



## The Control of Non-Linear Chaotic System Including Noise Using Genetic Based Algorithm

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### Abstract

*In this study, the Lorenz chaotic system including the nonlinear equations has been analyzed against the noise changes. The discrete-time state feedback control method and discrete-time PI controller method have been used for these noisy system controls. Optimization of the controller parameters was performed using a genetic algorithm method. The study was implemented in MATLAB-Simulink simulation program. In terms of noise, the Gaussian white noise was used as a system and measurement noise. Noise has emerged as a significant impact on the Lorenz chaotic system. Changes associated with this condition are given in detail on the graphics. The discrete-time PI controller method was shown to better performance when compared to discrete-time state feedback control method for the system control. Furthermore, the error performance change was given in the literature as an important parameter.*

**Keywords:** Lorenz chaotic system, Gaussian white noise, Discrete-time PID controller, Discrete-time state feedback control.

### 1. Introduction

Chaotic signal behaviors may make significant differences depending on the initial conditions [1]. Thus, it is highly difficult to predict these chaotic system behaviors. In order to better understand these relevant behaviors, many novel chaotic systems have been developed in recent years and most of them have been realized via electronic circuits [2,3].

Chaotic signals may be seen both as positive behaviors and negative behaviors that need to be destroyed. This depends on the condition of chaotic behavior use. Its use for the purpose of enciphering information in a communication system may be seen as a positive behavior whereas the removal of the noise of internal combustion car engine is considered as a chaotic behavior that needs to be destroyed.

In order to be able to better analyze the chaotic systems behaviors, first of all these systems should be modeled closer to the real ones. Chaotic systems are modeled with mathematical expressions. Then, by using these models chaotic systems are easily computerized. The noise factor should definitely be integrated into this system in order for real life problems to be analyzed more accurately and to be able to consider the disruptive effects during the working of the elements in electronic circuits. The integrated noise may have both positive and negative effects on chaotic systems. The noise positively increases the synchronization in the loosely-connected systems [4]. Negatively, the noise may significantly destroy the structure of chaotic systems and decrease the data synchronization.

There has been substantial amount of studies in the literature in order to analyze the effects of noise on chaotic systems. Some of them are as follows: Geng et al. have analyzed the dynamic behaviors of nonlinear chaotic systems on which statistical multiple inhibitor has been applied [5]. Majhi et al. have investigated the effects of noise-induced functional delay on chaotic synchronization [6]. Longtin has studied the effects of noise on harmonic resonance [7]. Behara et al. have used artificial neural network for active noise control [8].

In chaotic systems, controllers are used to provide desired synchronization behaviors. In this study, discrete time state feedback control method and discrete time PI controller method have been used to carry the chaotic systems into the desired equilibrium points and to analyze system resistance against noise.

Optimal coefficient calculations have been made to maximize the performances of both control systems abovementioned. Optimal values of these control methods coefficients have been found by using genetic algorithm. Although a number of novel optimization algorithms have been developed nowadays, genetic algorithm has been still widely used.

In this study, behaviors of Lorenz chaotic system that constitute the milestone of chaotic systems against white Gauss system noise and white Gauss measurement noise have been analyzed. In order for the results to be compared, two different controller methods have been used. These are discrete time state feedback control method and discrete time PI control method. According to the results obtained, discrete time PI control method has been revealed to outperform relative to discrete time state feedback control method.

## 2. Important mathematical definitions in closed-loop system

The mathematical model of the Lorenz chaotic system has given in Eq. (1) where  $x$ ,  $y$  and  $z$  are state variables;  $a$ ,  $b$  and  $c$  are positive constant parameters.

$$\left. \begin{aligned} \frac{\partial x}{\partial t} &= a(y - x) \\ \frac{\partial y}{\partial t} &= cx - xz - y \\ \frac{\partial z}{\partial t} &= xy - bz \end{aligned} \right\} \quad (1)$$

The system has been added to the controller state variable ( $U_y(s)$ ) for controlling of the Lorenz chaotic system given in Eq. 1. If the system and measurement noise is added to the Lorenz chaotic system, and then the Laplace transform is applied to the system, as a result, Eq. 2 is obtained.

$$\left. \begin{aligned} X(s) &= \frac{1}{s} \left[ aY(s) - aX(s) + U_d(s) \right] + U_{\delta}(s) \\ Y(s) &= \frac{1}{s} \left[ X(s)(c - Z(s)) - Y(s) + U_y(s) + U_d(s) \right] + U_{\delta}(s) \\ Z(s) &= \frac{1}{s} \left[ X(s)Y(s) - bZ(s) + U_d(s) \right] + U_{\delta}(s) \end{aligned} \right\} \quad (2)$$

Before the integral operators, the controller signal has been integrated with the closed-loop control system. Non-linear Lorenz chaotic system structure block diagram with the inclusion of the system and measurement noise is shown in Figure 1.

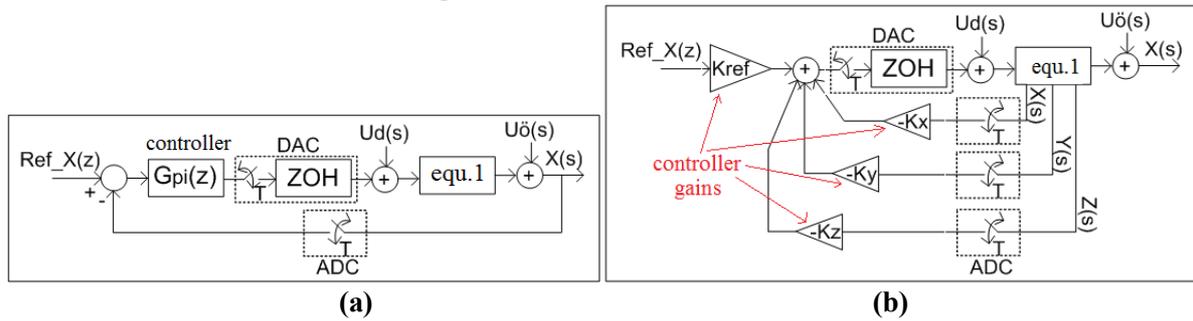


Fig. 1. Lorenz chaos control model with the noise, (a) PI, (b) state feedback control model

The model given in Eq. 2 has been transformed the Z-domain using the S-domain integral operator method. Using the given operator (forward difference method) with the Eq. 3, this conversion was performed. Here,  $T_s$  show the sampling period [9].

$$s \approx \frac{1}{T_s} \left( \frac{1-z^{-1}}{z^{-1}} \right) \quad (3)$$

The located the closed-loop control system structure in Figure 1 which are Gaussian white measurement noise  $U_d(s)$  and white Gaussian noise  $U_{\delta}(s)$  is shown in Figure 2 and Figure 3, respectively. While, the amplitude of the white Gaussian noise varies in the range of  $\mp 10$ , the amplitude of white Gaussian noise measurement change in the range of  $\mp 0.2$ . Both the noise was applied to the system from 15 seconds.

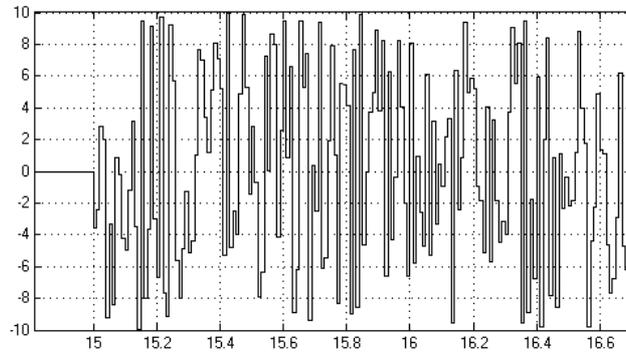


Fig. 2. White Gaussian the system noise ( $U_d(s)$ )

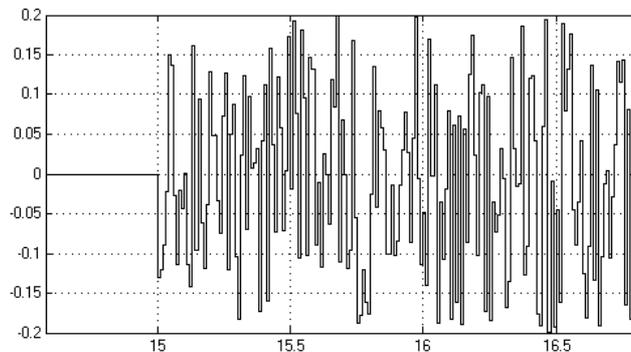


Fig. 3. White Gaussian the measurement noise ( $U_{\delta}(s)$ )

## 2.1 Genetic Algorithm

The studies related to genetic algorithm are described in detail in conducted a study earlier. In this study; genetic algorithm coefficients, flow diagram and the obtained results can be studied in detail [10].

## 3. Simulation Results

The simulation results are shown in Figures 4, 5, 6, 7, 8, 9, 10, and 11. Uncontrolled and noiseless version of the  $x$  state variable in the Figure 4, the version of the  $x$  state variable uncontrolled and system noise in the Figure 5, the version of the  $x$  state variable uncontrolled and the measurement noise in the Figure 6 are shown. It has emerged a significant impact on the chaotic signal of noise.

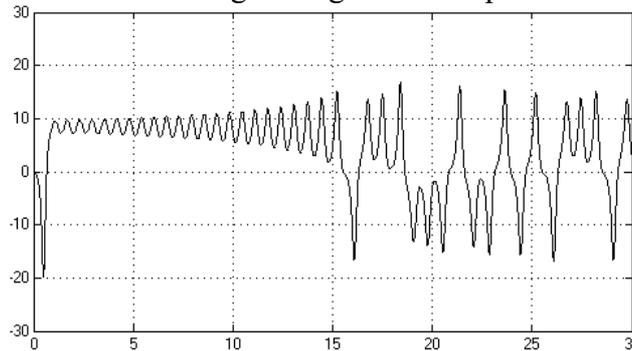


Fig. 4. Uncontrolled and noiseless version of the  $x$  state variable

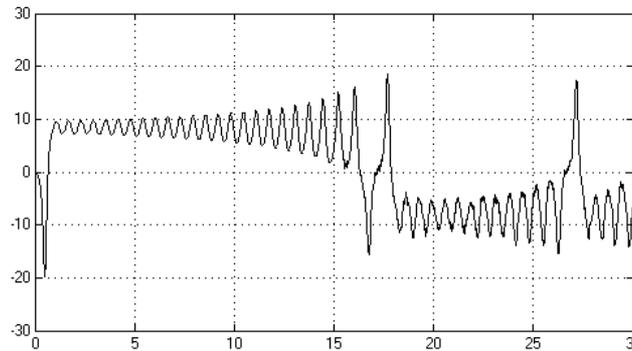


Fig. 5. The version of the  $x$  state variable uncontrolled and system noise

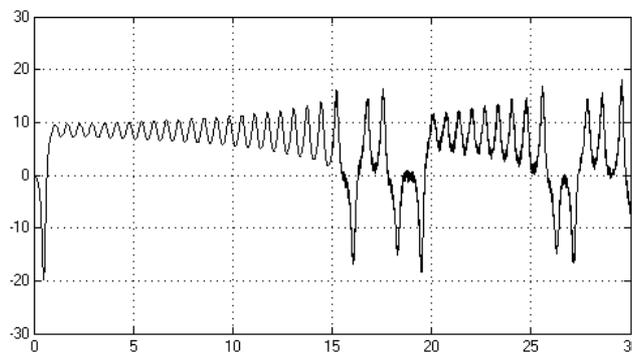


Fig. 6. The version of the  $x$  state variable uncontrolled and the measurement noise

The variations of the state variables obtained against noise by applying closed-loop control system the discrete-time state feedback control method are shown in the Figure 7. When the method is also applied to discrete-time PI controller, change of the state variables are given in Figure 8. While, the noise has been activated for the 15 seconds, the control has been activated for the 25 seconds.

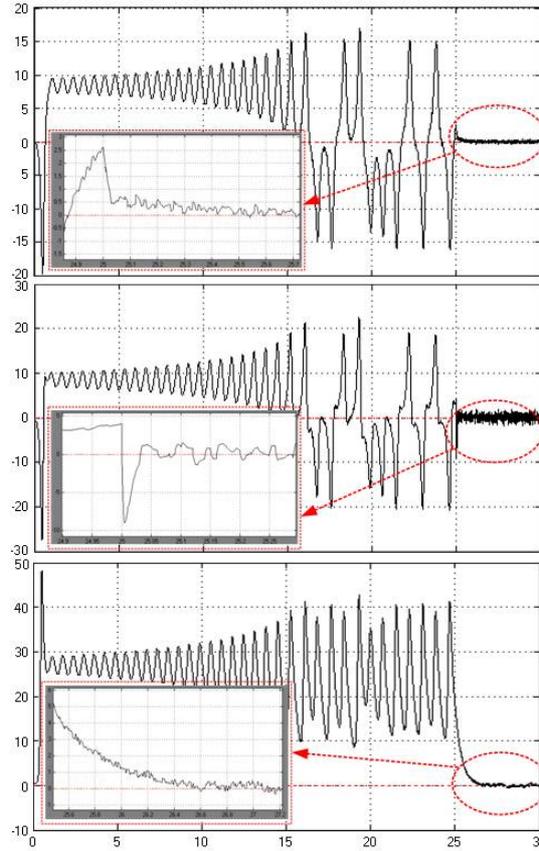


Fig. 7. Under discrete-time feedback situation control methods, the  $x, y$  and  $z$  state variables change, respectively

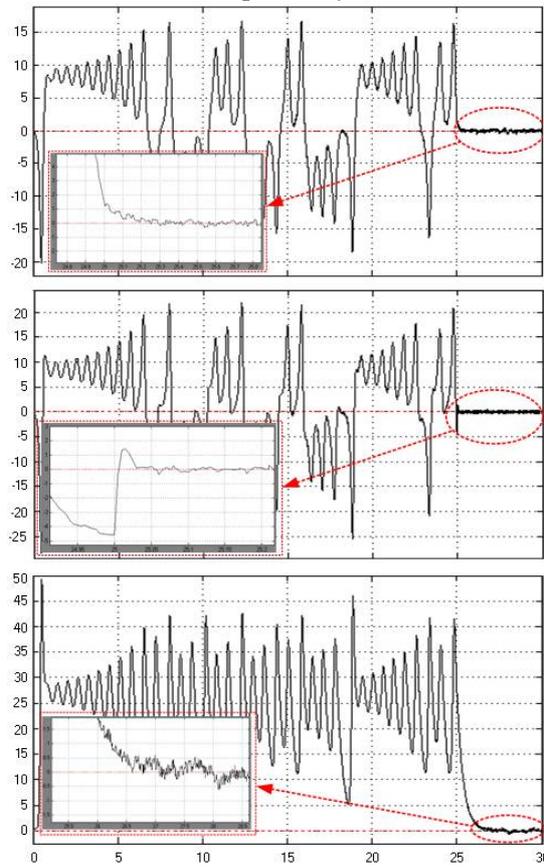


Fig. 8. Under PI control methods, the  $x, y$  and  $z$  state variables change, respectively

According to the results obtained in the Figure 7 and 8, controller performance criteria for both controllers are given in Table 1.

Table 1. Controller performance criteria

State Variable	Controller	Settling Time	Steady-State Error
$x$	State Feedback	200 (ms)	$0.5 \bar{\pi}$
	PI	150 (ms)	$0.1 \bar{\pi}$
$y$	State Feedback	25 (ms)	$1.0 \bar{\pi}$
	PI	25 (ms)	$0.25 \bar{\pi}$
$z$	State Feedback	1400 (ms)	$0.5 \bar{\pi}$
	PI	160 (ms)	$0.5 \bar{\pi}$

Additionally, the results obtained using both control method,  $xy, xz$  and  $yz$  the phase portraits are shown in Figures 9 and 10.

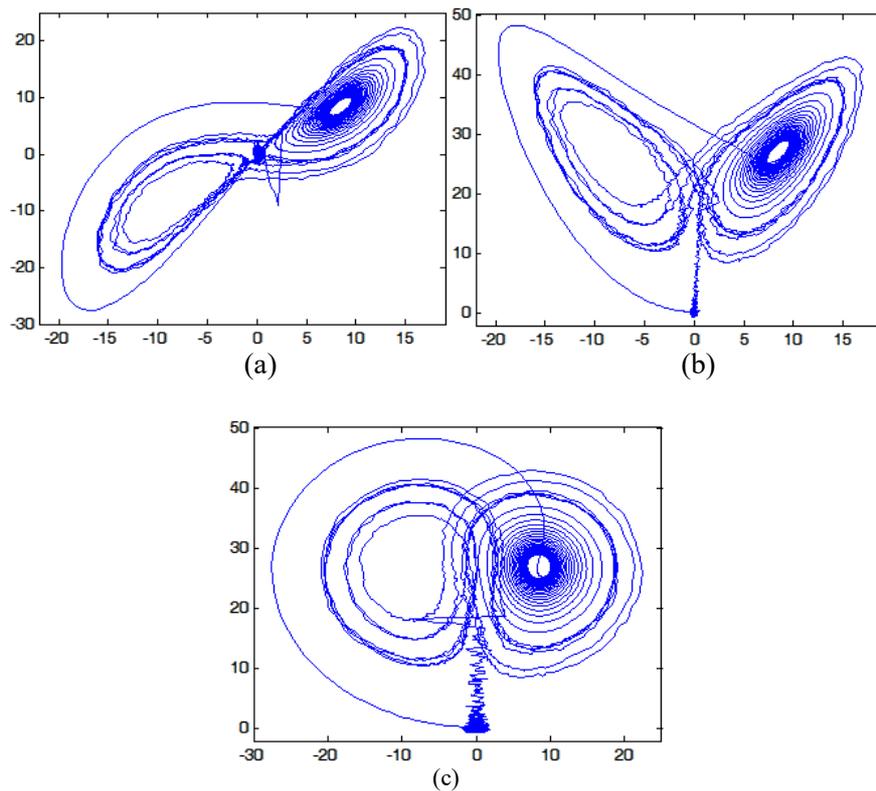


Fig. 9. Under discrete-time feedback situation control methods, (a)  $xy$ , (b)  $xz$  and (c)  $yz$  phase portraits

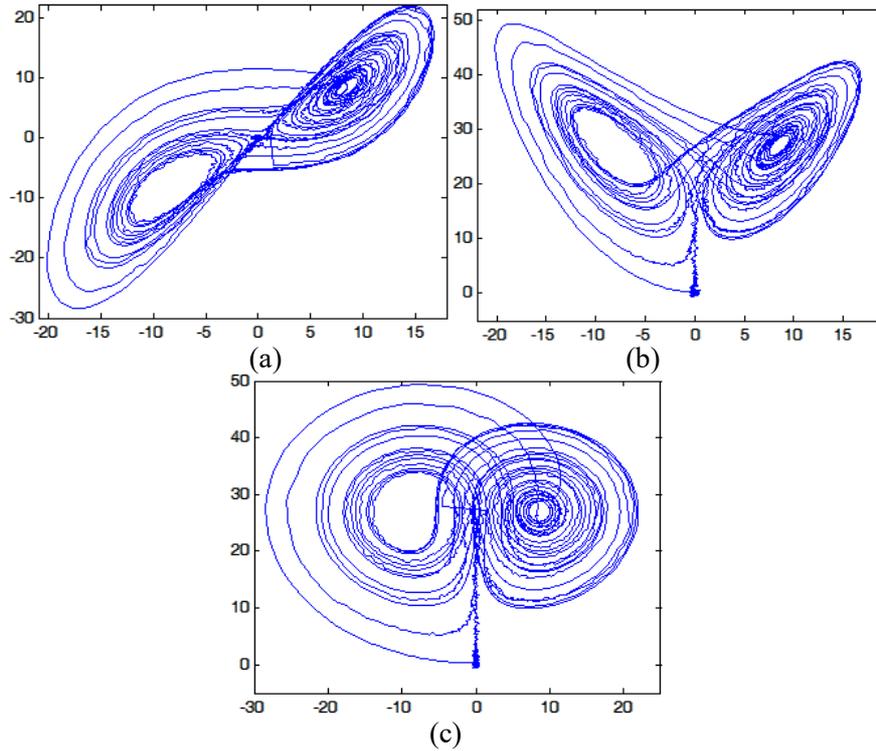


Fig. 10. Under discrete-time PI control methods, (a)  $xy$ , (b)  $xz$  and (c)  $yz$  phase portraits

The error performance indicators frequently used in the literature are an important indicator to determine the performance of the controller. Therefore, Matlab / Simulink structure of error performance criteria is shown in the Figure 11, 12 and 13. Also, the exchange of error is given in Table 2. The integral of absolute error (IAE), integral of squared error (ISE), integral of time multiplied by absolute error (ITAE) has been calculated according to the Equation (4), (5) and (6), respectively.

$$IAE = \int |e(t)| dt \tag{4}$$

$$ISE = \int e^2(t) dt \tag{5}$$

$$ITAE = \int t|e(t)| dt \tag{6}$$

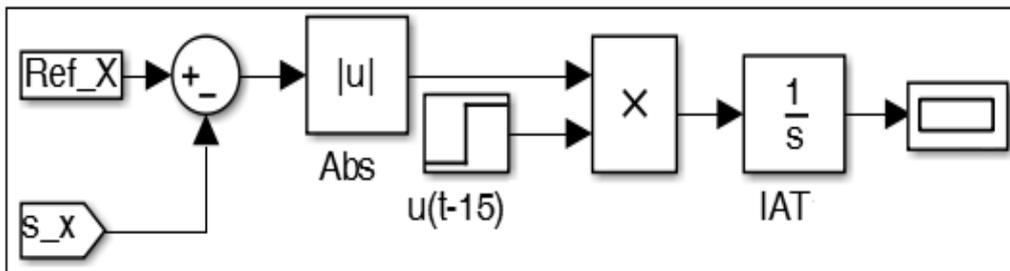


Fig. 11. Matlab/Simulink structure of the integral of absolute error (IAE)

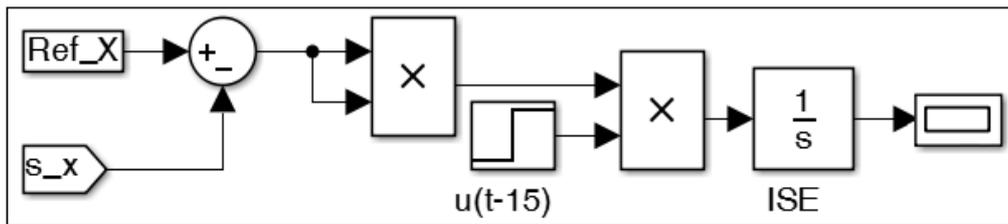


Fig. 12. Matlab/Simulink structure of the integral of squared error (ISE)

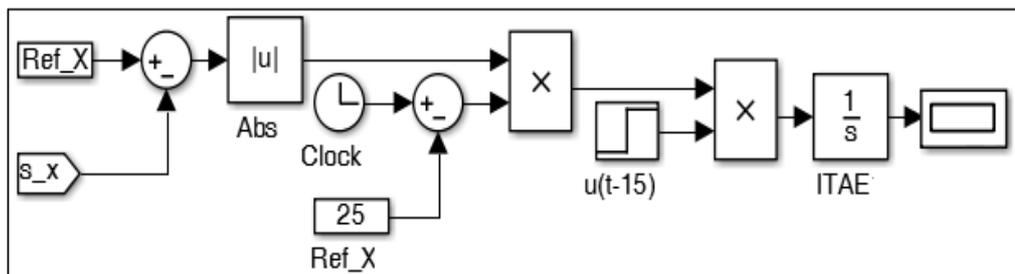


Fig. 13. Matlab/Sim. structure of the integral of time multiplied by absolute error (ITAE)

Table 2. The error performance criteria of controller

		IAE	ISE	ITEA
x	State Feedback	0.6464	0.211	1.217
	PI	0.7539	0.2327	1.637
y	State Feedback	2.866	2.994	6.784
	PI	0.382	0.1073	0.865
z	State Feedback	7.431	64.15	4.447
	PI	11.01	147	5.796

#### 4. Conclusions

In this study, the behavior of the Lorenz chaotic system against white Gaussian noise and white Gaussian measurement noise has been analyzed. Noise has emerged a significant impact on the chaotic signal. Two different controller methods had been used to compare the results of performance analysis. These methods were discrete-time state feedback control method and discrete-time PI controller method. The discrete-time PI control method has emerged a better performance than the feedback controller method as only using controller of the Y state variable. PI controller has given worse results than state feedback controller for the state variables X and Z.

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