, 2, 5-7, 9, 16-17] were studied relatively very large in proportion to the discs having different profiles such as parabolic and hyperbolic types. Eraslan and Akış [13] used two variants of a parabolic function for disks made of functionally graded materials. Bayat et al. [3], based on the power-law distribution, studied both analytically and semi-analytically the elastic response of rotating hollow discs having parabolic and hyperbolic thickness profiles. However, they consider a variable thickness disk as a combination of sub-uniform discs with different thicknesses. Ghorbani [26] also divided a variable thickness disk into virtual sub-uniform disks. Tütüncü and Temel [23], Zheng et al. [21], Yıldırım [4], Boğa and Yıldırım [24], and Yıldırım and Kacar [25] considered continuously varying discs in their studies.

In the present study, for a simple power-law graded annulus having a continuously varying hyperbolic profile, an investigation of the variation of the elastic field with some material and geometrical parameters is fastened on. To do this Yıldırım's [4] closed-form solutions are exploited.

2. Mathematical Background

Yıldırım [4] studied analytically exact elastic response of a convergent/divergent hyperbolic rotating disc made of a power-law graded material under four different boundary conditions such as a stationary disc subjected to internal/external pressures, a rotating disc whose surfaces may expand freely, a rotating disc mounted a rigid shaft with/without a rigid casing located at the outer surface.

In this study the following differential equation which governs the axisymmetric static behavior of a hyperbolic annulus subjected to both the inner and outer pressures and made of a linear elastic power-law graded material (Fig. 1) is used [4].

$$\frac{(-1+m\nu+\beta\nu)u_r}{r^2}+\frac{(1+m+\beta)u_r{}'}{r}+u_r''=0 \tag{1}$$

Where r is the radial coordinate, u_r is the radial displacement (Fig. 1), β is the inhomogeneity constant of a power-graded material with the following Young's modulus

$$\mathrm{E(r)}=E_a\left(\frac{r}{a}\right)^{\beta} \tag{2}$$

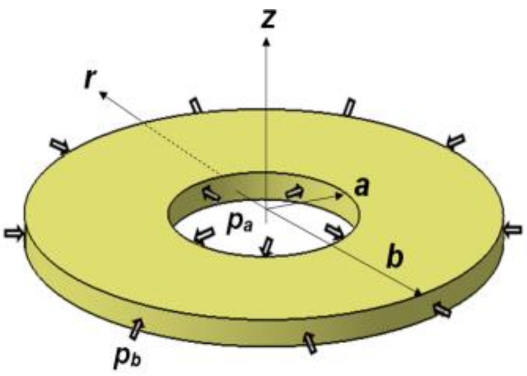
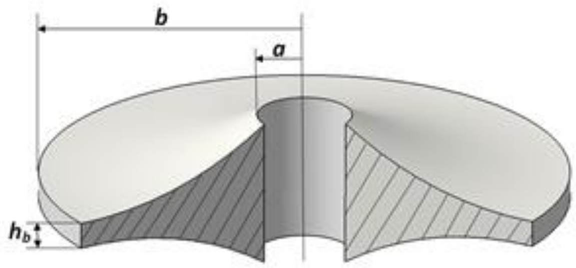


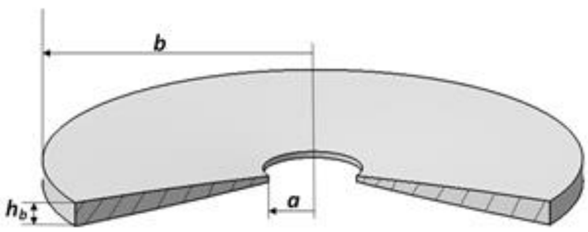
Fig. 1. Loading and geometry of a pressurized uniform annulus

and, m is the profile/thickness parameter of a disc with hyperbolically varying thickness (Fig. 2).

$$h(r) = h_a \left(\frac{r}{a}\right)^m \tag{3}$$



Convergent ($m<0$)



Divergent ($m>0$)

Fig. 2. A hyperbolic annulus

Resulting disc profiles with the changing profile thickness parameters, m, are illustrated in Fig. 3 for both convergent and divergent hyperbolic discs ($a=0.02m$, $b=0.1m$, $h_a = a$).

Solution of Eq. (1), which is derived from stress-strain relations, strain-displacement relations, and the equilibrium equation in the radial direction under axisymmetric assumption, is given by Yıldırım [4] as

$$u_r(\mathrm{r}) = r^{\frac{1}{2}(-m-\beta-\xi)}(C_1 + C_2 r^{\xi}) \tag{4}$$

Where

$$\xi = \sqrt{\left(4 + (m+\beta)(m+\beta-4\nu)\right)} \tag{5}$$

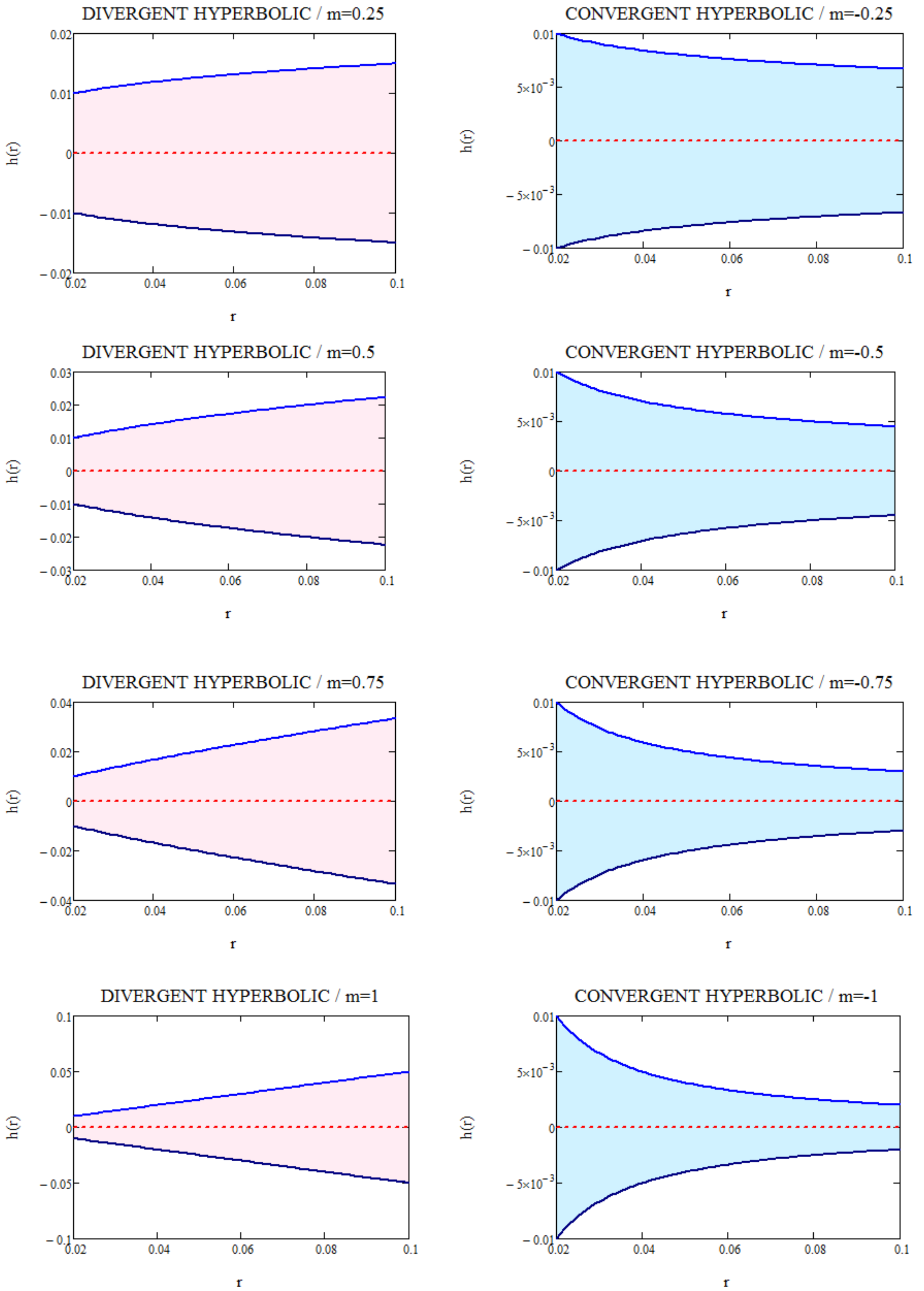


Fig. 3. The chosen thickness parameters and corresponding profiles considered in the present study

Poisson's ratio, ν, is assumed to be constant along the radial direction. With the help of the Hooke's law, the radial stress, σ_r, and hoop stress, σ_θ , are found as

$$\sigma_r = -\frac{1}{2}C_{11}r^{\frac{1}{2}(-2-m-\beta-\xi)}\left(C_2r^{\xi}(m+\beta-2\nu-\xi)+C_1(m+\beta-2\nu+\xi)\right)$$

$$\sigma_\theta = \frac{1}{2}r^{\frac{1}{2}(-2-m-\beta-\xi)}\left(C_2C_{11}r^{\xi}\left(2-\nu(m+\beta-\xi)\right)-C_1C_{11}\left(-2+\nu(m+\beta+\xi)\right)\right) \tag{6}$$

Where

$$C_{11} = \frac{E_a\left(\frac{r}{a}\right)^{\beta}}{1-\nu^2} \tag{7}$$

In Eq. (7) E_a stands for Young's modulus of the material located at the inner surface. The boundary conditions at both surfaces are defined by $\sigma_r(a) = -p_a$, $\sigma_r(b) = -p_b$ (Fig. 1). The integration constants in Eqs. (4) and (6) are expressed by

$$C_1 = \frac{2(\nu^2-1)a^{\frac{\xi-\beta}{2}}b^{\frac{\xi-\beta}{2}}}{\frac{E_a}{a^{\beta}}(a^{\xi}-b^{\xi})(\beta+m-2\nu+\xi)}\left(p_a a^{\frac{m}{2}+1}b^{\frac{\beta+\xi}{2}}-p_b b^{\frac{m}{2}+1}a^{\frac{\beta+\xi}{2}}\right) \tag{8a}$$

$$C_2 = a^{-\beta/2}b^{-\beta/2}\left(p_b a^{\beta/2}b^{\frac{1}{2}(m+\xi+2)}-p_a b^{\beta/2}a^{\frac{1}{2}(m+\xi+2)}\right)\frac{2(\nu^2-1)}{\frac{E_a}{a^{\beta}}(a^{\xi}-b^{\xi})(\beta+m-2\nu-\xi)} \tag{8b}$$

In the above, p_a and p_b denote the inner and outer pressures, respectively (Fig. 1). Yıldırım's [4] study comprises explicit form of elastic responses in terms of integration constants as seen in Eqs. (4), (7) and (8). Now, it is time to expand those formulas [4] for the present study. Substitution of integration constants C_1 and C_2 into Eqs. (4) and (6) gives the following closed-form formulas

$$\begin{aligned} u_r = {} & \frac{1}{\frac{\mathrm{E}_a}{a^{\beta}}(a^{\xi}-b^{\xi})(\beta+m-2\nu-\xi)(\beta+m-2\nu+\xi)}\Bigg(2(\nu^2 \\ & -1)p_a a^{\frac{1}{2}(-\beta+m+\xi+2)}r^{\frac{1}{2}(-\beta-m-\xi)}\left(b^{\xi}(\beta+m-2\nu-\xi)-r^{\xi}(\beta+m-2\nu+\xi)\right)\Bigg) \\ & + \frac{1}{\frac{\mathrm{E}_a}{a^{\beta}}(b^{\xi}-a^{\xi})(\beta+m-2\nu-\xi)(\beta+m-2\nu+\xi)}\Bigg(2(\nu^2 \\ & -1)p_b b^{\frac{1}{2}(-\beta+m+\xi+2)}r^{\frac{1}{2}(-\beta-m-\xi)}\left(a^{\xi}(\beta+m-2\nu-\xi)-r^{\xi}(\beta+m-2\nu+\xi)\right)\Bigg) \end{aligned} \tag{9a}$$

$$\sigma_r=\left\{\frac{p_a\left(b^{\xi}-r^{\xi}\right)a^{\frac{1}{2}(-\beta+m+\xi+2)}r^{\frac{1}{2}(\beta-m-\xi-2)}}{a^{\xi}-b^{\xi}}\right\}+\left\{\frac{p_b\left(a^{\xi}-r^{\xi}\right)b^{\frac{1}{2}(-\beta+m+\xi+2)}r^{\frac{1}{2}(\beta-m-\xi-2)}}{b^{\xi}-a^{\xi}}\right\}$$

(9b)

$$\sigma_\theta = \left\{\frac{1}{(a^{\xi}-b^{\xi})(\beta+m-2\nu-\xi)(\beta+m-2\nu+\xi)}\left(p_a a^{\frac{1}{2}(-\beta+m+\xi+2)}r^{\frac{1}{2}(\beta-m-\xi-2)}\left(b^{\xi}(\beta+m-2\nu-\xi)(\nu(\beta+m+\xi)-2)-r^{\xi}(\nu(\beta+m-\xi)-2)(\beta+m-2\nu+\xi)\right)\right)\right\}$$
$$+\left\{\frac{1}{(b^{\xi}-a^{\xi})(\beta+m-2\nu-\xi)(\beta+m-2\nu+\xi)}\left(p_b b^{\frac{1}{2}(-\beta+m+\xi+2)}r^{\frac{1}{2}(\beta-m-\xi-2)}\left(a^{\xi}(\beta+m-2\nu-\xi)(\nu(\beta+m+\xi)-2)-r^{\xi}(\nu(\beta+m-\xi)-2)(\beta+m-2\nu+\xi)\right)\right)\right\}$$

(9c)

In the above, $m=0$ gives uniform disk profiles that is unchanging thickness along the radial coordinate. For uniform discs made of such kind of materials, one may easily derive the following radial stress from Eq. (9b) by eliminating the thickness profile

$$\sigma_{r(m=0)}=\frac{a^{\frac{1}{2}(-\beta+\xi+2)}p_a r^{\frac{1}{2}(\beta-\xi-2)}\left(b^{\xi}-r^{\xi}\right)}{a^{\xi}-b^{\xi}}+\frac{b^{\frac{1}{2}(-\beta+\xi+2)}p_b r^{\frac{1}{2}(\beta-\xi-2)}\left(a^{\xi}-r^{\xi}\right)}{b^{\xi}-a^{\xi}} \tag{10}$$

Where (see Eq. (5))

$$\xi=\sqrt{4+\beta^2-4\beta\nu} \tag{11}$$

Horgan and Chan [1] proposed formulas for linear elastic response of uniform cylinders or stress-free discs made of a power-graded material. Horgan and Chan's [1] equation for radial stress is rewritten here by using the present notation

$$\sigma_{r-HORGAN}=-\frac{a^{\frac{-\beta}{2}}b^{\frac{-\beta}{2}}r^{\frac{1}{2}(-2-\xi+\beta)}}{b^{\xi}-a^{\xi}}\left(-a^{\xi+\frac{\beta}{2}}b^{1+\frac{\xi}{2}}p_b+a^{\frac{\beta}{2}}b^{1+\frac{\xi}{2}}p_b r^{\xi}+b^{\frac{\beta}{2}}a^{1+\frac{\xi}{2}}p_a\left(b^{\xi}-r^{\xi}\right)\right) \tag{12}$$

or in the form of

$$\sigma_{r-HORGAN}=\frac{a^{1+\frac{\xi}{2}-\frac{\beta}{2}}r^{\frac{1}{2}(-2-\xi+\beta)}}{a^{\xi}-b^{\xi}}p_a\left(b^{\xi}-r^{\xi}\right)+\frac{p_b b^{1+\frac{\xi}{2}-\frac{\beta}{2}}r^{\frac{1}{2}(-2-\xi+\beta)}}{b^{\xi}-a^{\xi}}\left(-r^{\xi}+a^{\xi}\right) \tag{13}$$

Yıldırım [4] validated that Eqs. (10), (12) and (13) are identical in his study. Moreover he also verified that Roark's formulas [31] for uniform discs made of an isotropic and homogeneous material may be obtained from Eq. (9) by using $C_{11} = \frac{\mathrm{E}}{1-\nu^2}$, $\beta = m = 0$, and $\xi = 2$ as follows

$$
\begin{aligned}
u_r &= -\frac{a^2 p_a (b^2(\nu+1) - (\nu-1)r^2)}{Er(a^2-b^2)} + \frac{b^2 p_b (a^2(\nu+1) - (\nu-1)r^2)}{Er(a^2-b^2)} \\
\sigma_r &= \frac{a^2 p_a (b^2 - r^2)}{r^2(a^2-b^2)} + \frac{b^2 p_b (a-r)(a+r)}{r^2(b^2-a^2)} \\
\sigma_\theta &= -\frac{a^2 p_a (b^2 + r^2)}{r^2(a^2-b^2)} + \frac{b^2 p_b (a^2 + r^2)}{r^2(a^2-b^2)}
\end{aligned}
\tag{14}
$$

3. Numerical Examples

The geometrical and material features are chosen as: $a = 0.02\,m; b = 0.1\,m;\ \nu = 0.3; E_a = 200\ \mathrm{G}Pa; p_a = 1\ \mathrm{G}Pa; p_b = 0.1\ \mathrm{G}Pa.$

From the definition of β in Eq. (2), a positive inhomogeneity index, $\beta > 0$, means that $E_a < E_b$ that is one of the materials having smaller Young's modulus is located at the inner surface and Young's modulus of the mixture material continuously increase towards the outer surface. If the inhomogeneity index is negative, $\beta < 0$, a material whose Young's modulus is higher than the other is placed at the inner surface.

3.1. Effect of the thickness parameter

Variation of the elastic quantities with the profile parameter (see Fig. 3) is shown in Figs. 4-6 for the particular values of inhomogeneity parameters $\beta = -5$, $\beta = 0$, and $\beta = 5$.

- $\beta = 5$ gives positive radial displacements for all profile indexes, converse is true for $\beta = -5$. For $\beta = 0$, that is for isotropic and homogeneous discs, divergent profiles give negative radial displacements towards the outer surface.
- Positive inhomogeneity indexes with divergent profiles give the smallest radial displacements.
- The radial stress is in compression for all profiles and inhomogeneity indexes. For both divergent, convergent and uniform disc profiles, on the other hand, the maximum radial stress is at the inner surface for all inhomogeneity indexes due to the boundary conditions.
- The radial stress with $m \geq 1$ seems to offer higher stresses than the initial stress at the vicinity of the inner surface.
- The hoop stress may be either in tension or in compression for all profiles and inhomogeneity indexes. For an isotropic and homogeneous disc, in general the maximum hoop stress is located at the inner surface as Horgan and Chan [1] stated.
- The divergent profiles offer smaller hoop stresses for all profiles.
- $\beta = -5$ presents the smallest hoop stress in magnitude for both divergent, convergent and discs profiles including uniform ones. On the other hand, the absolute maximum hoop stress is at the outer surface for $\beta = 5$, while it is at the inner surface for $\beta = -5$.
- For a negative inhomogeneity index and a convergent disc, maximum hoop stress is located at the inner surface and it is tension in character. However for a negative inhomogeneity index

and a divergent disc with $m \geq 1$, maximum hoop stress is set at the inner surface as in compression.

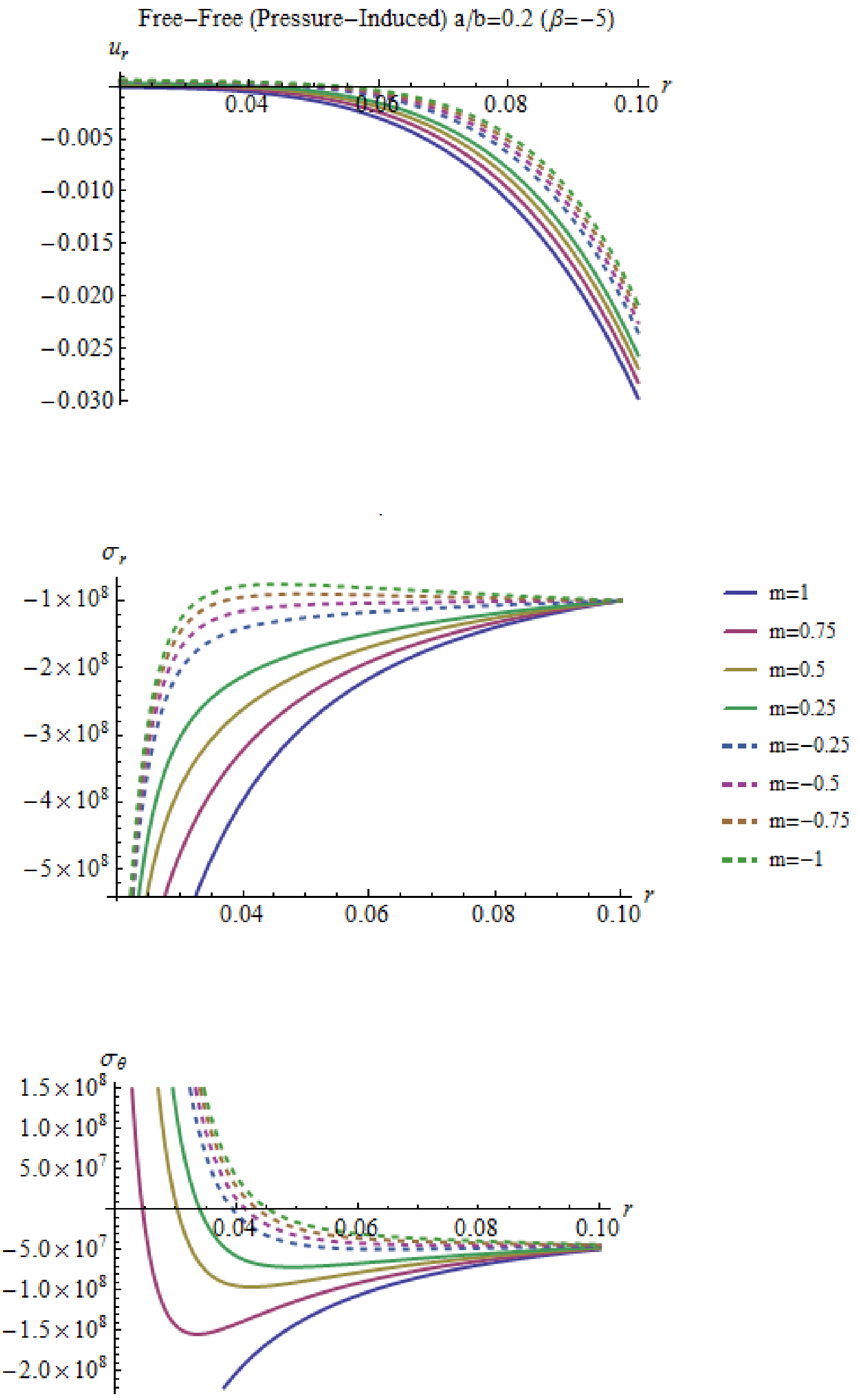


Fig. 4. Variation of pressure-induced elastic responses with profile parameters for free-free boundary condition and $\beta = -5$

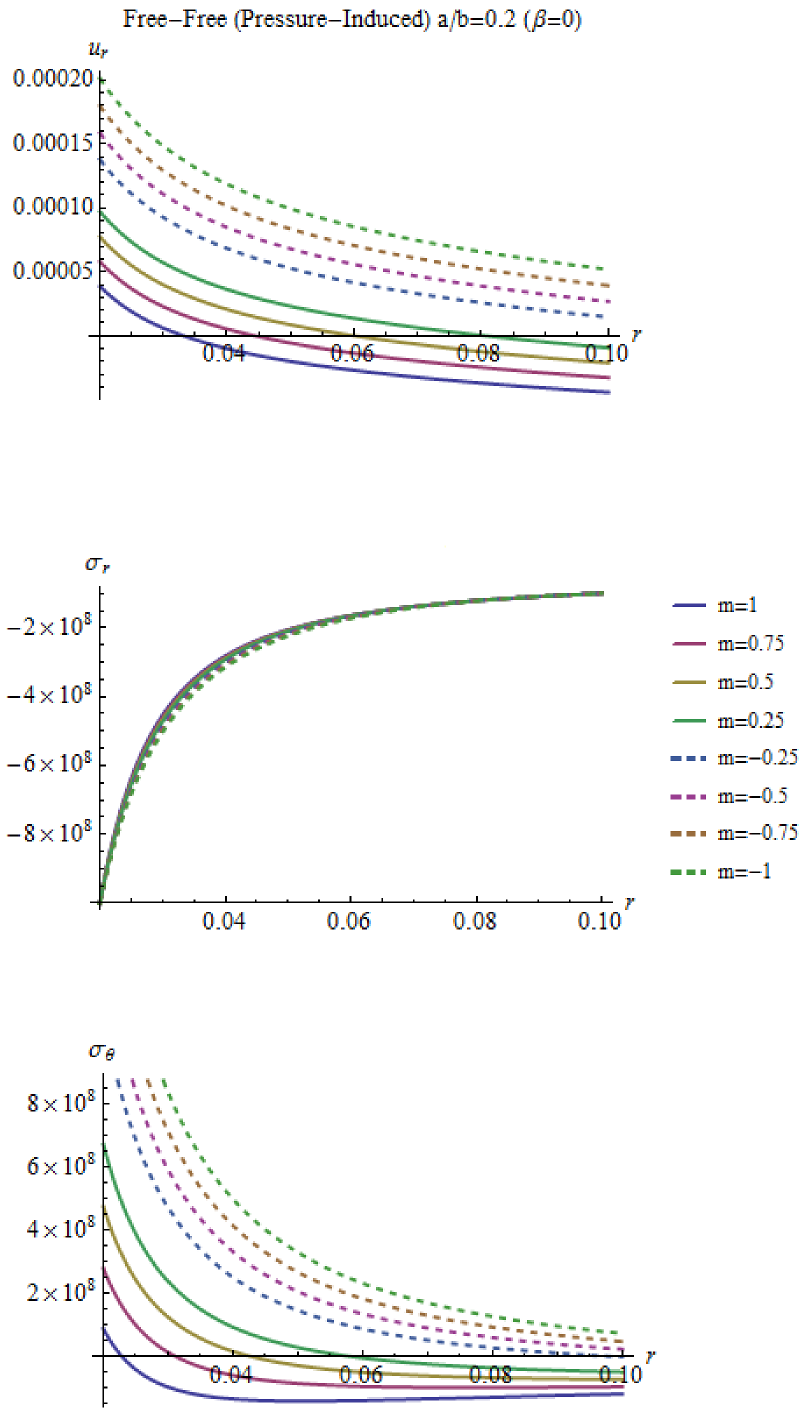


Fig. 5. Variation of pressure-induced elastic responses with profile parameters for free-free boundary condition and $\beta = 0$

3.2. Effect of the inhomogeneity parameter

Variation of the elastic quantities with the inhomogeneity parameter is shown in Figs. 7-9 for the values of $m = -1$, $m = 0$, and $m = 1$. From Figs. 7-9 the followings may be concluded

- For convergent, divergent and uniform disc profiles the plausible radial displacements are obtained with positive inhomogeneity indexes.
- For all inhomogeneity indexes, a divergent profile offers a close radial stress variation along the radial coordinate.
- Convergent disc profiles give radial stresses in compression for all inhomogeneity indexes.

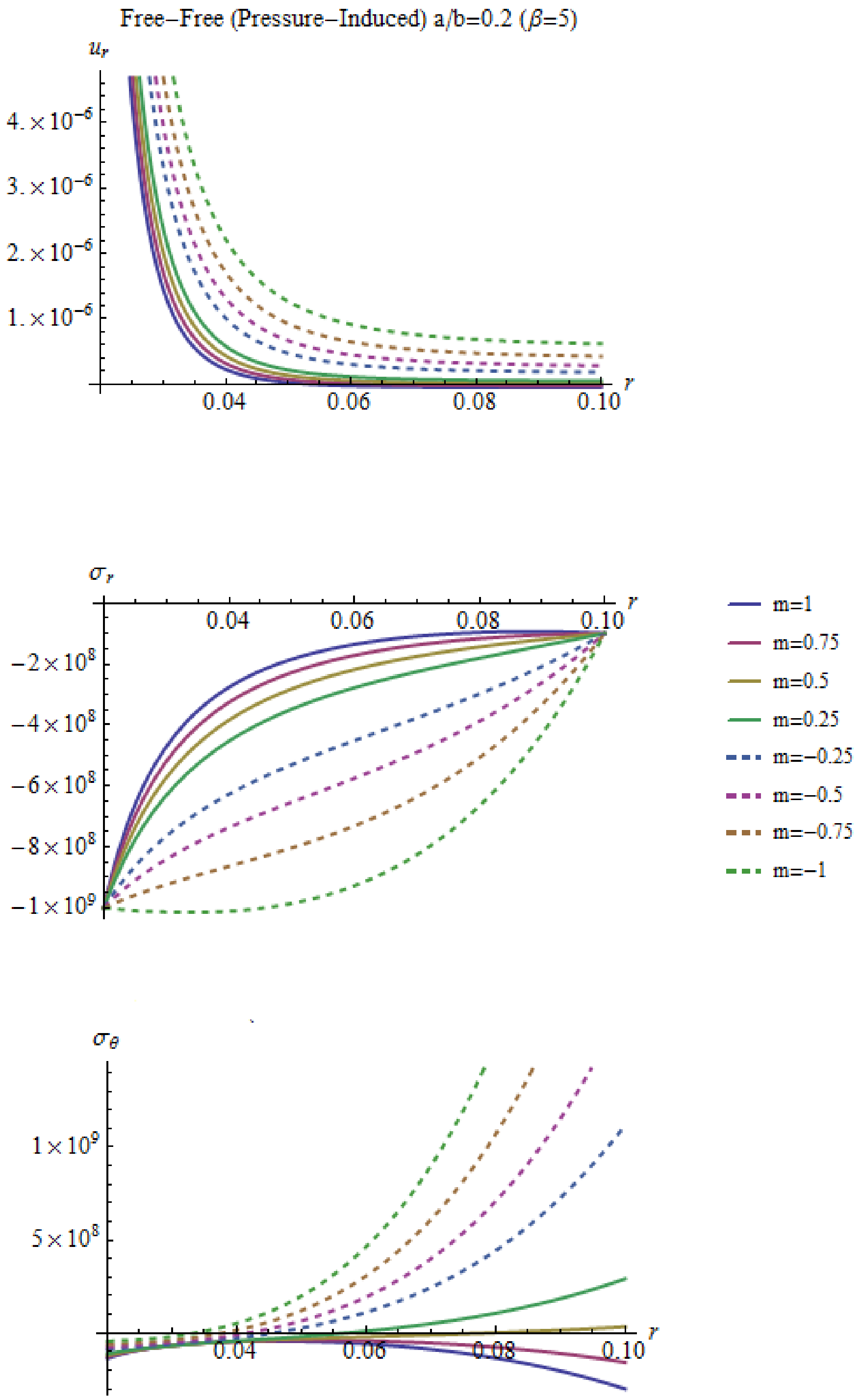


Fig. 6. Variation of pressure-induced elastic responses with profile parameters for free-free boundary condition and $\beta = 5$

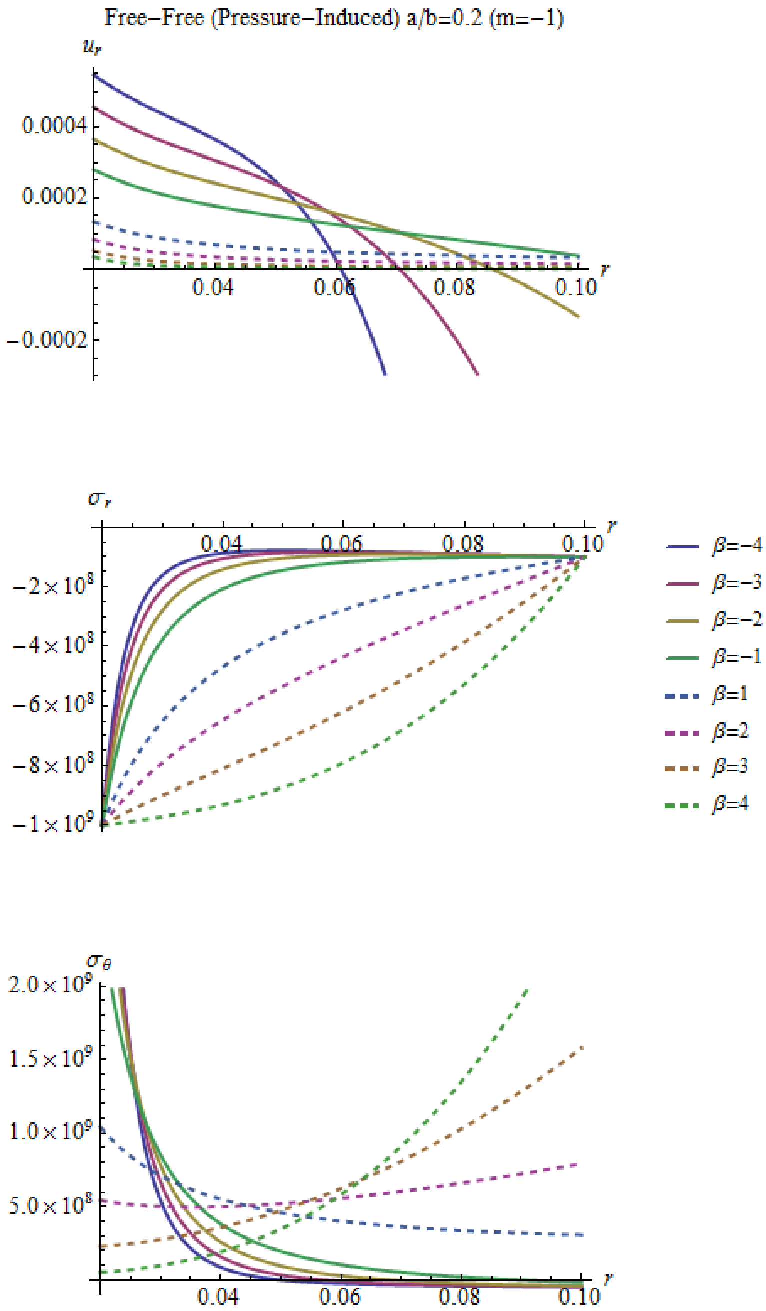


Fig. 7. Variation of pressure-induced elastic responses with inhomogeneity parameters for free-free boundary condition and $m = -1$

- Divergent profiles offer much smaller hoop stresses for all inhomogeneity indexes.
- Maximum absolute hoop stress is located at the inner surface for negative inhomogeneity indexes and convergent profiles. For convergent discs and positive inhomogeneity indexes, maximum hoop stress is located either at the inner or the outer surface.
- Maximum absolute hoop stress is set at the outer surface for positive inhomogeneity indexes and divergent profiles.
- For divergent profiles and negative inhomogeneity indexes, maximum hoop stress is build up either at the inner or vicinity of the inner surface.

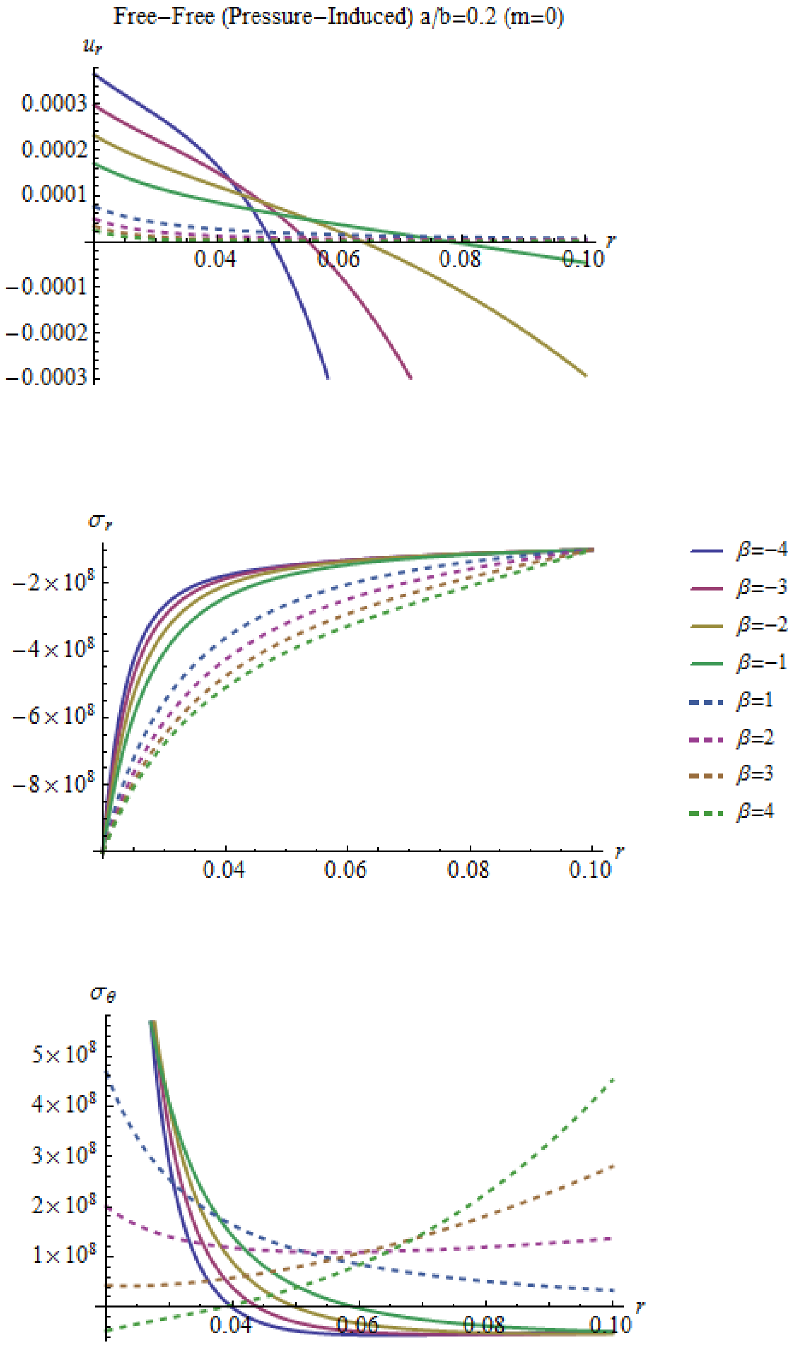


Fig. 8. Variation of pressure-induced elastic responses with inhomogeneity parameters for free-free boundary condition and $m = 0$

4. Conclusions

For an annulus having continuously varying thickness and pressurized at both surfaces, a parametric study is performed to observe the effects of both the inhomogeneity index of a simple power law material grading, and the thickness parameter. Analytical formulas of late published by the author are tailored for the present parametric study. The thickness parameters presenting either convergent or divergent disc profiles are assumed to be in the range of $-1 \leq m \leq 1$. Inhomogeneity indexes for the material grading rule are also chosen in a wide range as $-4 \leq \beta \leq 4$.

The effects of those parameters on the variation of the radial displacement, the radial and hoop stresses are all graphically illustrated and discussed. As expected, it is observed that those parameters have considerable influence on the static response of such an annulus. Those variations are obviously observed for the hoop stresses. In other words, the variation of the hoop stress in radial coordinate is closely sensible to variation of those parameters. That is its amplitude and sign may be drastically changed with those parameters.

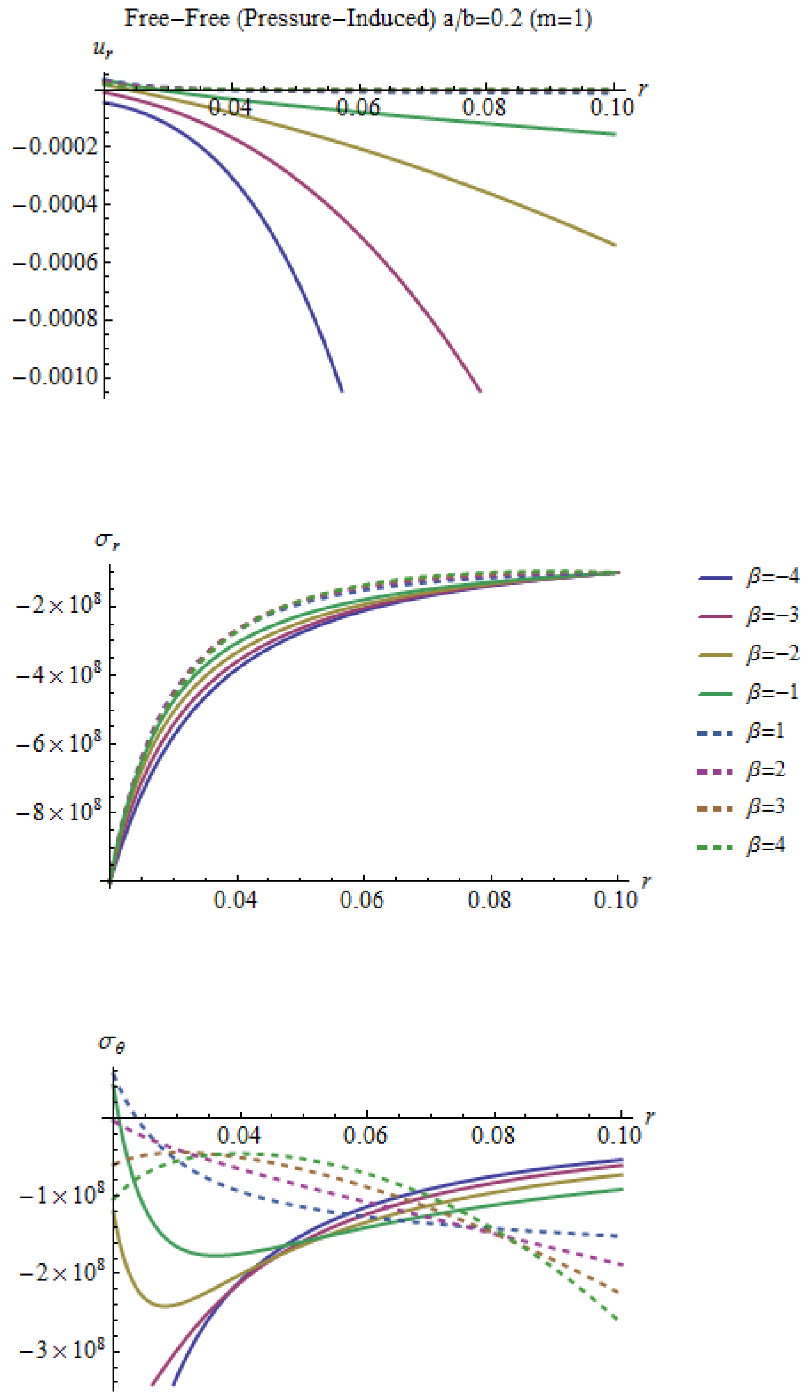


Fig. 9. Variation of pressure-induced elastic responses with inhomogeneity parameters for free-free boundary condition and *m=1*

For the chosen problems in the present study it may be concluded that

- Positive inhomogeneity indexes always offer smaller radial displacements.
- If $E_a < E_b$ $(\beta > 0)$ and a divergent profile is used, then one may get much smaller radial displacements.
- $E_a > E_b$ that is $\beta < 0$ is better together with divergent profiles than $E_a < E_b$ $(\beta > 0)$ for getting much smaller hoop stresses. That is to say a material having higher elasticity modulus is better to locate at the inner surface.
- Divergent profiles offer much smaller hoop stresses for all inhomogeneity indexes.

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