International Journal of Engineering & Applied Sciences (IJEAS)
Vol.8, Issue 4 (2016) 75-86

Effect of Rotary Inertia on Vibrational Response of Embedded Graphene Sheets

Bekir Akgöz

Akdeniz University, Faculty of Engineering, Civil Engineering Department,
Division of Mechanics, Antalya-TURKIYE
E-mail address: bekirakgoz@akdeniz.edu.tr

Received date: December 2016
Accepted date: December 2016

Abstract:

In the present study, influence of rotary inertia on the size-dependent free vibration analysis of embedded single-layered graphene sheets is examined based on modified couple stress theory. Governing differential equations and corresponding boundary conditions in motion are derived by implementing Hamilton's principle on the basis of Kirchhoff thin plate theory. Also, effect of elastic foundation is taken into consideration by using a two-parameter elastic foundation model. Influences of additional material length scale parameter, mode number, elastic foundation and rotary inertia on the natural frequencies are investigated in detail.

Keywords: Scale effect, silicone matrix, sector graphene, discrete singular convolution, vibration.

1. Introduction

In the recent years, small-sized structural elements like rod, beam, plate and shells have a wide range of applications in micro- and nano-electro mechanical systems (MEMS and NEMS) with the rapid developments in nanotechnology. Carbon nanotube has a one-dimensional structure and is an allotrope of carbon, and one of the key structures in the nanotechnology applications. Carbon nanotubes were firstly discovered by Iijima, in 1991 as multi-walled carbon nanotubes (MWCNTs) [1]. Graphene sheets (GSs) have a two-dimensional structure and possess advanced mechanical, electrical, and thermal properties. Due to their unique properties, graphene based micro or nano-devices are generally used in MEMSs and NEMSs for high frequency and high sensitive purposes.

It has been observed from some experiments that there is a size effect on the deformation behavior of the micro-/nano-sized structures [2–4]. Because of the various difficulties in experimental studies, the researchers have tended to continuum mechanics approaches. However, the classical continuum models which are successful for modelling of macro-sized structural elements [5–22] fail to predict the mechanical behavior characteristics of small-sized structures. Consequently, several non-classical continuum theories have been developed such as couple stress theory [23–25], micropolar theory [26], nonlocal elasticity theory [27,28] and strain gradient theories [29–32] to determine the mechanical responses of such structures.

Modified strain gradient theory (MSGT) was introduced by Lam et al. [3] in which the strain energy density includes higher-order deformation gradient tensors besides classical symmetric strain tensor. For linear elastic isotropic materials, the formulations and governing equations contain three additional material length scale parameters relevant to higher-order deformation gradients in addition to two classical ones. This non-classical theory has been frequently employed to investigate the microstructure-dependent mechanical responses of beam [33–49] and plates [50–55].

It is noted that if dilatation gradients and deviatoric stretch gradients are omitted, the formulation and governing equations of modified strain gradient theory will be turned into those of modified couple stress theory (MCST) elaborated by Yang et al. [56]. The difference between classical couple stress theory and this modified version is defined as the couple stress tensor is symmetric and only one material length scale parameter is included in MCST. This simpler theory has been utilized to analyze the bending, buckling and free vibration behaviors of microbeams [57–61] and microplates [62–72]. On the other hand, there are many studies in the scientific literature on the mechanical behavior characteristics of carbon nanotubes, microtubules, nanobeams and nanoplates based on nonlocal elasticity theory [73–86].

The purpose of this study is to examine the influence of rotary inertia on the micro-dependent free vibration analysis of embedded single-layered graphene sheets (SLGSs). Governing differential equations and corresponding boundary conditions in motion are derived by implementing Hamilton's principle on the basis of Kirchhoff thin plate theory in conjunction with modified couple stress theory. Also, effect of elastic foundation is taken into consideration via Pasternak elastic foundation model. Influences of additional material length scale parameter, mode number, elastic foundation and rotary inertia on the natural frequencies are investigated in detail.

2. Modified couple stress theory

The modified couple stress theory was elaborated by Yang et al. [56]. Unlike the classical continuum theory, this popular theory contains an additional material length scale parameter associated with the rotation gradient tensor for prediction the microstructure-dependent deformation behaviors. In the framework of this elasticity theory, the strain energy U in a linear elastic isotropic material occupying a volume V is

$$U = \frac{1}{2}\int_V \left(\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij}\right)dV \tag{1}$$

where σ_{ij}, ε_{ij}, m_{ij}, χ_{ij} are the components of classical stress and strain tensors, deviatoric part of the couple stress tensor, and symmetric curvature tensor, respectively [56]

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2G\varepsilon_{ij} \tag{2}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{3}$$

$$m_{ij} = 2Gl^2\chi_{ij} \tag{4}$$

$$\chi_{ij} = \frac{1}{2}\left(\theta_{i,j} + \theta_{j,i}\right) \tag{5}$$

$$\theta_i = \frac{1}{2} e_{ijk} u_{k,j} \tag{6}$$

where λ and G are the well-known Lamé constants, δ_{ij} is the Kronecker delta, e_{ijk} is the alternating symbol, l is the additional material length scale parameter, u_i and θ_i are the components of displacement vector **u** and rotation vector **θ**, respectively.

3. Dynamical model for a rectangular microplate

The displacement fields for time-dependent deformations based on Kirchhoff's plate theory can be expressed by considering a rectangular graphene sheet (see Fig. 1) as

$$u(x,y,z,t) = -z\frac{\partial w}{\partial x},\; v(x,y,z,t) = -z\frac{\partial w}{\partial y},\; w(x,y,z,t) = w(x,y,t) \tag{7}$$

where u, v, and w denote the displacement components in x, y, and z directions, respectively. Substitution of Eq. (7) in Eqs. (3), (5) and (6) yields the non-zero strain and rotation components of the SLGS as

$$\varepsilon_{xx} = -z\frac{\partial^2 w}{\partial x^2},\; \varepsilon_{yy} = -z\frac{\partial^2 w}{\partial y^2},\; \varepsilon_{xy} = \varepsilon_{yx} = -z\frac{\partial^2 w}{\partial x \partial y} \tag{8}$$

$$\theta_x = \frac{\partial w}{\partial y}, \theta_y = -\frac{\partial w}{\partial x} \tag{9}$$

$$\chi_{xx} = \frac{\partial^2 w}{\partial x \partial y},\; \chi_{yy} = -\frac{\partial^2 w}{\partial x \partial y},\; \chi_{xy} = \chi_{yx} = -\frac{1}{2}\left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial y^2}\right) \tag{10}$$

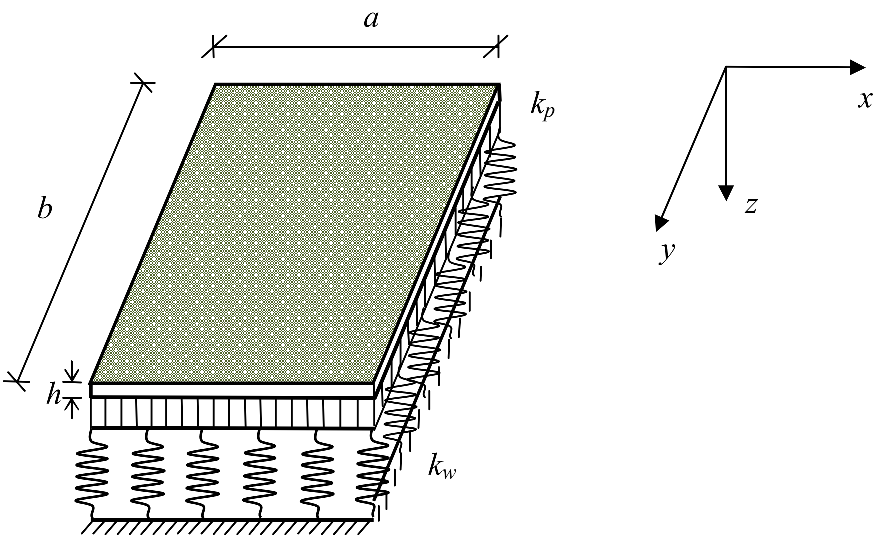

Fig. 1. Coordinate system and continuum model of an embedded single-layered graphene sheet

The governing differential equations and associated boundary conditions for embedded SLGS can be obtained by implementing Hamilton's principle [5]

$$0=\int_0^T\left(\delta T-\left(\delta U-\delta W\right)\right)dt \tag{11}$$

where δT, δU and δW are the first variations of kinetic energy, strain energy, and work done by external applied forces, respectively. In view of Eq. (11), the following expression can be written on the time interval $\left[0,T\right]$ as [62]

$$\begin{aligned}0=\int_0^T\Bigg\{\int_\Omega\Bigg[&-\left(\frac{\partial^2 M_{xx}}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_{yy}}{\partial y^2}-\frac{\partial^2 Y_{xx}}{\partial x\partial y}+\frac{\partial^2 Y_{xy}}{\partial x^2}-\frac{\partial^2 Y_{xy}}{\partial y^2}+\frac{\partial^2 Y_{yy}}{\partial x\partial y}\right.\\&\left.-k_w w+k_p\nabla^2 w-I_0\frac{\partial^2 w}{\partial t^2}+I_2\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)\right)\delta w\Bigg]dxdy\\&+\oint_\Gamma\Bigg[\Bigg(\left(\frac{\partial M_{xx}}{\partial x}+\frac{\partial M_{xy}}{\partial y}-\frac{1}{2}\frac{\partial Y_{xx}}{\partial y}+\frac{\partial Y_{xy}}{\partial x}+\frac{1}{2}\frac{\partial Y_{yy}}{\partial y}+I_2\frac{\partial\ddot{w}}{\partial x}\right)n_x\\&+\left(\frac{\partial M_{yy}}{\partial y}+\frac{\partial M_{xy}}{\partial x}-\frac{1}{2}\frac{\partial Y_{xx}}{\partial x}-\frac{\partial Y_{xy}}{\partial y}+\frac{1}{2}\frac{\partial Y_{yy}}{\partial x}+I_2\frac{\partial\ddot{w}}{\partial y}\right)n_y\Bigg)\delta w\\&-\left(\left(M_{xx}+Y_{xy}\right)n_x+\left(M_{xy}-\frac{1}{2}Y_{xx}+\frac{1}{2}Y_{yy}\right)n_y\right)\frac{\partial\delta w}{\partial x}\\&-\left(\left(M_{xy}-\frac{1}{2}Y_{xx}+\frac{1}{2}Y_{yy}\right)n_x+\left(M_{yy}-Y_{xy}\right)n_y\right)\frac{\partial\delta w}{\partial y}\Bigg]ds\Bigg\}dt\end{aligned} \tag{12}$$

where Ω is the area of the plate middle surface, k_w and k_p are the Winkler and shear modulus of the elastic foundation, M_{xx},M_{xy},M_{yy} and Y_{xx},Y_{xy},Y_{yy} are the classical and non-classical moments, and also I_0 and I_2 the mass moments of inertia terms defined as [65]

$$\begin{Bmatrix}M_{xx}\\M_{yy}\\M_{xy}\end{Bmatrix}=-\frac{Eh^3}{12(1-v^2)}\begin{bmatrix}1&v&0\\v&1&0\\0&0&1-v\end{bmatrix}\begin{Bmatrix}\dfrac{\partial^2 w}{\partial x^2}\\\dfrac{\partial^2 w}{\partial y^2}\\\dfrac{\partial^2 w}{\partial x\partial y}\end{Bmatrix},\quad\begin{Bmatrix}Y_{xx}\\Y_{yy}\\Y_{xy}\end{Bmatrix}=Gl^2h\begin{bmatrix}0&0&2\\0&0&-2\\-1&1&0\end{bmatrix}\begin{Bmatrix}\dfrac{\partial^2 w}{\partial x^2}\\\dfrac{\partial^2 w}{\partial y^2}\\\dfrac{\partial^2 w}{\partial x\partial y}\end{Bmatrix} \tag{13}$$

$$I_0 = \rho h \,,\ I_2 = \frac{\rho h^3}{12} \tag{14}$$

where h is the thickness of graphene sheet. After some mathematical manipulations, the equations of motion for the SLGSs in an elastic matrix can be expressed from Eq. (12) as:

$$\frac{\partial^2 M_{xx}}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} - \frac{\partial^2 Y_{xx}}{\partial x \partial y} + \frac{\partial^2 Y_{xy}}{\partial x^2} - \frac{\partial^2 Y_{xy}}{\partial y^2} + \frac{\partial^2 Y_{yy}}{\partial x \partial y}$$

$$- k_w w + k_p \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = I_0 \frac{\partial^2 w}{\partial t^2} - I_2 \frac{\partial^2}{\partial t^2} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \tag{15}$$

and the associated boundary conditions are defined as

$$\left(\frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - \frac{1}{2}\frac{\partial Y_{xx}}{\partial y} + \frac{\partial Y_{xy}}{\partial x} + \frac{1}{2}\frac{\partial Y_{yy}}{\partial y} \right) n_x$$

$$+ \left(\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y} - \frac{1}{2}\frac{\partial Y_{xx}}{\partial x} - \frac{\partial Y_{xy}}{\partial y} + \frac{1}{2}\frac{\partial Y_{yy}}{\partial x} \right) n_y \ \text{or}\ w = 0$$

$$-\left(M_{xx} + Y_{xy} \right) n_x + \left(- M_{xy} + \frac{1}{2} Y_{xx} - \frac{1}{2} Y_{yy} \right) n_y = 0 \ \text{or}\ \frac{\partial w}{\partial x} = 0$$

$$\left(- M_{xy} + \frac{1}{2} Y_{xx} - \frac{1}{2} Y_{yy} \right) n_x + \left(- M_{yy} + Y_{xy} \right) n_y = 0 \ \text{or}\ \frac{\partial w}{\partial y} = 0 \tag{16}$$

where (n_x, n_y) denote the direction cosines of the unit normal to the boundary of the middle plane. The governing equation of motion for embedded SLGS surrounded by an elastic medium can be rewritten in terms of displacements by taking into consideration the effect of rotary inertia as follows:

$$D\left(1 + \frac{6 l^2 (1-\nu)}{h^2} \right) \nabla^2 \nabla^2 w + k_w w - k_p \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)$$

$$= I_2 \frac{\partial^2}{\partial t^2} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) - I_0 \frac{\partial^2 w}{\partial t^2} \tag{17}$$

where D is the classical bending rigidity

$$D = \frac{E h^3}{12(1-\nu^2)} \tag{18}$$

4. Analytical solution of free vibration problem

In this section, the governing partial differential equation is analytically solved for free vibration of all edges simply supported embedded single-layered graphene sheets. In order to solve this equation, the following solution procedure is employed by using the separation of variables technique as

$$w(x,y,t) = W(x,y)\left(A\sin\omega\, t + B\cos\omega\, t\right) \tag{19}$$

where A and B are the integral constants. These constants are easily found by using the initial conditions. Also, $W(x,y)$ is shape function and ω is the natural frequency of the SLGS. Substitution of Eq. (19) into Eq. (17) yields

$$D\left(1+\frac{6l^2(1-\nu)}{h^2}\right)\nabla^4 W + k_w W - k_p\nabla^2 W - \omega^2 I_0 W + \omega^2 I_2\nabla^2 W = 0 \tag{20}$$

where ∇^4 is the biharmonic operator. On the other hand, the shape function can be expressed as the following Fourier series solutions

$$W(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}\sin\frac{m\pi\, x}{a}\sin\frac{n\pi\, y}{b} \tag{21}$$

where C_{mn} is the undetermined Fourier coefficient. This means that Eq. (21) must satisfy the associated boundary conditions. By using Eq. (21) into Eq. (20), one expression can be obtained as

$$D\left(1+\frac{6l^2(1-\nu)}{h^2}\right)\pi^4\lambda^2 + k_w + k_p\pi^2\lambda = \omega_{mn}^2\left(I_0 + I_2\pi^2\lambda\right) \tag{22}$$

where

$$\lambda = \frac{m^2}{a^2} + \frac{n^2}{b^2} \tag{23}$$

Finally, the natural frequency of SLGS with rotary inertia term (I_2) is expressed on the basis of modified couple stress theory as following

$$\omega_{mn} = \sqrt{\frac{D\left(1+\frac{6l^2(1-\nu)}{h^2}\right)\pi^4\lambda^2 + k_w + k_p\pi^2\lambda}{I_0 + I_2\pi^2\lambda}} \tag{24}$$

5. Numerical results and discussion

In this section, several illustrative examples are given to show the effects of size dependency, foundation parameters and rotary inertia on the natural frequencies of simply supported square single layered graphene sheets surrounded by an elastic medium. Following geometrical and material properties are used as: $E = 1.06\ TPa$, $\nu = 0.25$, $h = 0.34\ nm$, $\rho = 2250\ kg/m^3$ [62]. Also, the dimensionless foundation parameters are defined as $K_w = \frac{k_w a^4}{D}$ and $K_G = \frac{k_G a^2}{D}$.

Table 1. Effects of size dependency and elastic foundation parameters on the fundamental frequency values (*THz*) of square graphene sheet with rotary inertia effect (*a=b=15h, l=h, $I_2 \neq 0$*)

K_w	$K_G=0$		$K_G=15$		$K_G=30$	
	CT	MCST	CT	MCST	CT	MCST
0	0.2648	0.6210	0.3465	0.6600	0.4123	0.6968
20	0.2711	0.6237	0.3513	0.6625	0.4164	0.6992
60	0.2833	0.6291	0.3608	0.6676	0.4244	0.7044
100	0.2949	0.6344	0.3700	0.6726	0.4323	0.7088

Table 2. Effects of size dependency and elastic foundation parameters on the fundamental frequency values (*THz*) of square graphene sheet without rotary inertia effect (*a=b=15h, l=h, $I_2=0$*)

K_w	$K_G=0$		$K_G=15$		$K_G=30$	
	CT	MCST	CT	MCST	CT	MCST
0	0.2658	0.6232	0.3478	0.6624	0.4138	0.6993
20	0.2721	0.6260	0.3526	0.6649	0.4179	0.7017
60	0.2843	0.6314	0.3621	0.6700	0.4260	0.7066
100	0.2960	0.6367	0.3714	0.6751	0.4339	0.7114

Influences of size dependency and elastic foundation parameters on the fundamental frequency values of square graphene sheet with and without rotary inertia effect are tabulated in Tables 1 and 2, respectively. It is clearly seen that an increase in Winkler and Pasternak foundation parameters leads to an increment in the fundamental frequencies. Also, it is observed that the natural frequencies evaluated by MCST are larger than those predicted by CT. Moreover, it can be concluded from the tables that the rotary inertia term has a decreasing effect on the frequency values.

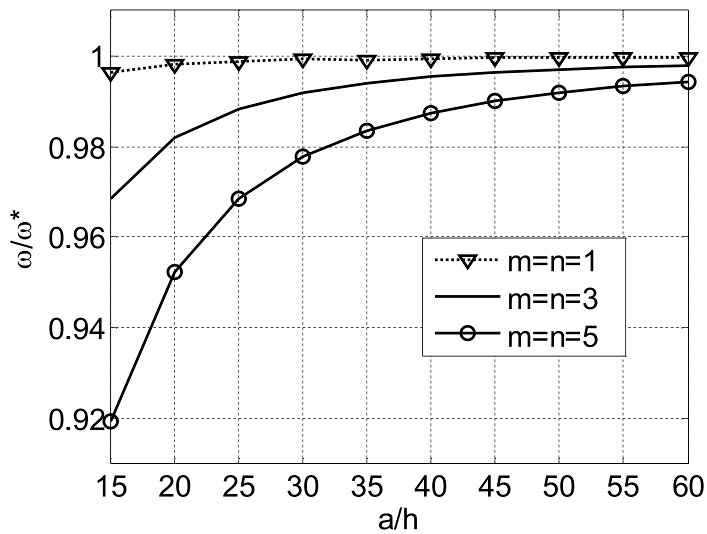

Fig. 2. Variation of the frequency ratio (with rotary inertia/without rotary inertia) with respect to a/h and mod number ($K_w=K_G=10$, *a=b, l=h*)

In Fig. 2, variation of the frequency ratio (with rotary inertia/without rotary inertia) is depicted for different *a/h* and mode number. It is observed that the ratio is almost equal to one for larger values of *a/h* while it tends to decrease by decreasing the length-to-thickness ratio. Also, it is found from the figure that the divergence between the frequencies is more considerable for greater mode numbers.

6. Conclusions

Effect of rotary inertia on the size-dependent free vibration analysis of embedded single-layered graphene sheets is investigated via modified couple stress theory. Governing differential equations and related boundary conditions in motion are derived with the aid of Hamilton's principle on the basis of Kirchhoff plate theory. Also, the interactions between the surrounding elastic medium and graphene sheet are simulated by Pasternak elastic foundation model. Influences of additional material length scale parameter, mode number, elastic foundation and rotary inertia on the natural frequencies are investigated in detail. It is concluded from the obtained results that the rotary inertia term has a decreasing effect on the natural frequency values of graphene sheets. In addition, it can be interpreted that this effect is more prominent for smaller *a/h* ratio and higher modes.

Acknowledgments

This study has been supported by The Scientific and Technological Research Council of Turkey (TÜBİTAK) with Project no: 112M879. This support is gratefully acknowledged.

References

[1] Iijima, S. Helical Microtubules of Graphitic Carbon. *Nature*, 354, 56–58, 1991.

[2] Poole, W.J., Ashby, M.F., Fleck, N.A. Micro-hardness of annealed and work- hardened copper polycrystals. *Scripta Mater.*, 34, 559–564, 1996.

[3] Lam, D.C.C., Yang, F., Chong, A.C.M., Wang, J., Tong, P. Experiments and theory in strain gradient elasticity. *J. Mech. Phys. Solids*, 51, 1477–1508, 2003.

[4] McFarland, A.W., Colton, J.S. Role of material microstructure in plate stiffness with relevance to microcantilever sensors. *J. Micromech. Microeng.*, 15, 1060–1067, 2005.

[5] Reddy, J.N., Energy Principles and Variational Methods in Applied Mechanics, John Wiley & Sons, 2nd Edition, 2002.

[6] Soedel,W., Vibrations of shells and plates, CRC Press, 3rd Edition, 2004.

[7] Viola, E., Tornabene, F., Fantuzzi, N. Static analysis of completely doubly-curved laminated shells and panels using general higher-order shear deformation theories. *Compos. Struct.,* 101, 59–93, 2013.

[8] Ferreira, A.J.M., Viola, E., Tornabene, F., Fantuzzi, N., Zenkour, A.M. Analysis of sandwich plates by generalized differential quadrature method. *Math. Probl. Eng.,* 964367, 2013.

[9] Civalek, Ö. Nonlinear dynamic response of laminated plates resting on nonlinear elastic foundations by the discrete singular convolution-differential quadrature coupled approaches. *Compos. Part B: Eng.,* 50, 171–179, 2013.

[10] Tornabene, F., Fantuzzi, N., Bacciocchi, M., Dimitri, R. Free vibrations of composite oval and elliptic cylinders by the generalized differential quadrature method. *Thin-Walled Struct.,* 97, 114–129, 2015.

[11] Pradhan, S.C., Loy, C.T., Lam, K.Y., Reddy, J.N. Vibration characteristics of functionally graded cylindrical shells under various boundary conditions. *Appl. Acoustics,* 61, 111–124, 2000.
[12] Baltacıoğlu, A.K., Civalek, Ö., Akgöz, B., Demir, F. Large deflection analysis of laminated composite plates resting on nonlinear elastic foundations by the method of discrete singular convolution. *Int. J. Press. Vessel. Pip.*, 88, 290–300, 2011.
[13] Civalek, Ö., Korkmaz, A., Demir, Ç. Discrete singular convolution approach for buckling analysis of rectangular Kirchhoff plates subjected to compressive loads on two-opposite edges. *Adv. Eng. Softw.,* 41, 557–560, 2010.
[14] Akgöz, B., Civalek, Ö. Nonlinear vibration analysis of laminated plates resting on nonlinear two-parameters elastic foundations. *Steel and Compos. Struct.,* 11, 403–421, 2011.
[15] Civalek, Ö. Analysis of thick rectangular plates with symmetric cross-ply laminates based on first-order shear deformation theory. *J. Compos. Mater.*, 42, 2853–2867, 2008.
[16] Gürses, M., Civalek, Ö., Korkmaz, A., Ersoy, H. Free vibration analysis of symmetric laminated skew plates by discrete singular convolution technique based on first-order shear deformation theory. *Int. J. Numer. Meth. Eng.,* 79, 290–313, 2009.
[17] Baltacıoğlu, A.K., Akgöz, B., Civalek, Ö. Nonlinear static response of laminated composite plates by discrete singular convolution method. *Compos. Struct.,* 93, 153–161, 2010.
[18] Avcar, M. Free vibration analysis of beams considering different geometric characteristics and boundary conditions. *Int. J. Mech. Appl.,* 4, 94–100, 2014.
[19] Mercan, K., Demir, Ç., Civalek, Ö. Vibration analysis of FG cylindrical shells with power-law index using discrete singular convolution technique. *Curved and Layer. Struct.,* 3, 82–90, 2016.
[20] Avcar, M. Elastic buckling of steel columns under axial compression. *American J. Civ. Eng.,* 2, 102–108, 2014.
[21] Avcar, M. Effects of rotary inertia shear deformation and non-homogeneity on frequencies of beam. *Struct. Eng. Mech.,* 55, 871–884, 2015.
[22] Emsen, E., Mercan, K., Akgöz, B., Civalek, Ö. Modal Analysis of Tapered Beam-Column Embedded in Winkler Elastic Foundation. *Int. J. Eng. Appl. Sci.,* 7, 25–35, 2015.
[23] Mindlin, R.D., Tiersten, H.F. Effects of couple-stresses in linear elasticity. *Arch. Ration. Mech. Anal.,* 11, 415–448, 1962.
[24] Koiter, W.T. Couple stresses in the theory of elasticity: I and II. *Proc. K Ned. Akad. Wet. (B)*, 67, 17–44, 1964.
[25] Toupin, R.A. Theory of elasticity with couple stresses. *Arch. Ration. Mech. Anal.,* 17, 85–112, 1964.
[26] Eringen, A.C. Theory of micropolar plates. *Z .Angew. Math. Phys.,* 18, 12–30, 1967.
[27] Eringen, A.C. Nonlocal polar elastic continua. *Int. J. Eng. Sci.,* 10, 1–16, 1972.
[28] Eringen, A.C. On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves. *J. Appl. Phys.,* 54, 4703–4710, 1983.
[29] Vardoulakis, I., Sulem, J. Bifurcation Analysis in Geomechanics. London: Blackie/Chapman & Hall, 1995.
[30] Aifantis, E.C. Gradient deformation models at nano, micro, and macro scales. *J. Eng. Mater. Technol.,* 121, 189–202, 1999.
[31] Fleck, N.A., Hutchinson, J.W. A phenomenological theory for strain gradient effects in plasticity. *J. Mech. Phys. Solids,* 41, 1825–1857, 1993.
[32] Fleck, N.A., Hutchinson, J.W. A reformulation of strain gradient plasticity. *J. Mech. Phys. Solids,* 49, 2245–2271, 2001.

[33] Kong, S., Zhou, S., Nie, Z., Wang, K. Static and dynamic analysis of micro beams based on strain gradient elasticity theory. *Int. J. Eng. Sci.,* 47, 487–498, 2009.
[34] Wang, B., Zhao, J., Zhou, S. A micro scale Timoshenko beam model based on strain gradient elasticity theory. *Eur. J. Mech. A/Solids*, 29, 591-599, 2010.
[35] Akgöz, B., Civalek, Ö. Strain gradient elasticity and modified couple stress models for buckling analysis of axially loaded micro-scaled beams. *Int. J. Eng. Sci.,* 49, 1268–1280, 2011.
[36] Akgöz, B., Civalek, Ö. Analysis of micro-sized beams for various boundary conditions based on the strain gradient elasticity theory. *Arch. Appl. Mech.,* 82, 423–443, 2012.
[37] Kahrobaiyan, M.H., Rahaeifard, M., Tajalli, S.A., Ahmadian, M.T. A strain gradient functionally graded Euler-Bernoulli beam formulation. *Int. J. Eng. Sci.,* 52, 65–76, 2012.
[38] Akgöz, B., Civalek, Ö. Buckling analysis of functionally graded microbeams based on the strain gradient theory. *Acta Mech.,* 224, 2185–2201, 2013.
[39] Ansari, R., Gholami, R., Sahmani, S. Free vibration analysis of size-dependent functionally graded microbeams based on the strain gradient Timoshenko beam theory. *Compos. Struct.,* 94, 221–228, 2011.
[40] Akgöz, B., Civalek, Ö. Bending analysis of FG microbeams resting on Winkler elastic foundation via strain gradient elasticity. *Compos. Struct.,* 134, 294–301, 2015.
[41] Ghayesh, M.H., Amabili, M., Farokhi, H. Nonlinear forced vibrations of a microbeam based on the strain gradient elasticity theory. *Int. J. Eng. Sci.,* 63, 52–60, 2013.
[42] Akgöz, B., Civalek, Ö. Buckling analysis of linearly tapered micro-columns based on strain gradient elasticity. *Struct. Eng. Mech.,* 48, 195–205, 2013.
[43] Akgöz, B., Civalek, Ö. Investigation of size effects on static response of single-walled carbon nanotubes based on strain gradient elasticity. *Int. J. Comput. Methods,* 9, 1240032, 2012.
[44] Kahrobaiyan, M.H., Asghari, M., Ahmadian, M.T. Strain gradient beam element. *Finite Elem. Anal. Des.,* 68, 63–75, 2013.
[45] Akgöz, B., Civalek, Ö. Bending analysis of embedded carbon nanotubes resting on an elastic foundation using strain gradient theory. *Acta Astronaut.,* 119, 1–12, 2016.
[46] Akgöz, B., Civalek, Ö. A novel microstructure-dependent shear deformable beam model. *Int. J. Mech. Sci.,* 99, 10–20, 2015.
[47] Zhang, B., He, Y., Liu, D., Gan, Z., Shen, L. Non-classical Timoshenko beam element based on the strain gradient elasticity theory. *Finite Elem. Anal. Des.,* 79, 22–39, 2014.
[48] Akgöz, B., Civalek, Ö. A new trigonometric beam model for buckling of strain gradient microbeams. *Int. J. Mech. Sci.,* 81, 88–94, 2014.
[49] Lei, J., He, Y., Zhang, B., Gan, Z., Zeng, P. Bending and vibration of functionally graded sinusoidal microbeams based on the strain gradient elasticity theory. *Int. J. Eng. Sci.,* 72, 36–52, 2013.
[50] Akgöz, B., Civalek, Ö. A microstructure-dependent sinusoidal plate model based on the strain gradient elasticity theory. *Acta Mech.,* 226, 2277–2294, 2015.
[51] Wang, B., Zhou, S., Zhao, J., Chen, X. A size-dependent Kirchhoff micro-plate model based on strain gradient elasticity theory. *Eur. J. Mech. A Solids,* 30, 517–524, 2011.
[52] Movassagh, A.A., Mahmoodi, M.J. A micro-scale modeling of Kirchhoff plate based on modified strain-gradient elasticity theory. *Eur. J. Mech. A Solids,* 40, 50–59, 2013.
[53] Li, A., Zhou, S., Zhou, S., Wang, B. A size-dependent model for bi-layered Kirchhoff micro-plate based on strain gradient elasticity theory. *Compos. Struct.,* 113, 272–280, 2014.
[54] Sahmani, S., Ansari, R. On the free vibration response of functionally graded higher-order shear deformable microplates based on the strain gradient elasticity theory. *Compos. Struct.,* 95, 430–442, 2013.

[55] Ansari, R., Gholami, R., Faghih Shojaei, M., Mohammadi, V., Sahmani, S. Bending, buckling and free vibration analysis of size-dependent functionally graded circular/annular microplates based on the modified strain gradient elasticity theory. *Eur. J. Mech. A Solids,* 49, 251–267, 2015.
[56] Yang, F., Chong, A.C.M., Lam, D.C.C., Tong, P. Couple stress based strain gradient theory for elasticity. *Int. J. Solids Struct.,* 39, 2731–2743, 2002.
[57] Park, S.K., Gao, X.L. Bernoulli–Euler beam model based on a modified couple stress theory. *J. Micromech. Microeng.,* 16, 2355–2359, 2006.
[58] Kong, S., Zhou, S., Nie, Z., Wang, K. The size-dependent natural frequency of Bernoulli–Euler micro-beams. *Int. J. Eng. Sci.,* 46, 427–437, 2008.
[59] Ma, H.M., Gao, X.L., Reddy, J.N. A microstructure-dependent Timoshenko beam model based on a modified couple stress theory. *J. Mech. Phys. Solids,* 56, 3379–3391, 2008.
[60] Reddy, J.N. Microstructure-dependent couples stress theories of functionally graded beams. *J. Mech. Phys. Solids,* 59, 2382–2399, 2011.
[61] Akgöz, B., Civalek, Ö. Free vibration analysis of axially functionally graded tapered Bernoulli–Euler microbeams based on the modified couple stress theory. *Compos. Struct.,* 98, 314–322, 2013.
[62] Akgöz, B., Civalek, Ö. Free vibration analysis for single-layered graphene sheets in an elastic matrix via modified couple stress theory. *Mater. Des.,* 42, 164–171, 2012.
[63] Akgöz, B., Civalek, Ö. Modeling and analysis of micro-sized plates resting on elastic medium using the modified couple stress theory. *Meccanica,* 48, 863–873, 2013.
[64] Gao, X.-L., Huang, J.X., Reddy, J.N. A non-classical third-order shear deformation plate model based on a modified couple stress theory. *Acta Mech.,* 224, 2699–2718, 2013.
[65] Jomehzadeh, E., Noori, H.R., Saidi, A.R. The size-dependent vibration analysis of micro-plates based on a modified couple stress theory. *Phys. E,* 43, 877–883, 2011.
[66] Ke, L.L., Wang, Y.S., Yang, J., Kitipornchai, S. Free vibration of size-dependent Mindlin microplates based on the modified couple stress theory. *J. Sound Vib.,* 331, 94–106, 2012.
[67] Ma, H.M., Gao, X.L., Reddy, J.N. A non-classical Mindlin plate model based on a modified couple stress theory. *Acta Mech.,* 220, 217–235, 2011.
[68] Thai, H.T., Choi, D.H. Size-dependent functionally graded Kirchhoff and Mindlin plate models based on a modified couple stress theory. *Compos. Struct.,* 95, 142–153, 2013.
[69] Thai, H.T., Kim, S.E. A size-dependent functionally graded Reddy plate model based on a modified couple stress theory. *Compos. Part B: Eng.,* 45, 1636–1645, 2013.
[70] Thai, H.T., Vo, T.P. A size-dependent functionally graded sinusoidal plate model based on a modified couple stress theory. *Compos. Struct.,* 96, 376–383, 2013.
[71] Tsiatas, G.C. A new Kirchhoff plate model based on a modified couple stress theory. *Int. J. Solids Struct.,* 46, 2757–2764, 2009.
[72] Yin, L., Qian, Q., Wang, L., Xia, W. Vibration analysis of microscale plates based on modified couple stress theory. *Acta Mech. Solida Sin.,* 23, 386–393, 2010.
[73] Peddieson, J., Buchanan, G.R., McNitt, R.P. Application of nonlocal continuum models to nanotechnology. *Int. J. Eng. Sci.,* 41, 305–312, 2003.
[74] Civalek, Ö., Demir, Ç. A simple mathematical model of microtubules surrounded by an elastic matrix by nonlocal finite element method. *Appl. Math. Comput.,* 289, 335–352, 2016.
[75] Wang, Q., Liew, K.M. Application of nonlocal continuum mechanics to static analysis of micro and nano-structures. *Phys. Lett. A,* 363, 236–242, 2007.
[76] Civalek, Ö., Demir, Ç., Akgöz, B. Static analysis of single walled carbon nanotubes (SWCNT) based on Eringen's nonlocal elasticity theory. *Int. J. Eng. Appl. Sci.,* 1, 47–56, 2009.

[77] Demir, Ç., Civalek, Ö. Nonlocal Finite Element Formulation for Vibration. *Int. J. Eng. Appl. Sci.,* 8, 109–117, 2016.

[78] Reddy, J.N. Nonlocal theories for bending, buckling and vibration of beams. *Int. J. Eng. Sci.,* 45, 288–307, 2007.

[79] Thai, H.T. A nonlocal beam theory for bending, buckling, and vibration of nanobeams. *Int. J. Eng. Sci.,* 52, 56–64, 2012.

[80] Civalek, Ö., Akgöz, B. Vibration analysis of micro-scaled sector shaped graphene surrounded by an elastic matrix. *Comput. Mater. Sci.,* 77, 295–303, 2013.

[81] Gürses, M., Civalek, Ö., Akgöz, B. Mathematical modeling of vibration problem of nano-sized annular sector plates using the nonlocal continuum theory via eight-node discrete singular convolution transformation. *Appl. Math. Comput.,* 219, 3226–3240, 2012.

[82] Aydogdu, M. A general nonlocal beam theory: its application to nanobeam bending, buckling and vibration. *Phys. E,* 41, 1651–1655, 2009.

[83] Demir, Ç., Civalek, Ö. Nonlocal Deflection of Microtubules Under Point Load, *Int. J. Eng. Appl. Sci.,* 7, 33–39, 2015.

[84] Mercan, K., Civalek, Ö. Buckling Analysis of Silicon Carbide Nanotubes (SiCNTs). *Int. J. Eng. Appl. Sci.,* 8, 101–108, 2016.

[85] Civalek, Ö., Mercan, K., Demir, Ç., Akgöz, B. Coordinate Transformation for Sector and Annular Sector Shaped Graphene Sheets on Silicone Matrix. *Int. J. Eng. Appl. Sci.,* 7, 56–73, 2015.

[86] Arda, M., Aydogdu, M. Bending of CNTs under the Partial Uniform Load. *Int. J. Eng. Appl. Sci.,* 8, 21–29, 2016.